Goals and Rules in Central Bank Design[*]

Carl E. Walsh

This version: March 2015

Abstract

Beginning with the Reserve Bank of New Zealand Act of 1989, central banking reforms have focused on assigning clear goals for which monetary policy authorities can be held accountable. Inflation targeting regimes provide examples of such goal-based policy frameworks. An alternative approach, recently argued for by John Taylor (2012, 2014), relies on a rule-based framework in which the policy authorities are judged on whether they set their instrument in a manner consist with a legislated rule. I consider the performance of goal-based and rule-based frameworks. I first show analytically that both goal-based and rule-based systems balance a tradeoff between reducing sources of policy distortions and preserving policy flexibility. Then, using an estimated DSGE model, I find the optimal weights to place on goal-based and rule-based performance measures. When the rule is similar to that proposed recently in U.S. H.R. 5108, I find the optimal weight to assign to the rule-based performance measure is zero. However, when the rule is based on the output efficiency gap, it is generally optimal to make deviations from the rule a part of the central bank's performance measure.

JEL classification: E52, E61

1 Introduction

On 20 December 1989, the New Zealand Parliament gave unanimous approval to the Reserve Bank of New Zealand Act of 1989 (the Act), thereby formally inaugurating the world's first inflation targeting regime. The Act was part of a larger reform of governmental ministries, a reform designed to boost accountability by establishing clear objectives for government agencies. The assigned objective for the Reserve Bank was set out in Clause 8 of the Act:

> The primary function of the Bank is to formulate and implement monetary policy directed to the economic objective of achieving and maintaining stability in the general level of prices.

By establishing a numerical target for inflation, a process for communicating the target to the public through the Policy Target Agreement (PTA) between the government and the central bank, and a mechanism for accountability, the Act and the PTA contained all the key ingredients of inflation targeting.

The Act launched a global wave of central bank reforms that have clarified the policy responsibilities of central banks, increased their independence to implement policies consistent with their responsibilities, and provided clear measures of accountability against which their performance could be judged. These reforms have also promoted a greater level of transparency, transforming the way many central banks communicate their policy decisions and signal their future policy intentions. In general, accountability in inflation targeting regimes is strengthened by the public nature of the announced target and by the requirement that the central bank produce inflation reports or otherwise explain policy actions and their consistency with the announced target. Achieving the target becomes a measure of the central bank's performance.

Inflation targeting has now spread to almost 30 countries,[1] and many aspects that were pioneered in New Zealand – a public commitment to a target rate of inflation, high levels of transparency and accountability – are today considered best practice for monetary policy. The impact of New Zealand's reforms go beyond those central banks labeled as formal inflation targeters, as others, such as the Federal Reserve which has a dual mandate for price stability and maximum sustainable employment, now quantify the goal of price stability in terms of an announced, numerical goal for inflation. In fact, as many as 50 central banks now have quantitative targets or target ranges for inflation.[2] So the 25th anniversary of Reserve Bank of New Zealand Act of 1989 marks a landmark event in the history of central banking.

Inflation targeting itself has not remained a static policy framework since its birth. Further reforms in many countries, primarily related to increasing monetary policy transparency, have taken place, and experiences at the zero lower bound and with unconventional policy tools have forced some central banks to reconsider the way their policy decisions, and

---

[1]Combining the lists of Roger (2010) with that of Rose (2013) yields 28 inflation targeters.

[2]http://www.centralbanknews.info/p/inflation-targets.html

the information on which they are based, are conveyed to the public. Even away from the zero lower bound, developments in the theory of monetary policy have emphasized the importance of forward guidance (e.g., Woodford (2005), Woodford (2013)), and some inflation targeting central banks – here again the RBNZ has been in the forefront – provide information on the projected future path for the policy interest rate. Others, most notable the Federal Reserve and the Bank of England, have experimented with language designed to convey information about the circumstances that will trigger future increases in interest rates.

While widely adopted, inflation targeting has not won universal acceptance. Some critics have argued that inflation targeting has not mattered – that at least during the Great Moderation period, inflation targeters and non-targeters alike enjoyed similar improvements in macroeconomic performance.[3] Other critics argue it has mattered too much, blaming a focus on inflation as blinding central banks to the dangers of a finance crisis, thereby being part of the policy missteps that led to the global financial crisis of 2008-2009.

Proposals to reform inflation targeting or to replace it continue to be debated. Proposed reforms include giving the central bank new goals related to financial stability or replacing inflation as the primary goal with the price level or nominal income. These proposals are consistent with the general approach of inflation targeting in assigning goals to the central bank. They are also consistent with maintaining the central bank's independence to pursue its objectives, while the goals provide natural measures of performance that help ensure the central bank remains accountable.

A central bank's performance measure – the observable variable (or variables) by which the public and elected officials can judge whether the central bank has acted in a manner consistent with its charter – does not need to be based on an ultimate goal of monetary policy such as inflation. A central bank could be assigned and held accountable for achieving targets that are not themselves among the final goals of monetary policy. For example, in the 1970s, the U.S. Congress required the Federal Reserve to establish target growth rates for the money supply. Money growth rates are intermediate targets, neither an ultimate goal of policy nor something directly controlled as an instrument. Another alternative would be to judge the central bank's performance by comparing the central bank's instrument to the value prescribed by a legislated instrument rule. In fact, the U.S. House of Representatives recently held hearings on a bill that would establish an interest rate rule, with the Fed required to justify any deviations of the federal funds rate from the rule.[4] Taylor (2012) illustrates how an instrument rule can be used to assess ex post the Federal Reserve's policy.

Performance measures can differ, therefore, in terms of whether they focus on ultimate goals of macroeconomic policy while allowing for instrument independence, as is the case with inflation targeting, or whether they limit the instrument independence of the central

---

[3] An early paper to make this argument was Ball and Sheridan (2004).

[4] Hearings were held in July 2014. According to the Financial Times's report on Janet Yellen's February 25, 2015 testimony before the U.S. House Banking Committee, "the Fed chair swatted down calls from Republicans for the institution to be subject to mechanical rate-setting rules, saying she did not want its discretion to be 'chained'." Janet Yellen Defends US Central Bank Independence, 2/15/2015.

bank, as would be the case with a legislated instrument rule. Both inflation targeting and other goal-based regimes such as price-level targeting, speed limit policies, and nominal income targeting frameworks have been extensively analyzed in the literature.[5]. However, a similar analysis of regimes that base accountability on adherence to an instrument rule is absent from the literature, a gap the present paper seeks to fill.

Of course, there is a huge literature that studies the role of Taylor rules, and variants of Taylor's original rule (usually with the addition of the lagged interest rate) have become the standard method of specifying monetary policy to close general equilibrium models. Simple rules have played a large role in the literature on policy robustness (e.g., Levin and Williams (2003), Taylor and Williams (2010)). Ilbas et al. (2012) consider model uncertainty and show that including deviations of the policy rate from a simple rule can improve macroeconomic outcomes, allowing the central bank to cross-check it's policy against a rule that is potentially robust across a variety of different models.[6] However, they ignore any distortions to the central bank's objectives over inflation and the output gap that might arise from political pressures on monetary policy. These distortions play a central role in my analysis, while I ignore model uncertainty.

Tillmann (2012) is closest to the present paper in that he considers outcomes under discretion when the central bank minimizes a loss function that differs from social loss by the addition of a term reflecting deviations of the policy rate from the rate implied by a simple Taylor-type rule. He finds that some weight should be placed on this new terms when inflation shocks are serially correlated, a result similar to that of Clarida et al. (1999) who found a role for a Rogoff conservative central banker in a new Keynesian model only when inflation shocks were serially correlated. Walsh (2003a) shows that it can be optimal to place additional weight on inflation even when shocks are serially uncorrelated in the face of political distortions that cause the central bank's objectives to differ from those of society. These distortions generate a rationale for performance measures that is absent from the work of Tillmann (2012).

The rest of the paper is organized as follows. Section 2 reviews the objectives that central bank reforms such as the RBNZ Act were designed to address. Understanding the reason for reform is critical for evaluating the appropriate nature of any reform. An important distinction that arises is whether central bank reform is designed to constrain the central bank or to constrain the government. I then consider two forms of reform. The first (and standard) type emphasizes the assignment of goals to the central bank. The second approach proposes instrument rules that the central bank should follow. These two alternatives are illustrated using a simple model that allows analytic results to be derived.

---

[5]For example, Vestin (2006) provides an early analysis of price-level targeting, Walsh (2003b) compares price level targeting, output gap growth rate (speed-limit) policies, and nominal income policies, and Billi (2013) studies nominal income policies in the face of the zero lower bound on nominal interest rates.

[6]The monetary policy loss function incorporated into the Norges Bank's DSGE model (N.E.M.O.) actually adds a term of the form $(i_t - i_t^*)^2$. Previous version of N.E.M.O. set i_t^* equal to the value given by a simple instrument rule. Currently i_t^* is equal to the "normal" nominal interest rate, defined as the rate consistent with inflation equal to target and a zero output gap. This term is intended to add an implicity weight on financial imbalances in plicy determination. See Lund and Robstad (2012) and Evjen and Kloster (2012).

To evaluate the alternatives in a more realistic setting, a model incorporating sticky wages and sticky prices is employed in section 4. Parameter values and the relative volatility of alternative shocks, which the simple model showed are important for the evaluation, are obtained by estimating the model using Bayesian techniques.

The analytical results suggest both goal-based and rule-based systems must balance the same trade-off between reducing the impact of distortionary shocks to the central bank's policy objectives (arising, for example, from short-run political pressures) and allowing flexibility to pursue welfare-improving stabilization policies. The findings from the estimated DSGE model highlight the importance of the output measure used in the legislated rule. If the gap between output and its efficient level appears in the rule, judging performance by a comparison of inflation to its assigned target and the policy instrument to the recommendations of the rule both play a role in the optimal policy framework. When the rule takes the form proposed in the recent Congressional hearings, it is never optimal to use the rule to assess the central bank's performance. Conclusions are summarized in section 5.

