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INSTABILITY, IMPRECISION AND INCONSISTENT USE OF EQUILIBRIUM REAL INTEREST RATE ESTIMATES

Robert Beyer and Volker Wieland

MONETARY ECONOMICS AND FLUCTUATIONS

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Abstract

The current debate on monetary and fiscal policy is heavily influenced by estimates of the equilibrium real interest rate. In particular, this concerns estimates derived from a simple aggregate demand and Phillips curve model with time-varying components as proposed by Laubach and Williams (2003). For example, Summers (2014a) refers to these estimates as important evidence for a secular stagnation and the need for fiscal stimulus. Yellen (2015, 2017) has made use of such estimates in order to explain and justify why the Federal Reserve has held interest rates so low for so long. First, we re-estimate the U.S. equilibrium rate with the methodology of Laubach and Williams (2003). Then, we build on their approach and the modifications proposed in Mésonnier and Renne (2007) and Garnier and Wilhelmsen (2009) to provide new estimates for the United States, the euro area and Germany. Third, we subject these estimates to a battery of sensitivity tests. Due to the great uncertainty and sensitivity that accompany these equilibrium rate estimates, the observed decline in the estimates is not a reliable indicator of a need for expansionary monetary and fiscal policy. Yet, if these estimates are employed to determine the appropriate monetary policy stance, such estimates are better used together with the consistent estimate of the level of potential output.

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Instability, imprecision and inconsistent use of equilibrium real interest rate estimates[1]

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The current debate on monetary and fiscal policy is heavily influenced by estimates of the equilibrium real interest rate. In particular, this concerns estimates derived from a simple aggregate demand and Phillips curve model with time-varying components as proposed by Laubach and Williams (2003). For example, Summers (2014a) refers to these estimates as important evidence for a secular stagnation and the need for fiscal stimulus. Yellen (2015, 2017) has made use of such estimates in order to explain and justify why the Federal Reserve has held interest rates so low for so long. First, we re-estimate the U.S. equilibrium rate with the methodology of Laubach and Williams (2003). Then, we build on their approach and the modifications proposed in Mésonnier and Renne (2007) and Garnier and Wilhelmsen (2009) to provide new estimates for the United States, the euro area and Germany. Third, we subject these estimates to a battery of sensitivity tests. Due to the great uncertainty and sensitivity that accompany these equilibrium rate estimates, the observed decline in the estimates is not a reliable indicator of a need for expansionary monetary and fiscal policy. Yet, if these estimates are employed to determine the appropriate monetary policy stance, such estimates are better used together with the consistent estimate of the level of potential output.

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1. Introduction

Interest rates in leading industrial economies have been declining for about three decades. The economic recovery in the aftermath of the global financial crisis and the euro area debt crisis has been slow, while consumer price inflation remained close to zero until recently. Partly for these reasons, central banks have set near zero or negative policy rates and engaged in massive asset purchases. As a consequence, not only short- and medium-term money market rates remained either close to zero or negative, but also the rates for longer-term government bonds. In the euro area, for example, the yield curve of government bonds with an AAA-rating in the Euro Area estimated by the European Central Bank (ECB) has been negative for up to 10 years in the course of 2016. Taking into account that inflation expectations are positive and surveys for longer-term inflation near two percent, the real return tends to be negative as well.

Increasingly, central bankers argue that the key reason for keeping nominal interest rates so low for so long is a decline in the equilibrium real interest rate, for which desired savings would be equal to planned investment. Monetary policy -- it is said -- needs to steer real interest rates below the equilibrium in order to stimulate aggregate demand and inflation. FOMC Chair Janet Yellen has referred repeatedly to equilibrium real interest rate estimates near 0% that were obtained with a methodology proposed by Laubach and Williams (2003) (see Yellen, 2015, 2017 and Williams, 2015). In particular, she has argued that current FOMC policy is not deviating much from the prescriptions of Taylor-style interest rate rules if such an estimate of the equilibrium rate is used in the rules. Yellen speaks of "headwinds" that keep the equilibrium rate low for some time, for example, the unwinding of the high debt levels of private households. She expects the equilibrium rate to move and rise to a higher level in the future. By contrast, Lawrence Summers has cited Laubach-Williams type estimates of the equilibrium real interest rate as evidence for a secular stagnation. In his view, monetary policy is unable to raise growth and a large fiscal expansion is called for (see Summers 2014a,b,c). In the euro area, ECB President Draghi has also referred to a global excess of desired savings over planned investment as the main driver for the low interest rate environment (Draghi, 2016). The decline in equilibrium rate estimates with the methodology of Laubach and Williams (2003) is one of the reasons given for the massive quantitative easing started at the beginning of 2015 (see Constâncio, 2016).

In this paper, we conduct a comprehensive analysis of equilibrium interest rate estimates obtained within a simple aggregate demand and Phillips curve framework for the United States, Germany and the euro area. The associated equilibrium is best characterized as a medium-term concept. We start with a brief overview of the development of nominal and real interest rates in these three economies. Next, we employ the methodology of Laubach and Williams (2003) and a modified version due to Garnier and Wilhelmsen (2009) to obtain new estimates of the medium-term equilibrium real interest rate for these economies. Then, we examine the sensitivity of such estimates by considering different specifications regarding parameters, technical assumptions and data series. Finally, we explore to what extent such estimates are useful for assessing the appropriate stance of monetary and fiscal policy.

2. Trends in short-term nominal and real interest rates: 1967-2015

Nominal money market rates in the United States, in German and in the economies forming the euro area have trended downwards since the 1980s. Of course, nominal rates are closely linked to inflation and inflation expectations, which have to be taken into account in order to derive real rates, ex-post or ex-ante. **Figure 1** shows nominal and real interest rates since 1965. The underlying data for are three-month money market rates.

Figure 1 Nominal and real interest rates in the US, Germany and the Euro Area
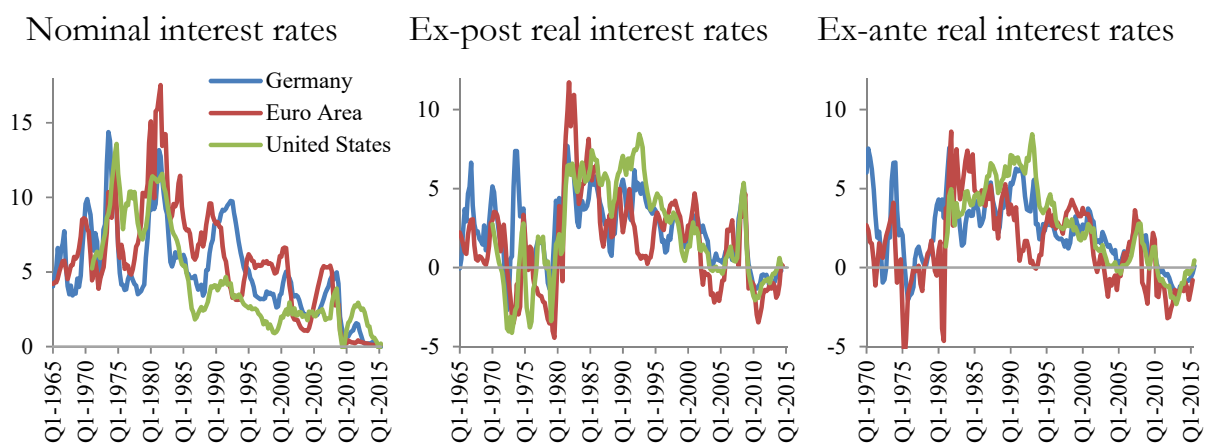
Sources: ECB, OECD, own calculations.

