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Improving Mathematics Achievement

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On February 14, 2005, the members of the Arkansas State Board of Education adopted the 2004 revision of their state's *Mathematics Framework* by unanimous vote. The board adopted the new *Framework* with little fanfare and with none of the explosiveness often found at mathematics education debates in recent years. The next day, the *Arkansas Democrat-Gazette* noted:

> Called "frameworks," the new standards describe what skills should be taught in each grade—kindergarten through eight—and in 14 different junior high and high school math courses beginning with algebra I and including pre-calculus, statistics and computer mathematics. In contrast, the 1998 math standards now in use do not break down the necessary skills by grade or by course but direct certain math concepts be covered during a span of grades, such as kindergarten through fourth grade, or ninth through 12th grades. Those 1998 standards have been criticized by some national organizations and some Arkansas teachers for being too vague and causing skills to be taught repeatedly or omitted altogether.[1]

1. Cynthia Howell, "Teaching of math in state refigured: New 'frameworks' zero out guesswork," *Arkansas Democrat-Gazette*, Feb. 15, 2005.

Standards for three grade spans fail to specify the grade level for any particular topic, and without grade-level specificity standards are rather worthless. They cannot be used effectively for accountability systems, and they provide only crude guidelines in designing tests to monitor progress, and they are not very useful in selecting textbooks.

With the move to grade-level and course-level specificity, Arkansas has taken a step in the correct direction. That step was sorely needed: The Fordham Foundation had given Arkansas mathematics standards dismal F grades.[2] *Education Week* gave Arkansas a D+ grade for standards and accountability across subject areas.[3] Either way, the consensus seemed to be that Arkansas had considerable room for improvement.

Just how good are the new Arkansas standards? What do they look like? Have they been significantly improved or just polished up a bit? Here we attempt to address these questions by looking at the Arkansas mathematics standards in a variety of ways. Our aim is to identify areas that have or have not improved, and offer insights that may be useful to Arkansas educators as well as those in other states.

The New Framework Structure for K–8

The new *Framework*[4] has a structure in K–8 that has grown out of the old framework structure in that it has five strands and

2. Ralph A. Raimi and Lawrence S. Braden, *State Mathematics Standards: An Appraisal of Math Standards in 46 States, the District of Columbia, and Japan* (Washington, D.C.: Thomas B. Fordham Foundation, 1998); and David Klein, et al., *The State of State Math Standards—2005* (Washington, D.C.: Thomas B. Fordham Foundation, 2005).

3. Editorial Projects in Education, “Quality Counts 2002: Building Blocks For Success,” *Education Week* (Jan 2002).

4. The frameworks are available online at http://arkedu.state.ar.us/curriculum/benchmarks.html#Math.

seventeen standards statements that run across all nine grade levels. The revised standard statements for K–8 are necessarily vague as a consequence. For example: “Standard 7. Analysis of Change: Students shall analyze change in various contexts.” The seventeen new K–8 Standards are now elaborated by 503 Student Learning Expectations (SLE’s). The SLE’s should be thought of as the actual standard statements. The number of SLE’s per standard varies considerably. For example, the Number Sense standard contains sixty-nine SLE’s while the Analysis of Change standard has only nine SLE’s. This is entirely appropriate because the quantity and difficulty of material to be learned necessarily varies across standards.

The number of SLE’s within any given standard is relatively constant across grade levels. This is worrisome as there is every reason to expect that some topics will develop in uneven spurts across the grade levels. Scattering a little bit of coordinate geometry across all grade levels starting in kindergarten seems at best artificial. Stretching seventeen topics across these grade levels can result in covering too many topics over and over again each year but never treating them in depth.

The new *Framework* groups together related SLE’s within standards. Herein we will call these groups *categories* since the *Framework* itself provides the names of the categories without actually supplying a label for this level of organization. The Number Sense standard, for example, is divided into two categories—whole numbers and rational numbers. The Numerical Operations and Estimation standard has five categories within it.

When broken down into categories within standards, we begin to see a more reasonable differentiation of emphases across grade levels. For example, Number Sense for whole numbers ends in grade 4. On the other hand, some of the structure seems less reasonable, such as addressing rational numbers across all grade levels and scattering a little on *Estimation* at each of seven grades.