2   Central bank reforms: goals, rules, independence, and accountability

Central bank reforms over the past 25 years have been aimed at removing, or at least reducing, the causes of poor monetary policy outcomes. Understanding the nature of the distortions that have produced poor policy is important for assessing the relative advantages or disadvantages of different types of reforms.

Three types of distortions have loomed large in monetary policy discussions. First, short-term political pressures, often related to a country's election cycle, can distort monetary policy decisions, resulting in an emphasis on near-term economic activity at the cost of longer-term objectives. Given that monetary policy operates with long lags, a central bank buffeted by short-term political pressures might have difficulty in achieving longer-term objectives, including low and stable inflation. And, if monetary policy has its primary effects on inflation through its influence on real economic activity, expansionary policies would first produce an economic boom, with inflation coming only later. This potentially creates an incentive for politicians to pressure central banks for expansionary policies timed to election cycles; a boom leading up to an election would benefit incumbents, while the inflationary costs would only be incurred later.[7] In this case, achieving medium-term inflation objectives would be incompatible with central banking regimes subject to political pressures.

Second, real economic distortions can cause inefficiencies that create a systematic bias towards policies aimed at expanding economic activity. For example, in standard new Keynesian models, monopolistic competition in goods and/or labor markets mean the economy's level of economic activity in a zero-inflation environment is too low relative to its efficient level. Real frictions in financial markets or in labor markets characterized by search and

---

[7]An extensive coverage of political business cycles models can be found in Drazen (2000).

matching frictions may also generate wedges between the economy's efficient allocation and the allocation arising with flexible prices and wages. While monetary policy can attempt to close these wedges in the short run by deviating from a policy of price stability, it cannot systematically and sustainably close them. Attempts to do so will ultimately fail, leaving the economy with excessively volatile inflation. Distortions arising from real economic inefficiencies and those due to political pressures on central banks may be closely related; the presence of real distortions may explain why politicians seek to influence monetary policy.

And third, even in the absence of political pressures or attempts to use monetary policy to achieve unachievable objectives, policy makers may lack the ability to commit credibly to future policies, leading to inefficient intertemporal policy responses to distortionary shocks. That is, even if the first two distortions are prevented from affecting monetary policy, the inability to commit to future actions will result in inefficient stabilization policies. The distortions resulting from discretionary policy played a large role in the academic literature seeking to explain why political pressures or the pursuit of unachievable objectives would lead to undesirably high inflation.[8] In the Barro-Gordon framework, popular at the time of the RBNZ Act in academic work on the inflation bias of discretion, removing short-term political pressures and assigning achievable goals to the central bank also succeeded in eliminating the distortion due to discretion. However, in new Keynesian models, with their emphasis on forward-looking expectations, discretion continues to produce inefficient outcomes even in the absence of political pressures or unsustainable goals.

Given these three potential sources of policy distortions, what types of central banking reforms might lead to improved monetary policy outcomes? I focus on two alternatives, both of which can be viewed as establishing a performance measure for the central bank. Performance measures provide metrics based on observable variables for evaluating the central bank's policy choices.[9] The definition of the performance measure is an important aspect of central bank reform: it affects the central bank's policy actions and is the basis for ensuring accountability in the conduct of policy.

The first type of reform, reforms such as inflation targeting, emphasize policy goals. An ultimate goal of policy serves as the measure of the central bank's performance. The second type emphasizes rules, with adherence to the rule the basis for assessing the central bank's performance. Using an instrument rules such as the Taylor Rule to evaluate the central bank is an example of a rule-based performance measure. In either case, the power of the performance measure indicates how important the measure is in the overall assessment of policy. For example, a strict inflation targeting regime in which the central bank is instructed to care only about achieving the target is an example of a high-powered regime.

The model of reform provided by the Reserve Bank of New Zealand Act and the Policy Targets Agreement focused on an ultimate goal that could be achieved by monetary policy.

---

[8] See Chapter 7 of Walsh (2010) for a survey of the literature on the inflation bias resulting from discretionary policies in models based on the time-inconsistency of optimal policy analysis of Kydland and Prescott (1977) as applied to monetary policy in the framework of Barro and Gordon (1983). See also Cukierman (1992).

[9] For the theory of performance measures, see Baker (1992), Baker et al. (1994), and Frankel (2014).

It did so by creating a contract between the elected government and the central bank designed to affect the policy choices of the Reserve Bank by altering the incentives of both the government and the central bank.[10] Incentives were affected by publicly establishing a clear policy goal, assigning responsibility for achieving it to the Reserve Bank, and establishing a system of accountability based on the goal. The elected government could alter the Bank's goal by changing the Policy Targets Agreement, but this had to be done in a public manner, and the government could not interfere in the implementation of monetary policy. The Act, together with the Policy Targets Agreement, created a performance measure for the Reserve Bank; it was to be evaluated on the basis of the consistency between its policy actions and the achievement of its inflation target.

A contract of this form could solve two and possibly all three of the distortions that had led to poor monetary policy. First, the public nature of the goal would help insulate the central bank from political pressures to pursue other objectives. By granting the Reserve Bank a high level of instrument independence to implement policy, the Act further limited the scope for short-term political factors to influence policy decisions. In other words, the Act served to constrain elected officials. In fact, in discussing the origins of inflation targeting in New Zealand, Sherwin (1999) credits the desire of Roger Douglas to make "monetary policy less susceptible to manipulation for short-term political ends." (p. 1).[11] The view ascribed to Douglas was consistent with empirical evidence pointing to a negative relationship among developed economies between average rates of inflation and measures of central bank independence.[12] Thus, a key characteristic of the reform was to increase central bank independence to constrain elected governments from influencing the implementation of monetary policy.[13]

While greater independence may shield monetary policy from political influences, it cannot ensure policy is only directed towards achieving obtainable goals. An independent monetary authority who wishes to promote social welfare may still face a temptation to pursue unsustainable objectives if, for example, real distortions imply steady-state output is inefficiently low.[14] So the Act assigned a specific goal to the Reserve Bank – price stability – that monetary policy could achieve. Sherwin (1999) quotes the report of the Parliamentary Finance and Expenditure Committee as stating "The Committee.... is firmly of the view that the primary function of monetary policy should be that set out in clause 8(i). (quoted

---

[10] Walsh (1995a) and Walsh (1995b).

[11] "The *process of delegation* through which the government assigns immediate responsibility for the conduct of monetary policy to a central bank is a means of restricting the strategy space available to the *government*." (Walsh (1995b) p. 240, emphasis in original)

[12] Important papers on this relationship include (Bade and Parkin (1984), Cukierman et al. (1992), Alesina and Summers (1993). See also Cukierman (1992). Criticism of the view that central bank independence is a solution to high inflation is argued by Posen (1993). The negative relationship between indexes of central bank independence and inflation held only for developed economies.

[13] Carlstrom and Fuerst (2009) find increases in central bank independence can account for 2/3rds of the better inflation performance among industrialized economies over past 20 years.

[14] The academic literature based on the model of Barro and Gordon (1983) generally did not distinguish between politically generated pressures for economic expansions and socially efficient but unsustainable attempts by the central bank to generate expansions. Both were captured by assuming that, even with flexible prices and wages, the economy's output would be below the desired level.

above) Members acknowledge that monetary policy should not be made to wear the cost of inappropriate fiscal and micro-economic policies. Monetary policy at the end of the day can only hope to achieve one objective, that is, price stability." Thus, the reforms instituted by the RBNZ Act focused on an achievable *goal* of monetary policy while allowing the central bank the independence to achieve this goal. The Act did not seek to constrain the Reserve Bank in its decisions about the appropriate policy stance required to achieve price stability. It instead removed from the Reserve Bank the authority to set its own goals. In the terminology of Debelle and Fischer (1994), the Act established a central bank that lacked goal independence but enjoyed instrument independence.

This type of reform – clear specification of goals together with greater central bank independence – became common during the 1990s.[15] Making the goals public helps to promote accountability, particularly if the central bank is assigned a single policy goal such as price stability or a target for inflation. Independence also has the potential to make the central bank less accountable, so Debelle and Fischer (1994) argued that independence needed to be limited and that independence to set instruments but not to define goals offered the best blue print for central bank reform.

Neither the assignment of goals nor instrument independence addresses directly the distortions that arise when policy makers are unable to commit to future actions. In the special case of the model of Barro and Gordon (1983), however, all three distortions could be addressed by giving the central bank instrument independence and holding it accountable based on the realized rate of inflation (Walsh (1995a)), or equivalently, by assigned it the right inflation target (Svensson (1997)). When private sector expectations are forward looking, inflation targeting alone does not solve the distortion that arises from discretionary policy. However, as policy makers and academics increasingly understood the important role expectations of future inflation play in controlling current inflation, and the role the expected future path of the policy interest rate plays in affecting the real economy, central banks placed greater emphasis on being transparent, systematic, and predictable in their actions. Doing so helped them gain greater influence over the private sector's expectations. Thus increases in transparency have been common (Crowe and Meade (2007), Blinder et al. (2008), Geraats (2009), Cukierman (2008), Dincer and Eichengreen (2014)). By being better able to influence future expectations, central banks are also partially able to overcome this third distortion.

To summarize, goal-based regimes are typically associated with instrument independence. Making goals public constrains the government, but if the central bank is judge only on the basis of the goal, as would the case with strict inflation targeting, it can also restrict the flexibility of the central bank. In the case of New Zealand, it is clear that the RBNZ is to be a flexible inflation targeter. This flexibility is reflected in the addition in 1999 of Clause 4(c) to the PTA; this clause states that "In pursuing its price stability objective, the

---

[15]The movement of many central banks towards greater independence and transparency is discussed by Crowe and Meade (2007) and Blinder et al. (2008). See Dincer and Eichengreen (2014) for an updates measiure of transparency that illustrates this trend.