Data concerning the United States and Germany are taken from the Organisation for Economic Co-operation and Development (OECD) Main Economic Indicators database. The euro area data stems from the Area Wide Model Database (AWM) of the European Central Bank. In this database, data from individual member countries prior to the establishment of the monetary union is aggregated in order to construct counter-factual euro area time series starting from the 1970s (Fagan et al., 2001). For inflation, we use indices of consumer prices from the same sources. Inflation expectations are estimated as in Hamilton et al. (2015) from an autoregressive model with a rolling window. The estimation starts with a window length of ten years that is extended gradually to 20 years.

Of course, nominal rates fluctuate a lot over this period but some trends are nevertheless apparent. From 1965 to 1980, interest rates tended to increase until reaching a peak in 1980. Then they declined again. The trend is more pronounced in the United States and in the euro area than it is in Germany. Currently, nominal interest rates are close to zero in all three economies. Nominal rates are constrained from falling much below zero percent, because cash provides an investment opportunity with a nominal return of 0% that is available to all savers. In other words, savers need not necessarily accept negative returns on savings held in bank accounts or other instruments.

High and increasing nominal rates during the 1970s are largely explained by high and rising inflation. As inflation rates in the United States, France, Italy and Span exceeded those in Germany, inflation also largely explains differences in nominal rates during this period. Accounting for inflation or inflation expectations, ex-post and ex-ante real interest rates declined and were negative on several occasions during the 1970s, in particular in the United States and several European economies excluding Germany.

Real interest rates increased after 1980. The increase is particularly pronounced for the ex-post real interest rate (middle panel) in the United States. The reason is that the tightening of monetary policy and subsequent extent of disinflation during Paul Volcker's tenure as Fed Chairman initially came as a surprise to market participants. Ex-ante real interest rates (right panel) also increased but leveled off around 5%. From the mid-1990s onwards real interest rates trended downwards. In 2009, they turned negative in the United States, the Euro Area and in Germany following substantial monetary easing. Most recently, the real return was again positive in the U.S. and in the Euro Area. Our estimates of ex-ante real rates exhibit relatively similar patterns to

ex-post rates, because the horizon of inflation expectations estimated following Hamilton et al. (2015) is only one quarter.

This brief comparison reveals that negative real interest rates already occurred before the global financial crisis and that the U.S. economy experienced negative real returns fairly often. Hence, they are not a new phenomenon. Indeed, Hamilton et al. (2015) find real returns in Germany previously turned negative for a while during the 19th century and during the two world wars, also in the United States.

Figure 2 provides estimates of trends in real interest rates using a moving average of ex-ante real rates with a window length of five years and the Hodrick-Prescott-Filter (1997). Both procedures deliver quite similar results. In the United States and Germany real rates initially trended downwards. From about 1975 to 1980, they were negative in the United States. During the first half of the 1980s, filtered rates in the United States increased quickly to 5% before starting a slow decline. In Germany, the initial increase and the subsequent decline were less pronounced. Nevertheless, current filtered rates are negative in Germany and the Euro Area as well.

Figure 2 Trends in ex-ante real interest rates
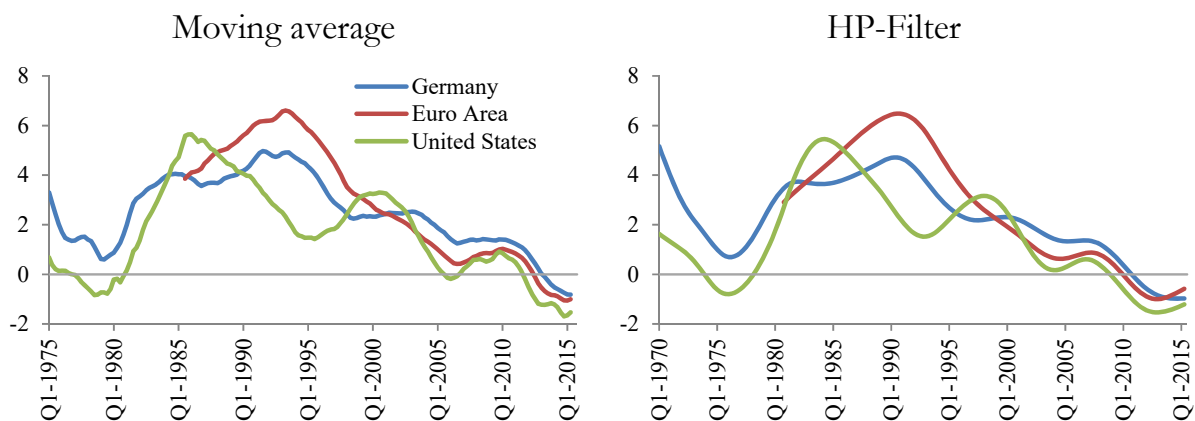
Note: The HP-Filter uses a smoothening parameter of 1600 and the moving average uses a five years window.

These trends in real rates could be interpreted to indicate movements in the equilibrium real rate over a medium term horizon. However, such measures are not informed by any theoretical concept of economic equilibrium. This requires a model of the economy.

3. Different concepts of equilibrium interest rates

The idea of an equilibrium interest rate goes back to Knut Wicksell (1898). In simple terms, the equilibrium interest rate is the real rate that is consistent with constant prices and for which GDP, in the long run, is equal to its potential. If a central bank pursues a positive inflation target, the equilibrium rate ensues when inflation is permanently at its target rate. Wicksell suggested that the level of the natural rate of interest changes over time and, for example, depends on factor productivity and the availability of productive factors.

Concepts of equilibrium employed to estimates equilibrium interest rates differ according to their time perspective. The long-run equilibrium rate obtains once all fluctuations due to cyclical and other temporary factors have died out. It is closely tied to the economic growth rate in steady state, that is, long-run potential growth. The influential Taylor (1993) interest rate rule makes use of a long-term equilibrium rate as a key reference point for central bank rate setting. Taylor (1993) sets it equal to trend growth from 1984 to 1992, which was 2 percent. Extending his estimate of trend growth to 2015 yields a value that is still close to 2 percent.