One of the most unusual categories is *Coordinate Geometry* because it is included starting in kindergarten. Inspection of the SLE's for grades K–5 shows that they begin with spacial relationships (over, under, behind, etc.), progress through rows and columns, and graduate to points on a coordinate grid. There is growth across the grade levels (except that the SLE's for grades 3 and 4 are identical). So, even if the category name, *Coordinate Geometry*, is somewhat misleading, the structural changes have led to more useful standards.

Before leaving the treatment of structure, however, there is yet another structural element in the new K–8 standards that deserves high commendation. Individual SLE's within any given category are physically arranged on the page such that similar topics are aligned across grade levels (at least across grade levels within the two grade spans since K–4 and 5–8 are listed separately). This horizontal alignment reflects what are often called vertical standards. This means that standards are aligned in a way that facilitates tracing the development of a topic across grade levels. Vertical standards help to assure that there is some progression year to year as the standards are written. Vertical standards help when the standards are interpreted to assure that year to year growth is attended to in practice. Vertical standards are very helpful in designing assessments because the growth in expectations across grade levels is more obvious.

Thus, the structure of the new standards spotlights both strengths and weaknesses. The new standards are improved; they are not only grade-specific but also more coherent and better organized. It is commendable that topics can now be traced vertically across grade levels. While this organization is beneficial, it also makes it easier to recognize when successive progress across grade levels is not specified sufficiently. The structure itself also suggests that many topics are spread out thinly across many grade levels rather than having a tighter focus at a few grade levels.

Sample Topic: Triangles

Here we investigate a few special topics having to do with triangles as illustrative cases. The first has to do with the formula for the area of a triangle. This topic is important because elementary students can actually derive the formula and see that it is necessarily correct. In so doing, they are laying the groundwork that will later develop into proofs in higher-level courses. There are a few ways to approach this topic, and it is perhaps easier if the area of parallelograms is addressed first. Fortunately, the Arkansas standards expect that students will be able to derive these area formulas rather than just memorize them.

Next, we look at the sum of the angle measures in a triangle. It may be possible for students to reason that the sum of the angles should equal the measure of a straight angle, but students at this level are unlikely to know about straight angles or their measure. The Arkansas SLE has students measure the angles and add up the measurements. Students will rarely come up with exactly 180 degrees (the correct answer), so such an exercise is likely to teach students that the sum of the measures of the interior angles is not a constant.

Finally, the Pythagorean Theorem (and distance formula) should be carefully addressed just before entry into introductory algebra. Students at this level should understand it and use it, but not be expected to prove it. They may be exposed to one or more proofs but constructing a proof should be deferred until high school geometry. Arkansas expects students to use the theorem rather than stressing a deep understanding. Arkansas also adds technology to this standard unnecessarily, as is frequently the case in other states.

As these examples illustrate, the new Arkansas standards have made improvements regarding important aspects of the relevant learning objectives. Arkansas, however, consistently lags behind states with more aggressive standards in the rate of progress across grades. The lag in progression becomes significant as students approach introductory algebra.

What Does a Good Standard Look Like?

Reports that evaluate mathematics standards across states typically establish a set of criteria and then rate standards according to the adopted criteria. The criteria selected vary from report to report, but in general, criteria suggest that each standard should be explicit, focused, and measurable. Each standard should identify a learning objective in a way that is clear and unambiguous. There should be little doubt as to what achievement falls within the standard and what does not. The learning objectives should not require interpretation or have different meanings to different readers. The content covered by an individual standard should be specific enough to guide both instruction and assessment. The standard should reference a learning objective, not a learning process or pedagogical preference. The best standards state expectations with sufficient specificity to give clear direction to those who develop assessments.

The new Arkansas K–8 framework contains some standards that fit these criteria well and others that don't come close. Here is an example of each:

- Grade 3—Select and/or write number sentences (equations) to find the unknown in problem-solving contexts involving two-digit times one-digit multiplication
- Grade 7—Investigate geometric properties and their relationships in one-, two-, and three-dimensional models, including convex and concave polygons

In the first case it is clear what the student should be able to do. It would be easy to differentiate between math problems that are consistent with the standard and those that are not. The learning objective is clear and assessment writers would know just what to do to measure this achievement.