Bank shall implement monetary policy in a sustainable, consistent and transparent manner and shall seek to avoid unnecessary instability in output, interest rates and the exchange rate." A further characteristic of goal-based regimes is that they are likely to be robust, as changes in the economy's structure may affect the monetary transmission process and alter the manner in which policy instruments are adjusted as functions of the state of the economy, but such changes do not alter the ultimate goals of policy.

Central bank reforms emphasizing goals, instrument independence, transparency and accountability are not the only shape reforms could have taken. An alternative could focus on assigning objectives that, unlike price stability, are not among the ultimate objectives of macroeconomic policy. For example, during the 1970s and 1980s, the role of intermediate targets in monetary policy implementation was widely discussed, and proposals for establishing target growth rates for various monetary aggregates were common. In 1975, a U.S. House of Representatives Concurrent Resolution called on the Federal Reserve to publicly announce monetary growth targets. The Full Employment Act of 1978 mandated publicly announced, annual growth targets for the money supply, and the Federal Reserve was required to report to Congress on its success in achieving the targets.[16] The Federal Reserve was assigned an objective – monetary growth targets – and in principle was held accountable for achieving these objectives, but the resulting targets were not among the ultimate goals of macroeconomic policy. However, the Fed was allowed to define its growth rate targets, weakening the target's role in constraining the Fed and in promoting accountability. Any constraining effect of announced monetary growth targets was further weakened by the Fed's practice of rebasing the level of the target path for monetary aggregates annually, ensuring that past target growth rate misses were compounded into the level of the monetary aggregates.[17]

Intermediate targets generally served as poor performance measures for monetary policy as the correlation between the targets and the ultimate objectives of monetary policy was often weak. In the U.S., rapid monetary growth combined with falling inflation in the early 1980s made the aggregate targets poor guides for policy, and the practice of base drift, while allowing the Fed greater flexibility in setting policy, weakened the usefulness of monetary growth rate targets as a means of ensuring policy accountability.[18]

Another alternative to making inflation the central bank's performance measure is to assess policy by comparing the central bank's setting of its instrument to a benchmark rule for the policy instrument. Such a rule-based system, in the extreme, eliminates any instrument independence and removes discretion from the policy process, directly solving any problems that arise from allowing policy makers discretion in implementing policy. In fact, Barro and Gordon (1983) and Canzoneri (1985) long ago argued that, absent private

---

[16]See Walsh (1987).

[17]For an analysis of base drift and the conditions under which it can be appropriate, see Walsh (1986). Inflation targeting leads to a similar situation in that the price level is allowed to be non-stationary. For some evidence that this is the practice in Australia, New Zealand, Sweden and the U.K. but not Canada, see Ruge-Murcia (2014).

[18]In a similar manner, inflation targeting weakens accountability if price stability is the actual goal, as it is in many central bank charters.

central bank information about the state of the economy, the central bank should have no discretion but instead be required to follow a rule that delineates the actions it should take as a function of the state of the economy.[19]

Some rules, such as the gold standard or an exchange rate peg, remove discretion completely from the hands of the central bank. But just as an inflation targeting regime does not need to be a regime of strict inflation targeting, a rule-based system does not need to be a strict (high powered) regime in the sense that the central bank is allowed absolutely no discretion. A flexible rule-based regime, much like flexible inflation targeting, would establish a rule but allow the central bank to deviate from the rule. Deviations would then need to be explained, or justified, by policy makers, just as a failure to meet an inflation target requires policy makers to explain why the target was missed. With the rule based on observable variables, such a system ensures accountability.[20] The power of the rule as a performance measure would depend on the weight given to such deviations in evaluating and holding accountable the central bank. The advantage of a rule-based system is that it increases the predictability of policy, is transparent, and simplifies the process of ensuring accountability.

Thus, if discretionary decisions by the central bank, and not political pressure from elected officials, are the source of poor monetary policy, reform must differ from the model provided by the RBNZ Act; it must constrain the central bank. As Tirole (1994) notes, rules are imposed when agents cannot be trusted with discretion. Legislating *rules* for the central bank to follow achieves this end by eliminating both goal and instrument independence. In a series of recent papers, John Taylor has argued that a commitment to a rule for monetary policy produces better outcomes than occur in regimes that emphasizes central bank independence (Taylor (2011), Taylor (2012), Taylor (2013)). He suggests overall macroeconomic performance was superior during periods in which the Federal Reserve acted in a systematic, predictable manner, and that forcing the Fed to adhere more closely to a rule would improve economic outcomes. After reviewing rules versus central bank independence, he concludes that "The policy implication is that we need to focus on ways to 'legislate' a more rule-based policy." p. 16, Taylor (2011))

Rule-based performance measures suffer from at least three potential problems. First, determining the right rule would be difficult. Even in quite simple theoretical models, the optimal instrument rule can be extremely complex (for example, see Woodford (2010)). A complex rule, even if known, might be hard to explain to the public, thereby reducing the ability of a rule-based performance measure to ensure policy transparency and accountability. Second, any optimal rule is optimal only with reference to a specific model, so changes in the economy's structure or our understanding of it will produce changes in the optimal rule. Third, it may not always be possible to characterize policy in terms of a single instrument

---

[19] Walsh (1995a) showed that aligning the central bank's incentives with observables such as inflation overcame the private information problem highlighted by Canzoneri (1985). Athey et al. (2005) revisit the rules verus discretion debate in the presence of private information.

[20] Taylor (2012) provides an example of how the Taylor Rule can be used to assess Federal Reserve performance.

rule. A rule for a short-term policy interest rate would no longer be meaningful if interest rates were at the zero lower bound, nor would it give guidance for balance sheet policies. Thus, instrument rules are likely to be less robust to structural changes than goal-based systems.[21] However, early work such as Levin et al. (1999) and Rudebusch (2002) suggested rules simple rules may be robust to model uncertainty. These considerations argue for adopting a simple but robust rule such as the Taylor Rule but one that also includes escape clauses.[22] Choosing which rule, and how accountability is to be maintained when the rule might not apply, must involve balancing the gains from limiting discretion against the costs of potentially forcing monetary policy to implement a bad rule.

Given the unprecedented actions by the Federal Reserve and other central banks during the financial crisis, it is not surprising that proposals have emerged for rule-based reforms designed to limit the Fed's discretion. In July 2014, hearings were held in the U.S. on H.R. 5018 which would impose several rule-based requirements on the Fed. First, the FOMC would be required to identify a Directive Policy Rule (DRP). The DRP would identify the policy instrument, "describe the strategy or rule of the Federal Open Market Committee for the systematic quantitative adjustment of the Policy Instrument Target to respond to a change in the Intermediate Policy Inputs." (Section 2C(c)(2). Intermediate Policy Inputs, defined in Section 2C(a)(4), include "any variable determined by the Federal Open Market Committee as a necessary input to guide open-market operations" but must include current inflation (together with its definition and method of calculation) and at least one of (i) an estimate of real, nominal or potential GDP, (ii) an estimate of a monetary aggregate, or (iii) an interactive variable involving the other listed variables. In addition, the Directive Policy Rule must "include a function that comprehensively models the interactive relationship between the Intermediate Policy Inputs (Section 2C(c)(3));" and "the coefficients of the Directive Policy Rule ... (Section 2C(c)(4))"

Perhaps more significantly in terms of constraining the Fed's flexibility, the proposed legislation also defines a *Reference Policy Rule* (RPR) and Section 2C(c)(6) requires that the FOMC must report "whether the Directive Policy Rule substantially conforms to the Reference Policy Rule". If it doesn't, the FOMC will need to provide a "detailed justification" for any deviation of the Directive Policy Rule and the Reference Policy Rule.

The proposed bill is quite specific about the Reference Policy Rule. Section 2C,(a) 9 defines the Reference Policy Rule as the federal funds rate given by

$$i_t^{RPR} = \pi_{t-1} + 0.5 \ln \left(\frac{GDP_t}{GDP_t^{potential}} \right) + 0.5(\pi_{t-1} - 2) + 2 \tag{1}$$

---

[21]But alterations in the economy's structure can also affect policy goals. For example, a change in price indexation would change the definition of inflation volatility that generates inefficiencies and that should appear in the measure of social welfare.

[22]See also Taylor and Williams (2010). Svensson (2003) provides a general critique of relying on Taylor Rules, while Benhabib J. et al. (2001) argue Taylor Rules do not rule out ZLB equilibria.

where
$$\pi_{t-1} = 100 \left(\frac{p_{t-1} - p_{t-5}}{p_{t-5}} \right)$$

is the inflation rate over the previous four quarter. This rule can be rewritten as

$$i_t^{RPR} = 4 + 1.5 \left(\pi_{t-1} - 2 \right) + 0.5 \ln \left(\frac{GDP_t}{GDP_t^{potential}} \right).$$

Written in this form makes clear that it is the Taylor Rule (Taylor (1993)). If average inflation is equal to 2% and the gap between GDP and potential is zero, then the policy rate will equal 4%. Thus, the rule assumes an inflation target of 2% and an average real interest rate of 2%.

Federal Reserve Chairwoman Janet Yellen said, in testimony before the House Financial Services Committee (July 16, 2014), that "It would be a grave mistake for the Fed to commit to conduct monetary policy according to a mathematical rule." In contrast, John Taylor in a Wall Street opinion piece (WSJ July 9, 2014) argued in favor of the bill. Section 2C(e)(1) does allow that the Act is not meant to require the FOMC to implement the strategy set out in the legislation if the "Committee determines that such plans cannot or should not be achieved due to changing market conditions." If such a situation occurs, the FOMC would have 48 hours to provide the U.S. Comptroller General and Congress with an explanation and an updated Directive Policy Rule. In turn, the Comptroller General would then have 48 hours to conduct an audit and issue a report to determine whether the FOMC's updated Directive Policy Rule is in compliance with the bill.