The relationship between the equilibrium real interest rate and economic growth can be derived theoretically from a neoclassical growth model in which prices are flexible (Sargent and Ljunqvist, 2004). In such a model, the equilibrium real rate follows from the consumption Euler equation, which characterizes optimal consumption and saving decisions. This equilibrium interest rate depends on the same technological factors that drive long-run economic growth. In addition, it is influenced by behavioral parameters that characterize the time preference and savings propensity of households, and by fiscal policy.

New Keynesian macroeconomic models with sticky prices feature a short-run equilibrium rate. It is the rate that would prevail if the price level were flexible (Woodford, 2003). The actual real interest rate deviates from this short-run equilibrium rate due to wage and price rigidities. The short-run equilibrium rate is often referred to as the "natural rate of interest" (Barsky et al., 2014; Curdia et al., 2014). It is influenced by all economic shocks and highly volatile. Consequently, it is not surprising that estimates of such a short-run equilibrium interest rate during the Great recession were for the most part negative. Within such New Keynesian models,

monetary policy can be used to bring the actual interest rate in line with the short-run equilibrium rate. Technically, this real interest rate would result from a monetary policy that solely aims at moving actual GDP to the value that would be realized if prices were fully flexible. This natural rate does not correspond to the long-run equilibrium rate that appears as a reference point in the Taylor rule. Rather, it is best understood as a prescription for setting the central bank rate that central banks could implement in lieu of a simple policy rule such as the Taylor rule. Instead, it is a rather complex policy rule that requires solving a complete structural macroeconomic model taking into account many different economic shocks.

By contrast, the equilibrium concept of Laubach and Williams (2003) (LW03 in the following) focuses on the medium-run and is tied less stringently to a structural model. It is meant to consider different factors that influence the equilibrium rate and change only slowly. The authors employ a simple Keynesian-style macroeconomic model augmented with various unobserved and time-varying components. An aggregate demand curve establishes a relationship between the deviation of the actual real interest rate from the equilibrium real rate and the output gap, that is, the deviation of actual GDP from its unobserved potential. A Phillips curve is used to link changes in the inflation rate to the output gap and various other factors including changes in oil and import prices. The equilibrium real interest rate is unobservable and varies over time. It depends on the growth rate of potential output and a number of other unobservable factors which are meant to capture what Yellen (2015, 2017) has termed 'headwinds'.

4. The econometric methodology of Laubach and Williams (2003)

4.1. The model

The simple macroeconomic model of LW03 consists of the following elements. An aggregate demand equation is given by

$$\tilde{y}_t = a_{y,1}\tilde{y}_{t-1} + a_{y,2}\tilde{y}_{t-2} + \frac{a_r}{2}\sum_{j=1}^{2}(r_{t-j} - r_{t-j}^*) + \epsilon_{1,t}, \tag{1}$$

where $\tilde{y}_t$ is the output gap and r_t denotes the real interest rate. The current output gap is determined by two lags of itself and the average of lagged deviations of the actual real interest rate from the equilibrium real interest rate r_t^*.

The second equation used is a Phillips curve. It links the current inflation rate π_t to the output gap in the previous period, to a weighted average of past inflation rates and the relative fluctuations of oil and import prices (π_t^I and π_{t-1}^O):

$$\pi_t = B_\pi(L)\pi_{t-1} + b_y\tilde{y}_{t-1} + b_I(\pi_t^I - \pi_t) + b_O(\pi_{t-1}^O - \pi_{t-1}) + \epsilon_{2,t}. \tag{2}$$

The third relationship is reminiscent of the neo-classical growth model that links the equilibrium real rate to the potential growth rate and behavioral parameters governing intertemporal utility maximization of households, ($r = \frac{1}{\sigma}g_c + \theta$), where r stands for the real interest rate, g_c stands for growth of per-capita consumption, σ stands for the intertemporal elasticity of substitution of consumption, and θ stands for the time preference. Accordingly LW03 use the following equation to link three unobservables, the equilibrium real interest rate, the potential growth rate and a third unobservable variable, z_t:

$$r_t^* = cg_t + z_t. \tag{3}$$

z_t is meant to capture factors like changes in the rate of time preference of households or trends in fiscal policy. In the following, we employ Yellen's term 'headwinds' to refer to these unobservable factors that cause medium-run fluctuations in the equilibrium real interest rate of this simple macroeconomic model. There will be different assumptions governing the law of motion of these headwinds. In the original specification of LW03, the headwinds follow a random walk:

$$z_t = D_z(L)z_{t-1} + \epsilon_{3,t}. \tag{4}$$

The level of potential output and the growth rate of potential output are also modelled as random walks:

$$y_t^* = y_{t-1}^* + g_{t-1} + \epsilon_{4,t} \tag{5}$$

$$g_t = g_{t-1} + \epsilon_{5,t} \tag{6}$$

LW03 employ the Kalman filter to estimate this model over time. Besides the equations given above, the estimation also requires assumptions concerning the structure of measurement errors regarding unobservables. In order to estimate these

signal-to-noise ratios consistently, LW03 rely on the procedure of Stock and Watson (1998). The signal-to-noise ratios govern the proportion of fluctuations attributed to the level and the trend of potential output, as well as the proportion of fluctuations attributed to the headwinds and the output gap.

4.2. The estimation procedure

The model is estimated in three steps. First, a simpler model is estimated, in which the real rate gap in the aggregate demand curve is omitted and trend growth is constant. This model is used to determine the signal-to-noise ratio for trend growth, ω_g, by translating the likelihood of a break in the trend into a parameter (Stock and Watson, 1998). In a second step, the model is estimated with this parameter and with constant headwinds in order to determine the signal-to-noise ratio of the latter, ω_z. Finally, all other parameters of the model are estimated by means of a maximum-likelihood procedure. Standard errors are computed using a Monte-Carlo-Simulation and consider both filter and parameter uncertainty (Hamilton, 1986).[2]

4.3. The specification of Garnier and Wilhelmsen (2009)

Garnier and Wilhelmsen (2009) (GW09) propose a modification of the econometric specification of LW03 in order to obtain more reasonable estimates than with the original LW03 specification, in particular for the euro area. Their specification differs in three aspects. Firstly, the headwinds variable, z_t follow a stationary process with a lag length of one. As Mésonnier und Renne (2007) as well as Garnier and Wilhelmsen (2009) already pointed out, this modification leads to more stable results. The results obtained under the stationarity assumption are also more easily interpretable than under the random walk assumption. Secondly, the variance of the headwinds is fixed. And thirdly, instead of ex-ante real interest rates, they propose using ex-post real interest rates. By avoiding to estimate inflation expectations, more data remains available for estimating the parameters of the model.