The second case is not at all clear. The standard is likely to be interpreted differently by various readers. Just what should be expected of a seventh-grade student to satisfy this standard? Perhaps this standard fails because it attempts to introduce investigation as a learning process rather than stating a learning objective. For whatever reason, it fails to make learning goals clear, and therefore, it isn't of use for guiding instruction or assessment.

High School Topics in the New Framework

In the 1998 *Framework*, Arkansas addressed topics from introductory algebra and geometry to cover the 9–12 grade span. The new *Framework* adds Algebra II, Algebra III, Statistics, and other courses to the mix. Thus, the most obvious improvement at the high school level in the new *Framework* is greater specificity for work beyond introductory Algebra and Geometry.

In the 1998 Fordham review,[5] the authors described the Arkansas standards as telegraphic and some of this style is found in the revision at the high school level. However, it should be noted that many of the secondary level standards are sufficiently clear and focused. Here is an Algebra I standard that manages to be an example of both points:

- Solve quadratic equations using the appropriate methods with and without technology

5. Ralph A. Raimi and Lawrence S. Braden, *State Mathematics Standards: An Appraisal of Math Standards in 46 States, the District of Columbia, and Japan* (Washington, D.C.: Thomas B. Fordham Foundation 1998).

- factoring
- quadratic formula with real number solutions

This standard is relatively clear. Weak Algebra I programs often include factoring of only simple cases where the coefficient of the squared term is 1, so the standard could be better if it explicitly included some more difficult cases. Again we see the use of calculators. Factoring quadratics is a skill that builds on number sense and the learning objective here shouldn't require a calculator. Similarly, it is more important for students to know and understand the quadratic formula and apply it to the extent of placing the coefficient terms into the formula correctly than to give estimated decimal values for some of the radicals that can result. Consequently, the emphasis on technology seen in K–8 continues to be excessive.

Another standard from Algebra I shows similar characteristics:

- Solve systems of two linear equations
  - numerically (from a table or guess and check)
  - algebraically (including the use of manipulatives)
  - graphically
  - technologically

Being able to solve the simple systems presents a clear objective. There are prescribed ways of doing this algebraically, which should be the focus. Making the algebraic solution one of only four approaches listed misguides both instruction and assessment. Further reducing the importance of the algebraic solutions by suggesting that they include the use of manipulatives just adds a little salt to the wound.

The objectives listed in the secondary standards are not always clear. For example, consider these Geometry standards:

- Describe relationships derived from geometric figures or figural patterns
- Investigate the measures of segments to determine the existence of triangles (triangle inequality theorem)

The first is far too vague to be useful. There aren't even any hints to suggest what sorts of relationships are to be included or what kind of descriptions are expected. By using the term "investigate," the second doesn't make clear exactly what is to be learned and how it should be measured.

Finally, here are two examples from Arkansas's new Statistics course:

- Analyze categorical data
- Use simulations to develop an understanding of the Central Limit Theorem and its importance in confidence intervals and tests of significance

The first suffers from being overly telegraphic. There are many ways to analyze categorical data, and some of them are clearly not appropriate in high school. The second introduces a pedagogical approach (simulations) and a process (develop understanding) rather than stating measurable objectives.

Common Problems

In the latest Fordham report,[6] David Klein et al. identified nine problems that they commonly found in state mathematics standards. The new Arkansas standards are still having difficulties in the first six of these common problem areas. Here we identify each of these six common problems and then comment on the new *Framework* with respect to that problem.

6. David Klein, et al., *The State of State Math Standards—2005* (Washington, D.C.: Thomas B. Fordham Foundation, 2005).

1. Calculators—One of the most debilitating trends in current state math standards is their excessive emphasis on calculators. Most standards documents call upon students to use them starting in the elementary grades, often beginning with kindergarten. Calculators enable students to do arithmetic quickly, without thinking about the numbers involved in a calculation.

The standards do not refer explicitly to *calculators*. However, fifty-seven of the 503 Student Learning Expectations refer to *technology*, which the Arkansas *Framework* often uses as a synonym for calculators. Most of these SLE's fall in the Number and Operations strand starting in kindergarten, just as Klein feared.