The type of rule-based accountability in the proposal contrasts sharply with goal-based accountability and central bank independence that has characterized most central bank reforms since the 1989 Reserve Bank of New Zealand Act. Under rule-based accountability, the central bank is required to specify clearly its instrument and the rule it uses to determine the setting of that instrument. Deviation from the rule are allowed, but the central bank is required to explain the rationale for any such deviations. Under goal-based accountability, the objectives of the central bank are made clear – if these are set by the government, the central bank lacks goal independence – but in the pursuit of these goals the central bank enjoys instrument independence. In this case, the central bank is required to explain how its actions are consistent with achieving the goals.

Table 1 summarizes the general characteristics of goal-based and rule-based reforms. I exclude examples of reforms based on intermediate targets such as money growth rates as they are inefficient systems both for achieving ultimate goals and for restricting the central bank's instrument setting. Goal-based and rule-based reforms have different implications for a central bank and for macroeconomic outcomes. They differ in terms of the type of the independence the central bank enjoys, and they differ in terms of who they are designed to constrain. Both can allow for flexibility and both provide the public with the ability to assess policy and, in principle, hold the central bank accountable.

Table 1: Types of Central Bank Reforms

	Goals Based	Rules Based
Examples	Inflation targeting	Exchange rate pegs
	Price level targeting	Gold standard
		Instrument rules (H.R. 5018)
CB independence		
Goal	Varied	Low
Instrument	High	Low
Constrains	Central Bank	Central Bank
	Government	
Flexibility	Varied	Varied
Transparency	Varied	High
Accountability	High	High
Robustness	High	Low

3 The performance of goal-based and rule-based regimes

In this section, a simple model is used to highlight the tensions that arise between accountability and flexibility under different performance measures and to explore how these tensions are addressed by goal-based and rule-based accountability. While the model used is quite simple, it helps to illustrate the effects of different policy regimes, leaving to the following section the use of an estimated model to evaluate goal-based and rule-based systems.

Let π^* be the socially optimal steady-state inflation rate, taken as exogenous and constant for simplicity, and define $\hat{\pi}_t \equiv \pi_t - \pi^*$ as actual inflation relative to the optimal rate. Assume social loss is given by

$$L_t^s = \frac{1}{2}\mathrm{E}_0 \sum \beta^i \left(\hat{\pi}_{t+i}^2 + \lambda x_{t+i}^2\right),\tag{2}$$

where $x_t \equiv x_t - x^*$ is the (log) gap between output and the socially efficient output level. Policy is delegated to a central bank with instrument independence but subject to possible political pressures that affect the goals the central bank pursues. Specifically, assume that absent any assignment of a performance measure, the central bank acts to minimize

$$L_t^{cb} = \frac{1}{2}\mathrm{E}_t^{cb} \sum \beta^i \left[\left(\hat{\pi}_{t+i} - \varphi_{t+i}\right)^2 + \lambda \left(x_{t+i} - u_{t+i}\right)^2\right]\tag{3}$$

where φ and u are mean zero stochastic shocks that represent deviations of the central bank's objectives from their socially optimal values. These can be thought of as representing unmodeled political pressures affecting the policy choices of the central bank or simply as distortions introduced by the preferences of the central bank policy authorities. In keeping with the now common practice in the analysis of monetary policy, I assume a fiscal tax/subsidy policy is in place that eliminates any steady-state inefficiencies. Thus, I ignore distortions arising from attempts to systematically affect the level of steady-state output.

The economy is characterized very simply by a new Keynesian Phillips curve given by

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa x_t + e_t, \tag{4}$$

and an expectational Euler equation given by

$$x_t = E_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(\hat{\imath}_t - E_t \hat{\pi}_{t+1} - \phi_t\right), \tag{5}$$

where ϕ_t and e_t are taken to be exogenous stochastic processes. Equation (4) is consistent with the standard Calvo model if firms who do not optimally choose their price instead index their price to π^*. Under optimal discretionary policy with i.i.d. shocks, the appendix shows that the unconditional expected social loss is

$$L_t^s = \frac{1}{2}\left(\frac{1}{1-\beta}\right)\left[\left(\frac{\lambda}{\lambda+\kappa^2}\right)\sigma_e^2 + \left(\lambda^3 + \kappa^2\right)\left(\frac{1}{\lambda+\kappa^2}\right)^2 \left(\lambda^2 \sigma_u^2 + \kappa^2 \sigma_\varphi^2\right)\right] \tag{6}$$

In the absence of political distortions represented by u and φ (and maintaining the assumption of i.i.d. shocks), social loss would be

$$\frac{1}{2}\left(\frac{1}{1-\beta}\right)\left(\frac{\lambda}{\lambda+\kappa^2}\right)\sigma_e^2 \leq L_t^s.$$

I next investigate whether holding the central bank accountable for achieve a goal such as the inflation rate or for adhering to a rule for setting the instrument can help lower social loss.

3.1   Delegation

Government in a pre-game stage defines a performance measure for the central bank. A goal-based regime specifies the central bank's objectives in terms of π and/or x, the two ultimate objectives on which social welfare depends. A rule-base regime specifies that assessment of the central bank's performance is based on a comparison of the policy instrument and the value implied by a simple instrument rule. I represent each type of regime by assuming the central bank continues to have preferences over actual outcomes given by (3) but is also concerned with minimizing deviations of outcomes from the bank's assigned performance measures. The weights attached to these additional performance measures represent the

power of the respective measure. Nesting both regimes, the central bank is assumed to set policy under discretion to minimize

$$L_t^{cb} = \frac{1}{2} \mathrm{E}_t^{cb} \sum \beta^i \left[\left(\hat{\pi}_{t+i} - \varphi_{t+i} \right)^2 + \lambda \left(x_{t+i} - x_{t+i}^* \right)^2 + \tau \hat{\pi}_{t+i}^2 + \delta \left(i_{t+i} - i_{t+i}^r \right)^2 \right], \qquad (7)$$

where τ is the implicit weight placed on achieving the inflation target (equivalently, the degree of central bank conservatism in the terminology of Rogoff (1985)) and δ is the weight placed on setting the interest rate equal to i^r, the rate implied by the rule.[23] We can rewrite L_t^{cb} as

$$L_t^{cb} = \frac{1}{2} \mathrm{E}_t^{cb} \sum \beta^i \left[(1+\tau) \hat{\pi}_{t+i}^2 - 2\varphi_{t+i} \hat{\pi}_{t+i} + \lambda x_{t+i}^2 - 2\lambda u_{t+i} x_{t+i} + \delta \left(i_{t+i} - i_{t+i}^r \right)^2 \right],$$

where terms independent of policy have been dropped.[24]

Since private agents are forward-looking in making decisions, optimal policy under discretion will result in lower social welfare than would the fully optimal commitment policy. The distortionary shocks φ_{t+i} and u_{t+i} also reduce welfare. The question for central bank design is whether a goal-based system with $\tau > 0$ or a rule-based system with $\delta > 0$ can, in an environment of discretionary decision making, improve welfare. In other words, in a pre-game stage, would the government choose non-zero values of τ and/or δ if it wished to minimize (2)?

I first consider the case of a goal-based regime in which $\delta = 0$ but τ is chosen optimally. Then the case of a rule-based regime with $\tau = 0$ and δ chosen optimal is analyzed. Finally, the case in which both τ and δ are jointly chosen is considered.

3.2 The assignment of goals

When the government assigns objectives to the central bank based on realized inflation, we have the case studied in Walsh (2003a). The analysis in that paper only considered distortionary shocks affecting the output objective of policy (i.e., $u \neq 0$ but $\varphi \equiv 0$) and also assumed the central bank had imperfect information about cost shocks, an extension I ignore here.

With $\delta = 0$, the central bank's problem under discretion can be written as

$$\min_{\hat{\pi}_t, x_t, \hat{i}_t} \frac{1}{2} (1+\tau) \hat{\pi}_t^2 - \varphi_t \hat{\pi}_t + \frac{1}{2} \lambda x_t^2 - \lambda u_t x_t$$

subject to (4) and (5). The nominal interest rate i is the instrument of monetary policy. Shocks are assumed to be i.i.d.[25] It is straightforward to show that equilibrium inflation

---

[23]For simplicity, I only consider goal-based regimes defined in terms of inflation and not the output gap.

[24]For evidence that the Fed has implicitly placed some weight on the Taylor rule, see Kahn (2012) and Ilbas et al. (2013).

[25]The case of serially correlated shocks is dealt with in the numerical analysis of section 4 based on an estimated model.

and the output gap are given by[26]

$$\hat{\pi}_t = \left[\frac{\kappa \lambda u_t + \kappa^2 \varphi_t + \lambda e_t}{\lambda + \kappa^2 (1 + \tau)} \right]$$

$$x_t = \left[\frac{\lambda u_t + \kappa \varphi_t - \kappa (1 + \tau) e_t}{\lambda + \kappa^2 (1 + \tau)} \right].$$

The central-bank-design problem is to pick τ to minimize the unconditional expectation of social loss. The appendix shows that the optimal value of τ is given by

$$\tau^* = \left(\frac{\lambda + \kappa^2}{\lambda^2} \right) \left(\frac{\lambda^2 \sigma_u^2 + \kappa^2 \sigma_\varphi^2}{\sigma_e^2} \right) \geq 0. \tag{8}$$

If $\varphi_t \equiv 0$, (8) reduces to the case considered in Walsh (2003a). In this case, $\tau^* = \left(\lambda + \kappa^2 \right) \left(\sigma_u^2 / \sigma_e^2 \right)$ increases linearly in λ and in the volatility of the distortionary shock to policy makers' goals (σ_u^2) relative to the volatility of cost shocks (σ_e^2). In the absence of both distortionary shocks u and φ, $\tau^* = 0$, consistent with the findings of Clarida et al. (1999), who showed there is no gain from appointing a Rogoff conservative central banker when the cost shock is serially uncorrelated. When distortionary shocks are present, τ^* is positive even when shocks are serially uncorrelated. The greater the variability of the political distortions represented by u and φ, the larger is the optimal τ and the more the central bank needs to be made accountable based on $\hat{\pi}_t$. Equivalently expressed, the more variable the wedge between social objectives and goals pursued by the central bank, the more high-powered (or the stricter) the inflation targeting regime needs to be.