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[2] To replicate the estimation of Laubach and Williams (2003) we use their original GAUSS code.

4.4. The data

In order to estimate the equilibrium real rate for the United States economy, we extend the original dataset of LW03 using time series from the Federal Reserve Economic Database (FRED). In addition, we extend the data until 2018 with FOMC forecasts. For Germany, we use the consumer price index, the real gross domestic product, and the three-month money market rate from the OECD Main Economic Indicator database. For oil- and import prices, we rely on data from the German Federal Statistical Office.

With regard to the Euro Area, we make use of the AWM database, which aggregates the time series of member countries prior the monetary union. This data covers the period from 1970 to 2014. Additionally, we cover the period from 1965 to 1970 with inflation and interest rates series from Cour-Thimann, Pilegaard, and Stracca (2001) and create an aggregate GDP ourselves. Using data from the OECD database, we update the time series up to the second quarter of 2015. Oil and import price data used does not cover the period from 1965 to 1970 and the most recent two quarters.

All time series are quarterly. Inflation expectations are estimated using an autoregressive process with a lag length of 20 years. Plots of the data used in estimation are shown in the appendix.

5. Empirical results for the United States economy

5.1. Estimates of the equilibrium real interest rate

Figure 3 reports the estimates obtained from the baseline specification of LW03. The dashed line refers to the estimates from a one-sided estimation, whereas the solid line refers to the estimates from a smoothed, that is, two-sided estimation. One-sided estimates are closer to what could be obtained in real-time. In this case, the estimation of the unobservable variables only makes use of past and current but not future data. Yet, the parameters of the model are first estimated over the full sample period. The two-sided estimation uses the full data set both for determining the parameters of the model, as well as for estimating the unobserved variables.

Until the mid-1970s, the estimated medium-run equilibrium real interest rate fluctuates around 5%. Thereafter, it slowly declines towards a level of 2% in 1998. Following a brief period with an increase, it decreases further until 2003. During the recession of 2008/09, the equilibrium rate estimate drops below 0%. In the second quarter of 2015, the rate is positive with 0.03%. The signal to noise ratios, ω_g and ω_z, are 0.0192 and 0.0428.

Figure 3 Estimates of the medium-term equilibrium real interest rate in the US
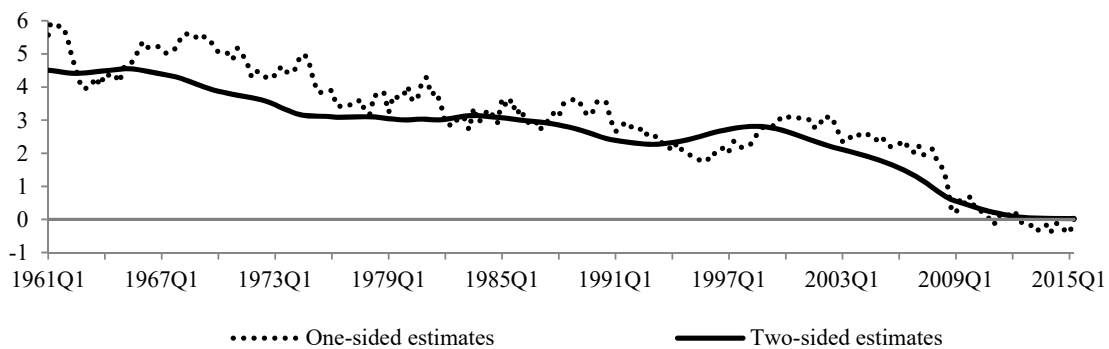
Figure 4 plots the other unobservable variables in the estimated model. The output gap fluctuates strongly. The lowest value is reached in the early 1980s. Following the recession of 2008/09, the output gap drops to -4%. Baseline, the estimated output gap from the baseline specification is back in positive territory at +1%.

Figure 4 Output gap, trend growth and other temporary factors in the US
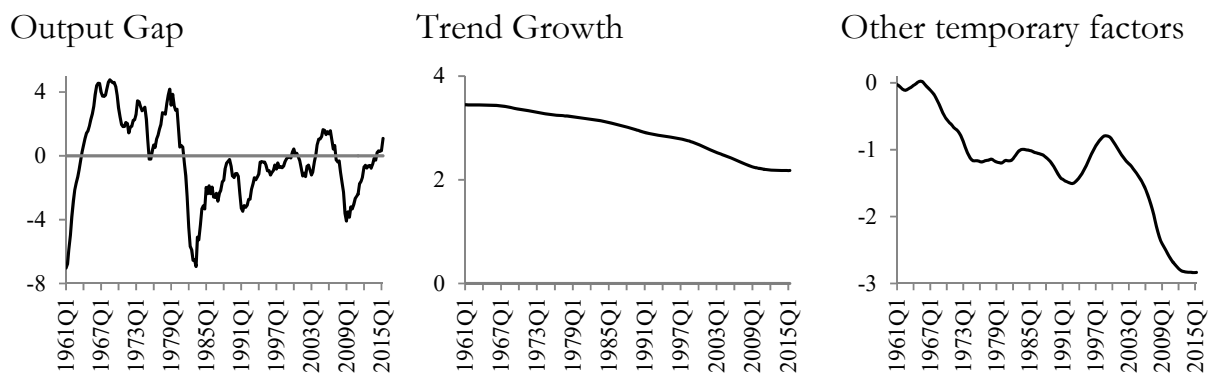
Trend growth declines from over 3.5% at the beginning of the 1960s to a little over 2% in 2009 and has remained at this level since then. Thus, changes in trend growth contribute to the decline in the equilibrium rate estimates. Yet, the headwinds variable, which is meant to capture all other temporary components, has a much larger impact. In fact, it reduces the equilibrium rate estimates over time by up to 3 percentage points. Though it is supposed to capture temporary stochastic factors, it takes on negative values for the full sample period.

5.2 High uncertainty and sensitivity of U.S. estimates

Laubach and Williams (2003) note that their estimates are characterized by a substantial degree of uncertainty. Yet this uncertainty is not mentioned in the influential policy contributions of Summers (2014a,b,c) and Yellen (2015, 2017). In our estimation, the average standard error amounts to 2.3%. **Figure 5** indicates 95% confidence bands. These bands are very wide, yet still do not reflect the full extent of uncertainty. Specifically, these standard errors neglect the uncertainty of the estimated signal-to-noise ratios and the greater uncertainty associated with more recent estimates. For example, the standard error of the estimate for the first period is 1.8%, while the standard error of the estimate for the most recent period is 3.3%.