2. Memorization of Basic Number Facts—There is no real mathematical fluency without memorization of the most basic facts. The many states that do not require such memorization of their students do them a disservice.

Many of the Arkansas standards are process-oriented, stating, for example, that students should develop strategies for basic additions facts rather than simply saying that students should know and be able to use the facts. The Arkansas standards do suggest that the basic facts should be mastered, but also indicate that students should add single digit numbers with a calculator.

3. The Standard Algorithms—Only a minority of states explicitly require knowledge of the standard algorithms of arithmetic for addition, subtraction, multiplication, and division. Many states identify no methods for arithmetic, or, worse, ask students to invent their own algorithms or rely on ad hoc methods.

Arkansas's new *Framework* does exactly what Klein warned against. It emphasizes developing algorithms *ad hoc,* making these invented algorithms at least as important (if not more important) than the standard algorithms.

4. Fraction Development—In general, too little attention is paid to the coherent development of fractions in the late elementary and early middle grades.

Arkansas refers to fractions in twenty-eight Student Learning Expectations which suggests that Arkansas gives adequate attention to the topic, but the Arkansas expectations promote distinctive pedagogical preferences rather than sticking closely to learning goals.

5. Patterns—The attention given to patterns in state standards verges on the obsessive. In a typical document, students are asked, across many grade levels, to create, identify, examine, describe, extend, and find "the rule" for repeating, growing, and shrinking patterns . . . the attention given to patterns is far out of balance with the actual importance of patterns in K–12 mathematics.

The presentation of patterns falls prey to a common error in school mathematics programs—failing to stipulate the necessary conditions for identifying missing terms. For example, consider the series 3, 5, 7, _. Most would say that the next value should be 9, and that is probably what educators following the Arkansas standards would want. However, we have no mathematical certainty what the next number should be. The sequence could be a list of primes, and the next value should be 11. In fact, any number is possible in the next position unless we add further stipulations, such as stating that it is a linear pattern or that it increases by a constant amount. The point is not trivial because mathematical logic is based on certainty, not guesswork. Some pattern fans have gone to the absurd length of asking things like, "What is most likely the next number in the pattern?"

6. Manipulatives—Manipulatives are physical objects intended to serve as teaching aids. They can be helpful in introducing new concepts for elementary pupils, but too much use of them runs the risk that students will focus on the manipulatives more than the math, and even come to depend on them.

Only two of the Arkansas SLE's refer to *manipulatives*. However, the term *objects* appears in forty-one of the Arkansas statements. *Objects* is often a synonym for manipulatives. If we inspect only the Number and Algebra strand there are eleven instances in Arkansas from grades 3–8. It is clear from the statements that students are expected to continue the use of manipulatives throughout the development of fluency in school mathematics.

Arkansas has been trapped by many of the pitfalls that Klein addressed in the most recent Fordham report. This is largely because the Arkansas standards and SLE's are not pedagogically neutral:

- The Arkansas standards tend to recommend teaching methods rather than only setting learning objectives. Some of the material in the Appendix to the Arkansas *Framework* goes the farthest in this regard, clearly promoting constructivist discovery-learning.
- The Arkansas standards rely too much on the use of calculators and manipulatives.
- The Arkansas standards have students develop their own algorithms, which is time-consuming and fraught with peril.
- The Arkansas standards could benefit from ensuring mastery of the standard algorithms and coherent treatment of the topic of fractions.

Recommendations

In six years, Arkansas is scheduled to produce yet another new math *Framework*. The next version should:

1. reduce the emphasis on technology and manipulatives;
2. focus on learning objectives rather than on pedagogy and process;
3. include deeper coverage of fraction development;
4. include mastery of the standard algorithms;
5. reduce the number of topics each year for greater focus on the most important topics; and
6. eliminate standards that are too vague to be useful.

Although clearly improved relative to the prior version, Arkansas educators should use the 2004 *Framework* with caution. Educators must now adjust curriculum development, assessment design, and instruction to the *Framework*. If they carefully select objectives and emphasize the right things, they will be able to overcome the limitations of the *Framework*. Because the new *Framework* is sometimes unclear and sometimes misguided, the opportunity exists to implement it wisely and produce effective results or to implement it poorly and yield little or no improvement.