A rise in the volatility of cost shocks increases the potential value of stabilization policy and so τ^* falls, as a more flexible flexible inflation targeting regime is desirable. With more potential gain from flexibility, the optimal regime assigns less weight to achieving the inflation target. Importantly, τ^* is independent of aggregate demand shocks operating through the expectational IS relationship, as the central bank always has an incentive to neutralize the impact of such shocks on inflation and the output gap.

3.3 The assignment of rules

Now suppose a legislated instrument rule is used to access the central bank's performance. In contrast to objectives based on an ultimate goal such as inflation, the central bank's objectives are distorted based on how it sets its actual policy instrument. In terms of (7), $\tau = 0$ but δ may be non-zero. The central bank's problem takes the form

$$\min_{\hat{\pi}, x, \hat{i}} \left[\frac{1}{2} \hat{\pi}_t^2 - \varphi_t \hat{\pi}_t + \frac{1}{2} \lambda x_t^2 - \lambda u_t x_t + \frac{1}{2} \delta (i_t - i_t^r)^2 \right]$$

subject to (4) and (5). Because the central bank is judged in part on how it sets its instrument, the expectational IS equation becomes relevant for its policy choice. Assume

---

[26]See the appendix for details.

that the reference rule is defined by

$$i_t^r = \psi_\pi \hat{\pi}_t + \psi_x x_t.$$

The appendix shows that the first order conditions for the central bank's problem imply

$$i_t = i_t^r + \frac{1}{a\delta} \left[\kappa \left(\hat{\pi}_t - \varphi_t \right) + \lambda \left(x_t - u_t \right) \right],$$

where

$$a \equiv \sigma + \psi_x + \kappa \psi_\pi.$$

In the absent of the rule-based performance measure, the central bank would set the term in brackets equal to zero. The greater the value of δ – that is, the more costly it becomes for the central bank to deviate from the reference policy rule – the smaller the role this unconstrained optimality condition plays in the setting of i_t and the closer i_t comes to equaling the benchmark rule value.

For the case of serially uncorrelated shocks, equilibrium inflation and the output gap are equal to

$$\hat{\pi}_t = \left[\frac{\kappa\alpha\delta\phi_t + \kappa\lambda u_t + \kappa^2\varphi_t}{\lambda + \kappa^2 + a^2\delta} \right] + \left[\frac{\lambda + a\delta\left(\sigma + \psi_x\right)}{\lambda + \kappa^2 + a^2\delta} \right] e_t$$

$$x_t = \frac{\alpha\delta\phi_t + \lambda u_t + \kappa\varphi_t - \left(\kappa + a\delta\psi_\pi\right)e_t}{\lambda + \kappa^2 + a^2\delta},$$

and social loss is

$$\mathcal{L} = \frac{1}{2}a^2\left(\lambda + \kappa^2\right)\left[\frac{\delta}{\lambda + \kappa^2 + a^2\delta}\right]^2\sigma_\phi^2 + \frac{1}{2}\lambda^2\left(\lambda + \kappa^2\right)\left[\frac{1}{\lambda + \kappa^2 + a^2\delta}\right]^2\sigma_u^2$$

$$+ \frac{1}{2}\kappa^2\left(\lambda + \kappa^2\right)\left[\frac{1}{\lambda + \kappa^2 + a^2\delta}\right]^2\sigma_\varphi^2 + \frac{1}{2}\left\{\frac{\left[\lambda + a\delta\left(\sigma + \psi_x\right)\right]^2 + \lambda\left[\kappa + a\delta\psi_x\right]^2}{\left[\lambda + \kappa^2 + a^2\delta\right]^2}\right\}\sigma_e^2.$$

Minimizing $\mathcal{L}$ with respect to δ implies the optimal weight on the rule-based objective is (see the appendix)

$$\delta^* = \frac{\left(\lambda + \kappa^2\right)\left(\lambda^2\sigma_u^2 + \kappa^2\sigma_\varphi^2\right)}{\left(\lambda + \kappa^2\right)^2\sigma_\phi^2 + \Lambda\sigma_e^2}, \tag{9}$$

where

$$\Lambda \equiv \left(\sigma + \psi_x\right)\left[\left(\sigma + \psi_x\right)\kappa^2 - \kappa\psi_\pi\lambda\right] + \lambda\psi_x\psi_x\left(\lambda + \kappa^2\right) - \left(\sigma + \psi_x + \kappa\psi_\pi\right)\lambda\psi_x\kappa. \tag{10}$$

To help interpret the expression for δ^*, assume initially that there are no aggregate demand shocks ($\phi \equiv 0$). In this special case,

$$\delta^* = \left(\frac{\lambda + \kappa^2}{\Lambda}\right)\left(\frac{\lambda^2\sigma_u^2 + \kappa^2\sigma_\varphi^2}{\sigma_e^2}\right). \tag{11}$$

Comparing (11) to (8) shows that both depend on $\left(\lambda + \kappa^2\right)\left(\lambda^2\sigma_u^2 + \kappa^2\sigma_\varphi^2\right)/\sigma_e^2$; as the vari-

ability of distortionary shocks u and φ increases relative to the variability of cost shocks e, the optimal τ^* and the optimal δ^* both increase. They do so for the same reason: allowing the central bank less flexibility becomes desirable when distortionary shifts in goals are more variable. The optimal τ^* and δ^* are both decreasing in the volatility of inflation shocks; as the scope for welfare-improving stabilization policy increases, the cost of distorting the central bank's objectives either by requiring it to place more weight on inflation variability or on matching the benchmark instrument rule becomes more costly.

The expression for δ^* given in (11) was derived for arbitrary policy response coefficients ψ_x and ψ_π. Suppose instead that these were optimally chosen. For example, continuing with the special case of no demand shocks and serially uncorrelated cost and distortionary shocks, the optimal interest rate rule can be expressed in terms of a reaction to either the output gap or to inflation, that is, only one response coefficient is needed. Let $\psi_x = 0$; the optimal response to inflation is then equal to $\psi_\pi^* = \sigma\kappa/\lambda$. One can show that

$$\lim_{\psi_\pi \to \psi_\pi^*} \delta^* \to \infty.$$

When the benchmark rule is equal to the optimal rule and there are no aggregate demand shocks, the central bank should not be allowed any flexibility.

Equation (11) applied when there were no shocks to the Euler equation, corresponding to the case of a constant equilibrium real interest rate. In the presence of shocks to the equilibrium real interest rate (i.e., $\phi \neq 0$), the optimal penalty on deviations from the rule can be written as

$$\delta^* = \left(\frac{\lambda + \kappa^2}{\Delta}\right) \left(\frac{\lambda^2\sigma_u^2 + \kappa^2\sigma_\varphi^2}{\sigma_e^2}\right) = \left(\frac{\lambda^2}{\Delta}\right)\tau^*,$$

where

$$\Delta \equiv \Lambda + \left(\lambda + \kappa^2\right)^2 \left(\frac{\sigma_\phi^2}{\sigma_e^2}\right) \geq \Lambda.$$

Thus, demand shocks ($\sigma_\phi^2 > 0$) call for putting less weight on deviations from the rule. This result is very intuitive – the specified rule does not allow for interest rate movements directly in response to demand shocks; an optimal policy would. Therefore, as demand shocks become a larger source of volatility, the optimal δ falls. If $\psi_x = 0$ and $\psi_\pi = \psi_\pi^*$ so that the assigned rule is consistent with the optimal response to inflation shocks, $\Lambda = 0$ and

$$\delta^* = \left(\frac{1}{\lambda + \kappa^2}\right) \left(\frac{\lambda^2\sigma_u^2 + \kappa^2\sigma_\varphi^2}{\sigma_\phi^2}\right) \geq 0.$$

In this case, the optimal value of δ is non-negative, independent of inflation shocks, but decreasing in the variance of demand shocks.

3.4 Jointly optimal goals- and rule-based regimes

The special cases just considered showed how setting τ and δ both involve a similar trade-off between the benefits of reducing flexibility to limit distortions and the costs of reducing the ability of the central bank to pursue socially desirable stabilization policies. The dependence of the power of goal-based and rule-based measures on the relative volatility of underlying shocks is reminiscent of the classic Poole results on instrument choice (Poole (1970)). Poole showed that an interest rate instrument performed better than a monetary aggregate instrument in the face of financial market shocks, while the reverse was true in the face of aggregate demand disturbances. In a similar manner, equations (8) and (9) suggest a goal-based performance measure may be best if shocks to aggregate demand dominate, while a rule-based measure may have advantages if shocks to inflation dominate. In general, Poole's analysis implies optimal simple rules will depend on the relative variances of the model's underlying shocks.[27] Similarly, one might expect that the weight to give to a goal-based performance measures relative to a rule-based measure may depend on the relative volatility of the model's shocks. The fact that, as shown by (8) and (9), the optimal τ is independent of demand shock volatility but decreasing in cost shock volatility while δ is decreasing in the volatility of demand shocks suggests there might be potential gains from using both forms of performance measures.

To assess the joint determination of the optimal values of τ and δ, I set $\kappa = 0.172$, consistent with a Calvo model of price adjustment with the fraction of non-optimally-adjusting firms equal to 75% per quarter combined with log utility ($\sigma = 1$) and a Frisch elasticity of labor supply of 1. For the baseline, I set the standard deviations of all the shocks equal to 0.025. The parameters of the rule are set equal to their Taylor-values of $\psi_\pi = 1.5$ and $\psi_x = 0.125$. I then solve numerically for the values of τ^* and δ^* that minimize the unconditional expectation of social loss, given by (2). I set λ equal to the value appropriate if (2) is interpreted as a second-order approximation to the welfare of the representative household.[28]. The analytic results for the optimal values of τ and δ taken individually showed that the variances of demand and cost shocks played a key role, so I investigate how variations in these variances affect the optimal power of the goal-based versus rule-based regimes.