Figure 5 Standard errors of the two-sided estimates in the US
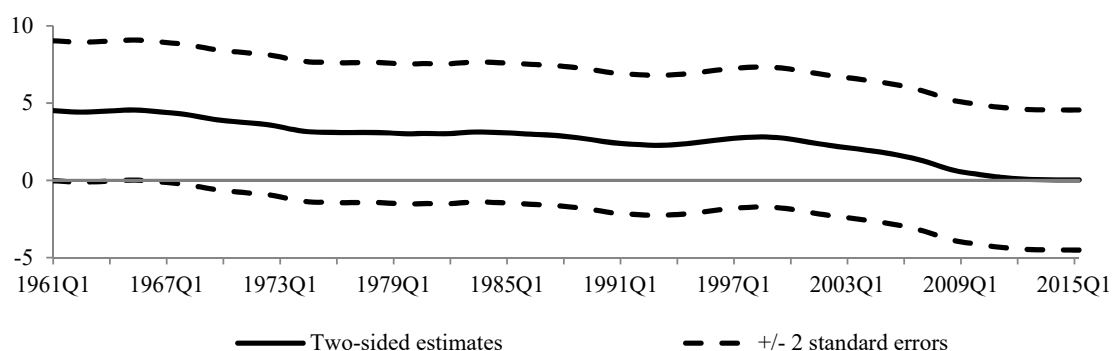
Moreover, Clark and Kozicki (2005) document that different models lead to very different equilibrium rate estimates and that real-time estimates are strongly influenced by data revisions. For this reason, we conduct a detailed sensitivity study. This investigation shows that estimates with the Laubach-Williams methodology change substantially when technical assumptions of the econometric procedure are modified and when other data series are used. Some of these variations were also reported in LW03.

Specifically, **Figure 6** compares the following variants of two-side estimates to the baseline specification: (i) estimates based on gross national income (GNI) instead of GDP; (ii) estimates based on a linear trend instead of a segmented linear trend as the starting value for the estimation; (iii) estimates based on the above-mentioned specification of Garnier and Wilhelmsen (2009) (GW09 in the following); (iv) estimates up to 2018 making use of FOMC forecasts.

Figure 6 Sensitivity Analysis: U.S. equilibrium rate estimates
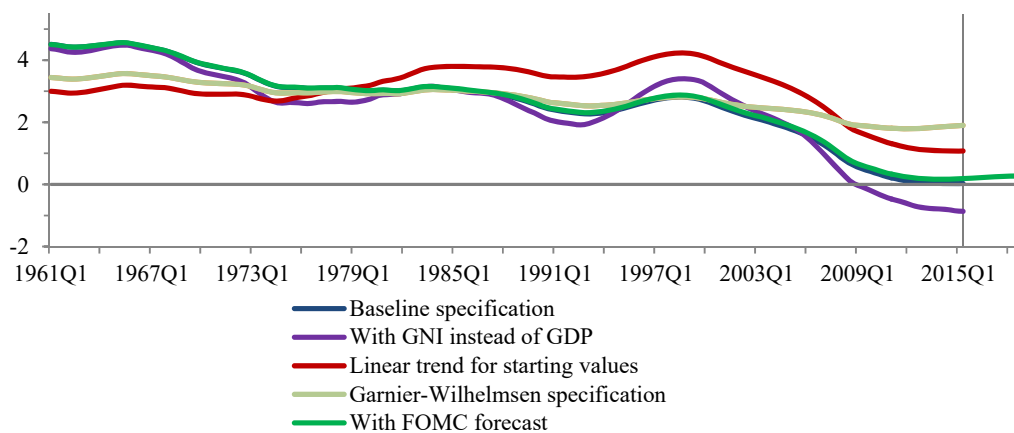
GNI instead of GDP

Williams (2014) replaces GDP by GNI. In case of GNI, factor incomes earned by foreign residents are added to GDP while income earned in the domestic economy by nonresidents is subtracted from GDP. For large economies such as the United States, differences between GDP and GNI should be minor. Still, changing the data series has a substantial impact on the results. With GNI the signal-to-noise ratios change and

attribute a stronger impact on the equilibrium real interest rate to the headwinds variable. As a result, the equilibrium rate declines more during the recession of 2008/09 down to about -1%.

Different starting values

Starting values are seemingly innocuous specification parameters, yet are found to matter for the results. Laubach and Williams (2003) use a segmented linear trend to define starting values for potential output. As an alternative, we consider a simple linear trend. The resulting equilibrium real rate estimates deviate substantially from the baseline specification. At the beginning of the estimation period, the rate is lower and the declining trend during the 1960s and 1990s vanishes. At the end of the period, the estimate using the alternative starting values for potential trend is 1 percentage point higher than in the baseline specification.

Garnier and Wilhelmsen Specification (2009)

To estimate the specification of GW09, we use the same signal-to-noise ratios as in the baseline specification. The resulting estimates change much less over time. The estimated equilibrium real interest rate in the second quarter of 2015 is 1.9% and thus, considerably higher than the estimate of the baseline specification.

Extending with forecasts until 2018

Lastly, we extend the data series until 2018 by employing FOMC forecasts. This allows us to evaluate the sensitivity of the two-sided estimates of the baseline specification at the end of the data window. Estimates towards the end of the sample rise a bit and the equilibrium real rate estimate at the end of 2018 is 0.25%.

5.3 Using the equilibrium rate estimates in the Taylor rule

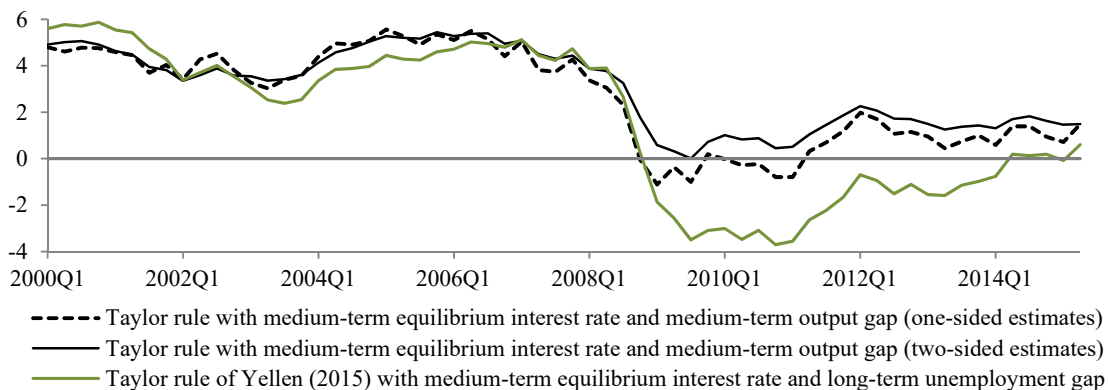As noted previously, FOMC Chair Janet Yellen has referred to estimates of the equilibrium real interest rate with the Laubach-Williams methodology (Yellen, 2015, 2017) in order to argue that the Fed's policy stance does not deviate much from the prescriptions of the Taylor rule. Specifically, she uses the current LW estimate of the medium-run equilibrium rate of 0% to replace the long-run estimate of 2% in the

original Taylor rule. As a result, interest rate prescriptions shift down by about 2 percentage points.