To assess the relative roles of τ and δ when both are chosen optimally, I report the ratio of their optimal values as the variances of the disturbances vary. Figure 1 plots τ^*/δ^* as a function of the variances of the fundamental demand and cost shocks σ_ϕ^2 and σ_e^2. Both τ^* and δ^* are positive, indicating a role for goals and rules, but as suggested by (8) and (9), the relative weight on goals as measured by τ rises as demand shocks increase in volatility, while the weight on rules as measured by δ rises as cost shocks become more volatile. For the parameters considered here, however, the weight given to deviations from the inflation

---

[27] See Walsh (2010), pp. 513-521.

[28] This implies a value of λ equal to $(\kappa/\theta^p)(1+\eta)/(1-a)$, where θ^p is the price elasticity of demand faced by firms, η is the inverse wage elasticity of labor supply, and $1-a$ is the elasticity of output with respect to labor. For $\theta^p = 9$, $\eta = 1$ and $a = 0.3$, this implies $\lambda = 0.0545$. See (21).

target in assessing the central bank's performance is much larger than the optimal weight placed on deviations from the Taylor rule.

According to (8) and (9), an increase in $\lambda^2 \sigma_u^2 + \kappa^2 \sigma_\varphi^2$ – that is an increase in the volatility of the distortionary shifts in objectives – would increase τ^* when $\delta = 0$ and δ^* when $\tau = 0$. In fact, these two equations imply the ratio between τ^* and δ^* is independent of the volatility of the distortionary shocks u and φ but depends on the relative variances of demand and cost shocks:

$$\frac{\tau^*}{\delta^*} = \left(\frac{\lambda + \kappa^2}{\lambda^2} \right) \left(\frac{\sigma_\phi^2}{\sigma_e^2} \right) + \frac{\Lambda}{\lambda^2}.$$

This continues to be true when τ and δ are optimally chosen jointly; they both increase with the volatility of the distortionary shocks u and φ, rising proportionately so that their ratio remains constant as $\lambda^2 \sigma_u^2 + \kappa^2 \sigma_\varphi^2$ increases. Thus, Figure 1 is independent of $\lambda^2 \sigma_u^2 + \kappa^2 \sigma_\varphi^2$. While the optimal measure of performance places some weight on deviations from the inflation goal and deviations from the interest rate rule, the fundamental choice between a goal-based and a rule-based performance measure depends on the relative importance of the underlying shocks to private sector consumption and price setting behavior.

3.5   Conclusions from the simple model

The simple model utilized in this section suggests that when political (or other) pressures cause transitory distortions to the objectives the central bank pursues relative to society's goals, there can be a role for both goal-based reforms and rule-based reforms. Both establish performance measures that affect the central bank's incentives and therefore affect policy choices. When each type of reform is considered in isolation, analytical expressions could be obtained for the optimal weight to place on achieving stable inflation and for punishing deviations from the Taylor Rule. These expression for τ^* and δ^* showed that increases in the variance of shocks that distorted the central bank's objectives called for increasing the power of both types of accountability measures. Increased volatility of cost shocks reduces the weight that should be placed on inflation goals as limiting the flexibility to respond to these shocks becomes more costly. Under goal-based accountability, demand shocks do not affect the optimal power as the central bank already has an incentive to neutralize demand shocks. In contrast, demand shocks reduce the optimal power of the rule-based system since the Taylor Rule does not allow for shifts in the equilibrium real rate of interest.

4   Goals and rules in an estimated model with sticky prices and wages

The previous section considered the use of goal-based and rule-based policy regimes using a very simply model in which some analytical results could be obtained and some results required a calibrated version of the model. In this section I consider the effects of τ and δ in an estimated new Keynesian model of sticky prices and wages based on Erceg et al.

(2000) (henceforth, EHL). As was clear from the expressions for τ^* and δ^* obtained in the previous section, their values will depend importantly on the relative volatility of different shocks. Thus, obtaining these values from an estimated model will provide a more realistic assessment of the performance of goal- versus rule-based incentive systems.

The basic model is standard and details of its derivation can be found in Erceg et al. (2000) or Chapter 6 of Galí (2008). The model takes the following form:

$$y_t = \mathrm{E}_t y_{t+1} - \left[i_t - \mathrm{E}_t \pi_{t+1} - \left(1 - \rho_\chi \right) \chi_t \right] \tag{12}$$

$$\left(1 + \beta \delta_p \right) \pi_t = \beta \mathrm{E}_t \pi_{t+1} + \delta_p \pi_{t-1} + \kappa_p \left(\omega_t - mpl_t + \mu_t^p \right) \tag{13}$$

$$\left(1 + \beta \delta_w \right) \pi_t^w = \beta \mathrm{E}_t \pi_{t+1}^w + \delta_w \pi_{t-1}^w + \kappa_w \left(mrs_t + \mu_t^w - \omega_t \right) \tag{14}$$

$$\omega_t = \omega_{t-1} + \pi_t^w - \pi_t + e_{z,t} \tag{15}$$

$$mpl_t = -a h_t \tag{16}$$

$$mrs_t = y_t + \eta h_t - \chi_t \tag{17}$$

$$y_t = \left(1 - a \right) h_t \tag{18}$$

$$g_t = y_t - y_{t-1} + e_{z,t}, \tag{19}$$

where y is output, ω the real wage, π inflation π^w wage inflation, mpl, the marginal product of labor, mrs the marginal rate of substitution between leisure and consumption, h hours and g is the growth rate of output. Aggregate productivity is assumed subject to a random walk process with innovation $e_{z,t}$, so output, the real wage, the marginal product of labor and the marginal rate of substitution between leisure and consumption are all defined as log deviations from the permanent component of productivity. Other variables are expressed as log deviation from their steady state values (including zero steady-state rates of price and wage inflation). χ, μ^p, and μ^w are stochastic shocks to the marginal utility of consumption, price markups and wage markups, all assumed to follow AR(1) processes with, for example, ρ_χ denoting the AR(1) coefficient for χ and $e_{\chi,t}$ denoting its innovation. The first equation is a standard Euler condition linking the marginal utility of consumption in periods t and $t + 1$. The next two equations are reduced form expressions for price and wage inflation, where δ_p and δ_w are the degrees of indexation in price and wage setting. The parameter η is the inverse wage elasticity of labor supply; $1 - a$ is the elasticity of output with respect to hours, the only variable input to production. To be consistent with the assumed unit

21

root process in productivity, the elasticity of intertemporal substitution in consumption is set equal to one.

The elasticity of inflation with respect to real marginal cost is equal to

$$\kappa_p = \frac{\left(1 - \varphi^p\right)\left(1 - \beta\varphi^p\right)}{\varphi^p} \frac{1 - a}{1 - a + a\theta^p}$$

where $1 - \varphi^p$ is the fraction of firms optimally adjusting price each period and θ^p is the price elasticity of demand facing individual firms. Similarly, the elasticity of wage inflation with respect to the gap between the marginal rate of substitution between leisure and consumption and the real wage is

$$\kappa_w = \frac{\left(1 - \varphi^w\right)\left(1 - \beta\varphi^w\right)}{\varphi^w} \frac{1}{1 + \eta\theta^w},$$

where $1 - \varphi^p$ is the fraction of wages optimally adjusting each period and θ^w is the wage elasticity of demand for individual labor types.

For estimation purposes, the model is closed with a specification of monetary policy, where the nominal interest rate i is treated as the policy instrument. I assume a standard Taylor Rule with inertia of the form

$$i_t = \rho_i i_{t-1} + \left(1 - \rho_i\right)\left(\phi_\pi \pi_t + \phi_g y_t\right) + v_t$$

where v is an exogenous policy shock.

4.1   Estimation

The model is estimated by Bayesian methods over the period 1984:1-2007:4, corresponding to the Great Moderation. A similar version of the EHL model has been estimated over 1984:1-2008:2 by Casares et al. (2011). I base my priors partially on their results, but I follow Chen et al. (2012) in choosing prior distributions of beta for parameters constrained to be between 0 and 1 and gamma for parameters that should be positive. Output growth, inflation, wage inflation and the nominal interest rate are treated as observables. Output is measured by chained real GDP deflated by the civilian population aged 16 and over. Inflation is measured by the log change in the GDP deflator, while wage inflation is the log change in hourly compensation in the non-farm business sector. The interest rate is the effective federal funds rate. All four observables are measured a quarterly rates. The values $\sigma = 1$, $\beta = 0.99$, $a = 0.36$, $\theta^p = 9$ and $\theta^w = 4.5$ were fixed, where the latter two values follow Galí (2013). Table 2 reports the prior distribution, means, and standard deviations,together with the posterior means and confidence intervals of the estimated parameters.[29]

---

[29]The estimation period is chosen to exclude the post 2008 period during which the federal funds rate was effectively at zero. The implications of the zero lower bound for goal-based and rule-based performance masures are discussed in the concluding section.

Table 2: Prior and posterior distributions: Structural parameters

	Priors			Posterior		
	prior dist.	mean	s.d.	mean	5%	95%
Structural parameters						
η	gamma	4.34	0.25	3.7812	2.6792	4.6645
δ_p	beta	0.5	0.15	0.3690	0.3090	0.4410
δ_w	beta	0.5	0.15	0.2325	0.2000	0.2606
φ_p	beta	0.75	0.1	0.2081	0.0914	0.3218
φ_w	beta	0.75	0.1	0.1891	0.0703	0.2946
Monetary policy						
ρ_i	beta	0.83	0.1	0.5144	0.5000	0.5329
ϕ_π	gamma	2	0.25	2.7303	2.4659	2.9993
ϕ_g	gamma	0.35	0.05	0.4404	0.3822	0.5000
Disturbances						
ρ_χ	beta	0.9	0.2	0.9015	0.8692	0.9350
ρ_{μ^p}	beta	0.9	0.2	0.9886	0.9646	0.9999
ρ_{μ^w}	beta	0.9	0.2	0.1421	0.0100	0.2937
ρ_v	beta	0.3	0.2	0.4634	0.3611	0.5595
σ_z	invg	1.0	0.2	0.6567	0.5766	0.7324
σ_χ	invg	1.0	0.2	1.1921	0.9488	1.3864
σ_v	invg	1.0	0.2	0.4412	0.4109	0.4705
σ_{μ^p}	invg	1.0	3	1.2011	1.0027	1.3801
σ_{μ^w}	invg	1.0	3	4.9443	3.9333	5.9998

4.2 Welfare measures

In viewing central bank design as an issue of delegation, the objectives pursued by the central bank may differ from those of society, either because the central bank's evaluation of economic outcomes differs inherently from society's or because the central bank has been assigned objectives that differ from those of society. The former case corresponds to Rogoff's conservative central banker, a policy maker whose preference for low and stable inflation is greater than that of the public. The latter is the case considered in this paper in which policy makers share society's preferences but have been assigned objectives that may differ from those of society. In either case, it is necessary to specify two sets of preferences – those taken to represent society's and those that underlie the central bank's policy choices.

In specifying these preferences, much of the monetary policy literature, including work on inflation targeting, takes the objectives of the central bank to be represented by a quadratic loss function in inflation squared (or squared deviations of inflation from target)

and an output gap squared. These objectives are then also implicitly identified with those of society. Under a delegation scheme, society's and the central bank's objectives could each be represented by ad hoc quadratic loss functions, but the two loss functions may differ. Alternatively, in models based on the preferences of the individual agents populating the economy, outcomes can be evaluated in terms of their implications for the welfare of the representative household. If a welfare-based measure is used to represent society's preferences, the objectives of the central bank could take one of two basic forms. One could still represent the central bank's objectives by a standard quadratic loss function augmented by the performance measures assigned to the bank. Or one could assume the policy maker cares about the welfare of the representative household, in addition to the performance measures they have been assigned. Each of these alternatives could then allow for distortionary shocks to the policy maker's output objective. Table 3 summarizes the combinations of objective functions that could be used to measure society's welfare and to represent the central bank's objectives. In the analysis of this section, six of the eight possible combinations of objectives will be considered; these combinations are indicated in the table. I have excluded the cases in which society's preferences are given by an ad hoc loss function while the central bank uses the welfare of the representative household to evaluate outcomes, as these combinations of preferences seem of limited relevance.

Table 3: Alternative welfare measures

		Society	
		Ad hoc	Welfare based
Central bank	Ad hoc	x	x
	Ad hoc w/ distorted output gap	x	x
	Welfare based		x
	Welfare based w/ distorted output gap		x

The ad hoc measure used to evaluate outcomes from society's perspective is taken to be

$$L_t^{s,adhoc} = \frac{1}{2}\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left(\hat{\pi}_{t+i}^2 + \lambda_x x_{t+i}^2 \right), \qquad (20)$$

while the welfare-based measures is taken to be a second-order approximation to the welfare of the representative household, where the approximation is taken around the economy's zero-inflation efficient equilibrium.[30] In the context of the sticky-price, sticky-wage model, this is given by (see Erceg et al. (2000))

$$L_t^{s,welf} = \frac{1}{2}\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[\left(\hat{\pi}_{t+i} - \delta_p \hat{\pi}_{t+i-1}\right)^2 + \lambda_x x_{t+i}^2 + \lambda_w \left(\hat{\pi}_{t+i}^w - \delta_w \hat{\pi}_{t+i-1}^w\right)^2 \right], \qquad (21)$$

---

[30]I assume fiscal taxes and/or subsidies are in place to ensure the steady-state allocation is efficient.

where

$$\lambda_x = \left(\frac{\kappa_p}{\theta^p}\right)\left(\frac{1+\eta}{1-a}\right)$$

$$\lambda_w = (1-a)\left(\frac{\kappa_p}{\kappa_w}\right)\left(\frac{\theta^w}{\theta^p}\right).$$

Since the weight on output gap volatility in $L_t^{s,adhoc}$ is ad hoc, I employ the same value for λ_x in (20) as for λ_x in (21). Based on the estimated parameters reported in Table 1, $\lambda_x = 0.1486$ and $\lambda_w = 0.4061$.

The central bank is assumed to minimize a loss function that is augmented by the performance measures which place additional weight on inflation volatility and deviations from an instrument rule:

$$L_t = L_t^{cb} + \frac{1}{2}\mathrm{E}_t\sum_{i=0}^{\infty}\beta^i\left[\tau\hat{\pi}_{t+i}^2 + \delta\left(i_{t+i} - i_{t+i}^r\right)^2\right],$$

where L_t^{cb} is the central bank's loss function in the absence of performance measures. Four alternative specifications for L_t^{cb} are used. These differ according to whether an ad hoc quadratic loss function or the welfare approximation is used and whether, for each of these loss functions, whether the central bank is concerned with x_{t+i}^2 or with the distorted gap $(x_{t+i} - u_{t+i})^2$. For example, if $u_t \equiv 0$ and the central bank employs an ad hoc quadratic loss function, policy will aim to minimize

$$\frac{1}{2}\mathrm{E}_t\sum_{i=0}^{\infty}\beta^i\left[\hat{\pi}_{t+i}^2 + \lambda_x x_{t+i}^2 + \tau\hat{\pi}_{t+i}^2 + \delta\left(i_{t+i} - i_{t+i}^r\right)^2\right]. \tag{22}$$

If the central bank's gap objective is distorted, policy will minimize

$$\frac{1}{2}\mathrm{E}_t\sum_{i=0}^{\infty}\beta^i\left[\hat{\pi}_{t+i}^2 + \lambda_x\left(x_{t+i} - u_{t+i}\right)^2 + \tau\hat{\pi}_{t+i}^2 + \delta\left(i_{t+i} - i_{t+i}^r\right)^2\right]. \tag{23}$$

A similar distinction will arise if the central bank is concerned with minimizing (21) or (21) with x_t^2 replaced by $(x_t - u_t)^2$.

Finally, the reference policy rule defining i_t^r is given by

$$i_t^r = 1.5\pi_t - 0.125z_t, \tag{24}$$

where z_t is a measure of real activity. Two alternatives for z_t will be considered: x_t, the gap between output and the efficient level of output and y_t, output relative to the permanent component of output, interpreted as corresponding to output relative to trend.

4.3 Results

As a starting point, consider the case in which social loss is measured by the standard quadratic loss function given by (20), and the central bank's objective is (22). Assume

$z_t = y_t$ in (24) so the reference policy rule includes inflation and the gap between output and potential as in the Reference Policy Rule proposed in H.R. 5018. The model given by (12) - (19) is solved over a grid of values for τ and δ under the optimal discretionary policy designed to minimize (22). For each combination, social loss measured by (20) is evaluated to obtain the values τ^* and δ^* that minimize social loss.

Row 1, column 1 of Table 4 shows that $\tau^* > 0$ but $\delta^* = 0$ when a standard quadratic loss function in inflation and the efficiency output gap is used to represent both social loss and the central bank's preferences. Because there is no distortion appearing directly in the central bank's loss function, i.e., $u_t \equiv 0$ and the central bank cares about $\hat{\pi}_t^2$ and x_t^2, the only role for the performance measures is to address the dynamic inefficiency of discretionary policy. Recall that Clarida et al. (1999) showed that in the presence of serially correlated cost shocks, as is the case here, having the central bank place more weight on its inflation goal (relative to the true social loss function) would lead to improved outcomes.[31] In contrast, the rule-based performance measure receives zero weight.

Table 4: Optimal τ and δ, Taylor Rule in π and y

		Social loss			
		(1)		(2)	
		Ad hoc (eq. 20)		Welfare (eq. 21)	
	Central bank loss	τ^*	δ^*	τ^*	δ^*
(1)	ad hoc: π, x	4.04	0	1.37	0
(2)	ad hoc: π, $x - u$	12.95	0	6.15	0
(3)	welfare			0.33	0
(4)	welfare in $x - u$			1.54	0

Now suppose the distortionary shock u_t that affects the output goal pursued by the central bank is added, so that the central bank seeks to minimize (23). Since shocks to the central bank's preferences were not incorporated into the estimated model, I arbitrarily set $\sigma_u = 1.0$ (one percent). Going from row 1, column 1 of Table 4 to row 1, column 2 shows that the optimal value of τ^* increases. As discretionary policy now suffers from the distortions in the central bank's output goal and those arising from discretion, the optimal power of the goal-based performance measure rises. As expected from the results of section 3, adding this distortion significantly increases τ^* (from 4.04 to 12.95). The optimal δ^* is still equal to zero.