As a measure of inflation Chair Yellen uses the personal consumption expenditures (PCE) prices excluding food and energy prices, as we do in our analysis. With regard to the output gap, she multiplies the deviation of the current unemployment rate from the long-run natural unemployment rate with an Okun's law factor of two in order to obtain an estimate of the long-run output gap. Taking these three elements together, she obtains a Taylor rate of 0.5%.

However, the long-run unemployment-based output gap is not consistent with the medium-run equilibrium rate estimate. The Laubach-Williams method estimates the level of potential output and its growth rate together with the equilibrium rate. Thus, it would be consistent to use the LW estimates of the equilibrium rate and the output gap together in the Taylor rule. **Figure 7** compares the Yellen-Taylor Rule from Yellen (2015) with the consistent version.

Figure 7 Taylor rule with LW equilibrium rate estimates

- - - - Taylor rule with medium-term equilibrium interest rate and medium-term output gap (one-sided estimates)
——— Taylor rule with medium-term equilibrium interest rate and medium-term output gap (two-sided estimates)
——— Taylor rule of Yellen (2015) with medium-term equilibrium interest rate and long-term unemployment gap

The Yellen-Taylor rule declines to about -4 % in the course of the recession of 2008/09. By the second quarter of 2015, the resulting interest rate prescription has just about risen to 0.6%. Yet, a consistent version of the rule that uses both the equilibrium real interest rate and the output gap estimate from the LW method, implies substantially higher interest rate prescriptions. For example, they decline only towards -1% in 2009 and return to positive territory in 2010. If the smoothed estimates are used, the rate prescriptions are even a bit higher. Hence, the consistent

version of the Taylor rule with the LW estimates of the equilibrium rate and the output gap calls for a tightening of the Federal Reserve's policy stance.

The LW estimate of the output gap does not decline as much as the long-run unemployment-based estimate from Yellen (2015) in the course of the recession of 2008/09. Furthermore, it turns positive in 2014. The reason is as follows. A reduction in the estimate of the potential growth rate, implies a lower equilibrium rate estimate but also a lower level of potential output.

6. Empirical results for Germany and the euro area

6.1. Estimates of the equilibrium real interest rate for Germany

When applying the LW method to data for Germany and the euro area, we encountered difficulties in achieving convergence of the estimation. Estimates turned out to be unstable and sensitive to starting values. Estimates for the euro area were not economically sensible. By contrast, using the simplified estimation procedure from Garnier and Wilhelmsen (2009), we obtain more stable and more plausible results.[3]

Figure 8 reports the estimation results. From 1965 and 1975, the estimated real equilibrium interest rate fluctuates between 1.5% and 3.3%. Subsequently, there is a long phase during which the rate stays around 2%. During the course of German re-unification, the estimates increase again to roughly 3%. From the second half of the 1990s onwards, however, the rate decreases steadily. After 2007, it varies between 0% and 1% reaching a value of 0.6% in 2015.

[3] We estimate the equilibrium real interest rate for Germany from 1965 onwards. While time series for Germany start later than for the United States, the simplified specification allows estimating the rate from the first data point onwards because it does not require a separate estimation of inflation expectations. The signal-to-noise ratios are taken from Garnier and Wilhelmsen (2009) s.t. ω_g=0.081 and ω_z=0.064.

Figure 8 Estimates of the medium-run equilibrium rate in Germany
(Garnier-Wilhelmsen specification)

.... One-sided estimates (Garnier-Wilhelmsen specification)
——— Two-sided estimates (Garnier-Wilhelmsen specification)
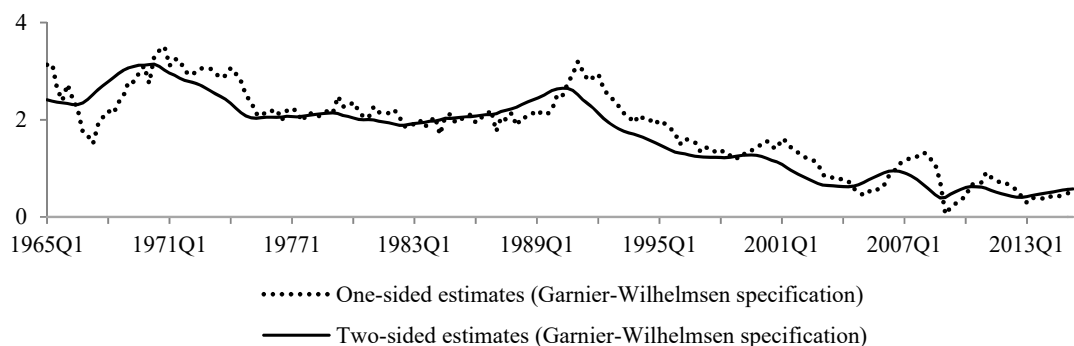
The degree of uncertainty is very high just as in the case of the LW specification for U.S. data. **Figure 9** shows a two-standard error bands. At 2.3%, the standard error is similar to the U.S. estimate. For Germany, however, filtering uncertainty is greater and recent estimates very unreliable. In the most recent period, the standard error is 7.9%.

Figure 9 Confidence bands for two-sided estimates with German data
(Garnier-Wilhelmsen specification)

——— Two-sided estimates (Garnier-Wilhelmsen specification)   — — — +/- 2 standard errors
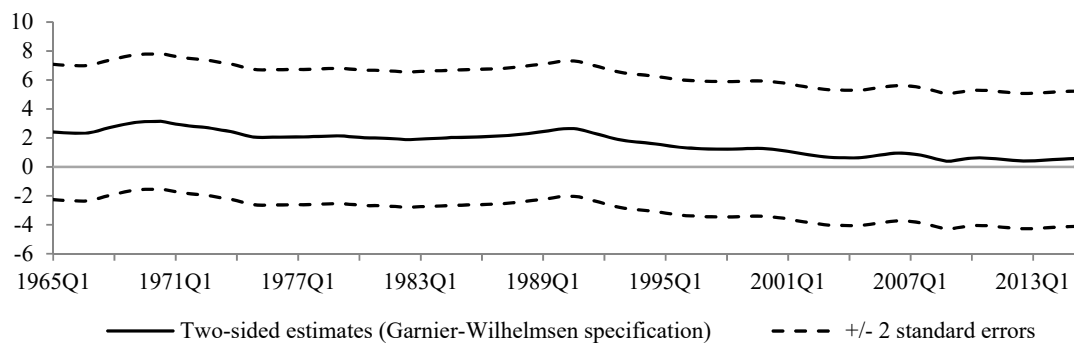
Estimates of the output gap, trend growth and temporary factors influencing the equilibrium rate are shown in **Figure 10**. The output gap estimate turns positive in 2013. It stands at 1.7% in the second quarter of 2015. Trend growth is estimated around 3% from 1965 up until German re-unification. Starting in the mid-1990s, it decreases steadily and is down to 0.7% in 2015. This estimate is somewhat lower than the estimate of the German Council of Economic Experts (SVR, 2015). The other components that influence the equilibrium real interest rate, i.e. the "headwinds", are rather small. This is most likely due to the stationarity assumption of Garnier and Wilhelmsen (2009).
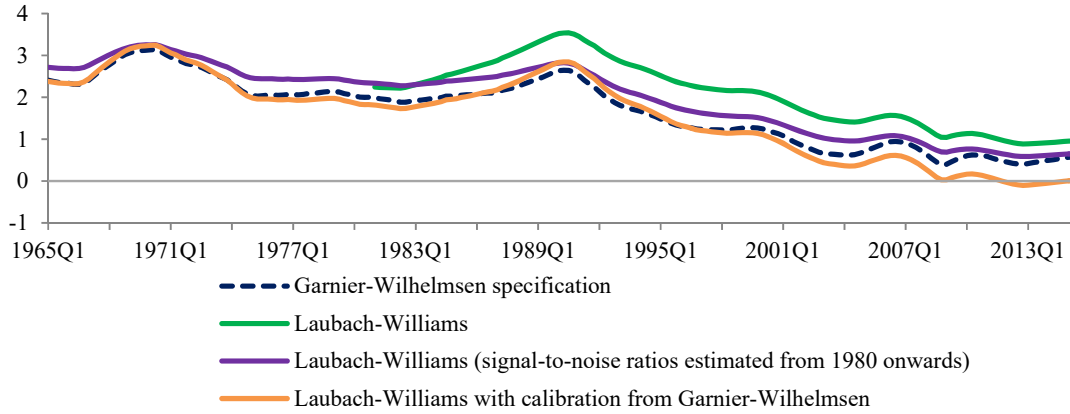
Figure 10 Output gap, trend growth and other temporary factors in Germany

Output Gap

Trend Growth

Other temporary factors
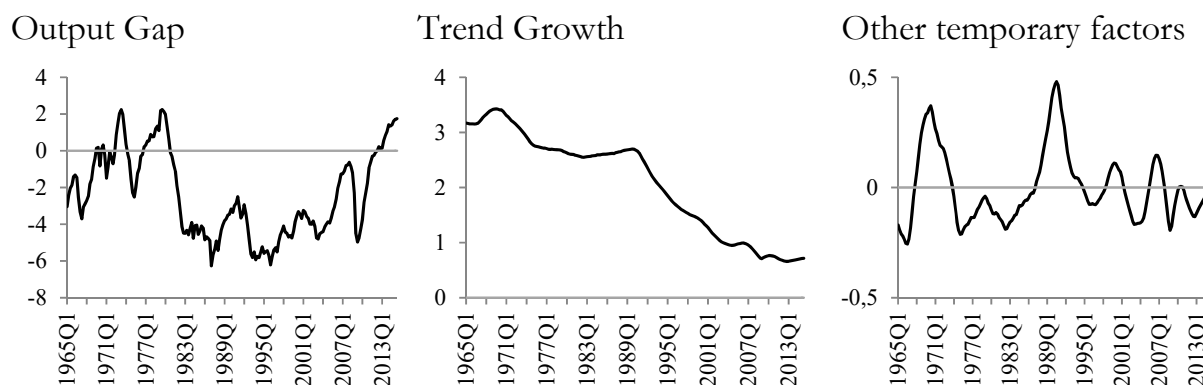
Our sensitivity analysis of estimates with German data arrives at the same conclusion as the one with U.S. data. Again, the estimation results are not robust. **Figure 11** compares the estimates of the Garnier-Wilhelmsen specification with three variants of the LW specification. First, we follow the U.S. baseline specification, but use ex-post instead of ex-ante real rates. With this specification, the estimation of the signal-to-noise ratios only converges from 1981 onwards. Secondly, we use these signal-to-noise ratios for estimating the model from 1965 onwards. Thirdly, we employ the signal-to-noise ratios of Garnier and Wilhelmsen (2009) in the LW specification.

LW equilibrium rate estimates lie partly above and partly below the GW specification. In the second quarter of 2015, they are between 0% and 1%. The estimation over the full period using the signal-to-noise ratios estimated from 1981 onwards delivers results that are very similar to those of the GW specification.

Figure 11 Medium-run equilibrium interest rate estimates for Germany
with different specifications

- - - - Garnier-Wilhelmsen specification
—— Laubach-Williams
—— Laubach-Williams (signal-to-noise ratios estimated from 1980 onwards)
—— Laubach-Williams with calibration from Garnier-Wilhemsen

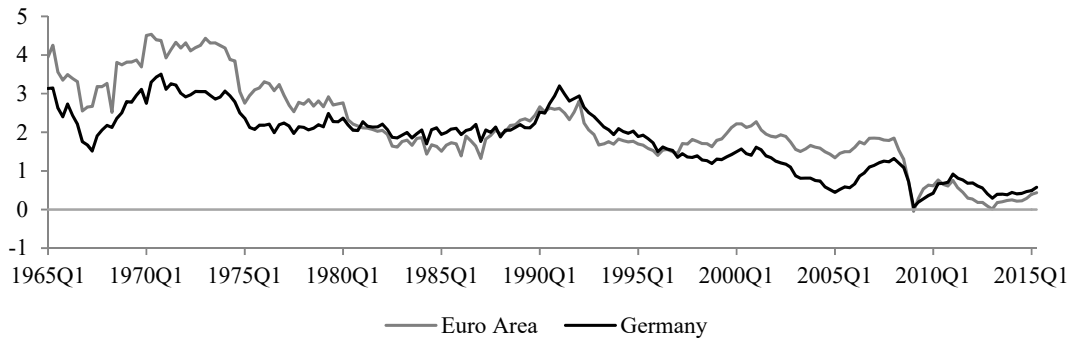6.2. Estimates of the equilibrium real interest rate for the Euro Area

For the Euro Area, we employ again the GW specification along with their signal-to-noise ratios. **Figure 12** reports one-sided and two-sided estimates. Initially, the estimated equilibrium real interest rate fluctuates around 4%. Then it declines fairly steadily to about 1%. Following the financial crisis, the rate falls quickly. The one-sided estimate in the first quarter of 2012 is close to 0%. Then it increases again. In the most recent period, it is 0.4%. The standard errors with euro area are considerably larger than with U.S. and German data.

Figure 12 Medium-run equilibrium rate estimates for the euro area
(Garnier-Wilhelmsen specification)

.... One-sided estimates (Garnier-Wilhemsen specification)
—— Two-sided (Garnier-Wilhemsen specification)
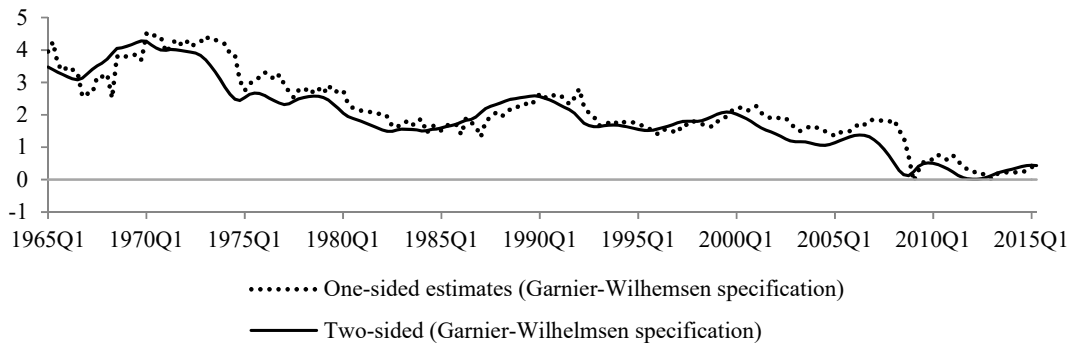
Figure 13 compares one-sided estimates from Germany and the euro area. Until 1980, the estimate for the euro area is above the one for Germany. From 1980 to the mid-1990s, they are quite similar. Following the financial crisis, it is lower in Germany.

Figure 13 Medium-run equilibrium rate estimates
for Germany and the euro area

7. Discussion

Equilibrium real interest rate estimates obtained with the Laubach-Williams (2003) method and related approaches are becoming increasingly influential in the monetary policy debate. Our empirical investigation and sensitivity analyses have delivered the following findings:

1. The estimates reported by Laubach and Williams (2003) and Williams (2014, 2015) for the United States can be replicated and updated without major difficulties when using the original specification and code of Laubach and Williams. Given the nature of the simple model that is used to identify the equilibrium rate, it is best considered a medium-run concept. The estimates obtained with the baseline specification tend to decrease during the last decades and decline somewhat below 0% in the course of the financial crisis.

2. However, these estimates with U.S. data are associated with a very high degree of uncertainty. Moreover, our sensitivity analyses show that relatively small and technical changes of the econometric specifications or the data series result in economically relevant changes in the estimates. Thus, these estimates are extremely imprecise and may well be biased. The reason for the sensitivity to small changes in the econometric specification is most likely due to the attempt to estimate several unobservable variables at the same time. These are the

medium-term equilibrium real interest rate, the level of potential output, the growth rate of potential output, and another unobservable variable that is capturing other factors that affect the equilibrium rate in the medium-term. The unobservable variables mostly depend on temporary and permanent shocks. It is therefore necessary to make assumptions to what degree forecast errors are due to temporary or permanent factors. If the parameters on which this decomposition depends, i.e. the signal-to-noise ratios, change even a little, equilibrium rate estimates often change a lot. Besides, the methodology requires very long data series in order to lead to fairly stable results. To be fair, Laubach and Williams (2003) noted the high degree of uncertainty and sensitivity of their estimates. We find that in spite of an additional 12 years of data, these features of the equilibrium rate estimates remain unchanged.

3. Applying the baseline specification of Laubach and Williams to data for Germany and the euro area yields even more discouraging results. For the most part, the length of available data samples is too short to obtain sensible and reliable estimates. We obtain more plausible results based on a modification of the econometric specification proposed by Garnier and Wilhelmsen (2009). It simplifies and restricts the modelling of unobserved factors other than the potential growth rate that influence the equilibrium rate. These factors represent what Yellen (2015) refers to as 'headwinds'. In the course of the financial crisis and recession, our estimates for Germany and the euro area drop as well. Yet, they remain mostly positive and the estimates are around 0.5% for 2015. Importantly, however, the estimates with German and euro area data are also very imprecise and very sensitive to small changes in the econometric specification.

4. Our findings were first circulated in a German language working paper (Beyer and Wieland, 2015) and an English language summary in GCEE (2015). Since then, Holston, Laubach and Williams (2017) have also provided estimates for the euro area along with estimates for the United Kingdom and Canada. Their specification differs in several ways from the baseline specification in Laubach and Williams (2003). The effect of trend growth on the equilibrium real rate is not estimated anymore. Instead it is restricted to unity. It would be considerably higher otherwise. Second, there are constraints on the estimated coefficients. Third, the estimation of inflation expectations is different. Nevertheless, the estimates remain very imprecise and highly sensitive.

With regard to the debate on monetary policy in a low rate environment and secular stagnation, we draw the following conclusions:

1. The decline in medium-run equilibrium rate estimates is much too uncertain and too sensitive to fairly arbitrary technical assumptions, to be used as the key argument for determining the appropriate monetary policy stance as in Yellen (2015, 2017). It does not provide reliable evidence for a secular stagnation as suggested in Summers (2014). These highly influential contributions to the debate on monetary and fiscal policy fail to highlight appropriately the imprecise nature of these estimates. Furthermore, there are a number of possibilities for omitted variable bias as noted by Taylor and Wieland (2016) and Cukierman (2016). These include structural factors reducing trend growth and potential, monetary policy deviations leading to low interest rates and the impact of financial stability concerns on policy and the economy.

2. While the Taylor (1993) rule uses an equilibrium rate in order to determine the appropriate level of monetary policy rates, this is a long-run concept. Estimates of the long-run equilibrium rate have not declined as much as the medium run concept from Laubach and Williams (see Taylor and Wieland, 2016). Even so, when using the LW estimate of the medium-run equilibrium rate in Taylor-style monetary policy rules, it is important to use the consistent medium-run output gap. When this is done, the Taylor-rule interest rate prescription is higher than with a long-run output gap as in Yellen (2015, 2017). The reason is that the medium-run equilibrium rate of near zero percent implies that output is near potential.

3. The high degree of uncertainty about the equilibrium rate suggests that it is worthwhile considering policy rules, which do not require information on the equilibrium rate. This is true, for example, for simple first-difference interest rate rules as considered by Orphanides and Wieland (2013). Such rules appear to perform well in model simulations under with noisy estimates of the equilibrium real interest rate according to Orphanides and Williams (2009). With regard to level rules, such as for example the Taylor (1993) rule, it is best to use an estimate of long-run equilibrium rate because these estimates are more precise and have not declined as much as the medium-run rate estimates.

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Appendix

Figure A1: Original data for the US, Germany, and the Euro Area