Results are similar when the welfare loss (21) is used to evaluation outcomes. Whether the central bank's objectives are based on the ad hoc loss function (22) (row 1, column 2) or (23) that includes a distorted output gap objective (row 2, column 2), it is optimal to rely solely on the goal-based performance measure ($\tau^* > 0$, $\delta^* = 0$). Now suppose the central

---

[31]See also Tillmann (2012).

bank cares about social welfare as well as its assigned performance measures. That is, the central bank attempts to minimize

$$\frac{1}{2}\mathrm{E}_t\sum_{i=0}^{\infty}\beta^i\left[(\hat{\pi}_{t+i}-\delta_p\hat{\pi}_{t+i-1})^2+\lambda_x x_{t+i}^2+\lambda_w\left(\hat{\pi}_{t+i}^w-\delta_w\hat{\pi}_{t+i-1}^w\right)^2+\tau\hat{\pi}_{t+i}^2+\delta\left(i_{t+i}-i_{t+i}^r\right)^2\right].$$
(25)

When the central bank cares about the welfare-based measure of loss, whether distorted by shocks to its output objective or not (rows 3 and 4, column 2), $\tau^* > 0$ and $\delta^* = 0$. Notice that the optimal power of the performance measure (τ^*) falls when the central bank cares about the welfare-based loss (compare row 1 and 2 to rows 3 and 4). Figure 2 shows how τ and δ affect welfare-based social loss when the central bank also cares about the welfare-based loss function but with distortions to its output objective (corresponding to row 4, column 2 of Table 5). Loss quickly becomes extremely large as δ increases above zero. It increases so quickly that the scale of the figure obscures the way loss varies with τ when δ is fixed at its optimal value of zero, making it hard to discern that $\tau^* = 1.54$. While setting τ equal to its optimal value reduces loss by 16% relative to the $\tau^* = \delta^* = 0$ case, increasing δ from 0 to just 0.05 when $\tau = 0$ leads to a increases social loss by a factor of almost 50.

The results reported in Table 4 can be summarized briefly; for all combinations of loss functions for the central bank and the measure of social loss, whether the central bank's output target is distorted or not, the optimal weight to place on the goal-based performance measure (τ) is positive while the optimal weight to place on the rule-based performance measure (δ) is zero.

Now assume $z_t = x_t$ in (24) so that the reference policy rule includes inflation and the gap between output and its efficient level. In this case, the reference rule is defined in manner that is more consistent with the underlying model. Results are shown in Table 5. Now, $\delta^* > 0$ for all six different combinations considered. Row 1, column 1 of Table 5 shows that when a standard quadratic loss function in inflation and the efficiency output gap is used to represent social loss and the central bank's preferences, it is optimal to employ both a goal-based system (i.e., $\tau^* > 0$) and a rule-based system $(\delta^* > 0)$. Both performance measures are used in this case to address the dynamic inefficiency of discretionary policy. Adding the distortion to the central bank's output goal (row 2, column 1) increases the power of both performance measures. For this case with two distortions, the two performance measures serve to some degree as substitutes. For example, if either τ or δ are set to zero, there is a large reduction in social loss as the other increases from zero. The gain from setting τ optimally when $\delta = 0$ is approximately the same as that obtained by setting δ optimally when $\tau = 0$. However, if either is set at their optimal value, the further gain from employing the other performance measure is relatively small.

Table 5: Optimal τ and δ, Taylor rule in π and x

		Social loss			
		(1)		(2)	
		Ad hoc (eq. 20)		Welfare (eq. 21)	
	Central bank loss	τ^*	δ^*	τ^*	δ^*
(1)	ad hoc: π, x	6.44	1.19	0.24	0.70
(2)	ad hoc: π, $x - u$	11.26	2.38	0	1.50
(3)	welfare			26.21	11.36
(4)	welfare in $x - u$			36.05	12.22

Rather than using an ad hoc loss function to assess outcomes as τ and δ vary, suppose the welfare-based loss function (21) is used to evaluate social loss. Assume policy is still determined by the central bank to minimize the ad hoc quadratic loss function (22) in $\hat{\pi}_t^2$ and x_t^2. Optimal values of τ and δ for this case are shown in rows 1 and 2, column 2 of Table 5. The weights on both the goal-based and the rule-based performance measures fall relative to the case when the ad hoc loss function was used to measure social loss. The reduction in τ^* when welfare is measured by (21) rather than the ad hoc (20) is large, from 6.44 to 0.24 when $u_t \equiv 0$, while δ^* falls by over 40%. But perhaps more interesting is the result in row 2, column 2. If the central bank's output gap target is subject to stochastic distortion as in (23), the optimal scheme involves only the rule-based performance measure ($\tau^* = 0$). This result is consistent with the idea that a rule-based performance measure is a means of restricting central bank discretion. Figure 3 shows the percent reduction in social loss as a function of τ and δ. Loss clearly declines as δ rises from zero; in contrast, the reduction in loss is is relatively flat as τ varies for a fixed δ. In any case, the effects on loss as τ and δ vary is small. The results from section 3.4 indicated τ^* and δ^* would depend on the relative volatilities of the underlying shocks. Redoing the case corresponding to row 2, column 2 of Table 5 with the standard deviation of aggregate demand shocks doubled causes τ^* to rise from 0 to 2.70 while δ^* falls to 0.70. The percent reduction in social loss as τ and δ vary for the case of more volatile demand shocks is shown in Figure 4. Now, it is optimal to rely on both the goal-based measure and the rule-based measure of performance. This suggests the optimal performance measure may be highly dependent on the properties of the model's stochastic disturbances.

Rows 3 and 4 report results when the central bank cares about the welfare-based loss function (25). In the absence of a distorted output gap objective, both τ^* and δ^* are positive (Table 5, row 3, column 2), and both are large. If the output gap target the central bank focuses on is distorted by u shocks so that $x_t - u_t$ rather than just x_t appears in the central bank's loss function, the optimal values of τ^* and δ^* both increase (see row 4, column 2), and in the case of τ^*, it increases quite significantly. Interestingly, when each performance measure is considered in isolation, the optimal weights are relatively small. For example, if

$\delta = 0$ so that only the inflation measure is employed, the optimal weight to place on the goal-based measure is 1.45; when δ is also set optimally, $\tau^* = 36.05$. Similarly, if $\tau = 0$, the optimal value of δ is only 0.40; it increases to 12.22 when τ is set optimally. This is show for δ in Figure 5 while plots the change in social welfare as a function of δ for $\tau = 0$ and $\tau = \tau^*$. Notice that if only the rule-based performance measure is employed (i.e., $\tau = 0$), social loss is higher than would occur with no performance measure ($\tau = \delta = 0$) for all $\delta > 5.4$.

In general, the findings in Table 5 suggest a role for both types of performance measures. However, in evaluating these results, an important consideration to bear in mind is that the rule-based performance measure analyzed here was taken to be the basic Taylor Rule, with the coefficients on inflation and the output measure set equal to Taylor's original values. If these coefficients were optimized for the specific model used, it is likely that the optimal weight to put on the rule-based performance measure would rise.

5 Extensions and conclusions

The central banking reforms initiated by the RBNZ Act of 1989 emphasized the importance of defining clear and sustainable goals for the central bank, combined with instrument independence in the conduct of policy. Such a system promotes accountability by establishing goals that are clearly defined and by giving the central bank the responsibility and ability to achieve these goals. Accountability has been further enhanced by trends towards greater transparency as central banks have concluded that policy is more effective when it is clearly understood by the public. Goal-based systems were motivated, in part, by a desire to constrain governments in their ability to influence monetary policy while allowing flexibility in the actual implementation of policy.

Reforms based on goals are not the only possibility for central banks. An alternative approach focuses on constraining the central bank by establishing instrument rules as the means of measuring the central bank's performance. Requiring a central bank to justify its policy actions with reference to a specific instrument rule is a means of strengthening accountability by limiting the central bank's flexibility.

In a simple analytical exercise, I showed that stochastic distortions to the central bank's goals, which could arise either from pressures external to the central bank or from the pursuit by the central bank of goals that differ from society's, justify a role for goal-based *and* rule-based performance measures. In using either performance measure, the need to limit distortionary shifts in objectives from affecting output and inflation must be balanced against the cost of reducing the bank's ability to engage in stabilization policies. Using a calibrated version of the simple model, I showed that an increase in the volatility of demand shocks relative to cost shocks increased the optimal weight to place on the goal-based performance measure relative to the rule-based measure.

The two approaches to central bank design were then evaluated using an estimated DSGE model with sticky prices and wages. Using the basic Taylor Rule as the reference

29

policy rule in the rule-based performance measure, along with Taylor's original coefficients on inflation and the measure of real economic activity, I find the definition of real activity used in the rule is crucial. When the rule is based on output deviations from potential, as in the recent proposal in the U.S. House of Representatives, the optimal weight to place on deviations from the rule-based performance measure was always zero. In contrast, it was always optimal to employ a goal-based inflation performance measure. When the measure of real activity in the reference policy rule was the gap between output and its efficient level, it was generally optimal to place weight on both the goal-based and the rule-based measures of performance.

An important consideration in establishing any performance measure is its robustness. A reference policy rule, such as the one analyzed in this paper, that does not allow for shifts in the equilibrium real rate of interest is likely to produce poor outcomes if such shifts are an important source of macroeconomic volatility. An optimal rule would overcome this particular problem, but operational rules must be based on observable variables if they are to be of practical relevance, and the equilibrium real interest rate consistent with efficient production is unobservable. Optimal rules are also unlikely to be robust to model misspecification, an issue not addressed here. A reference policy rule that is optimal for a given model will presumably serve as a good performance measure within that model but may lead to poor results if the model is wrong or if the economic structure changes over time. Rule-based performance measures based on a rule optimized for a specific model would need, therefore, to be of low power. Of course, a simple rule, such as the Taylor Rule, may be more robust across models and in the face of structure change than rules optimized for a specific model, and so a simple rule may serve as a useful, robust reference rule.

To simplify the analysis of the paper, I have ignored the constraint imposed by the zero lower bound (ZLB) on nominal interest rates.[32] The presence of the ZLB poses difficulties for both the goal-based and the rule-based performance measures. Neither provides a clear metric for what the central bank should do doing, or for how it's performance should be judged, when the policy rate is at zero. This difficulty may, however, be less significant for the goal-based measure. A goal-based regime such as inflation targeting establishes a goal for the central bank but does not tie the hands of policy makers in terms of how policy is implemented to achieve the goal. For example, if the policy rate were at its lower bound and inflation below target, then a goal-based performance measure creates an incentive for the central bank to seek out new policy instruments in an effort to achieve its goal. A rule-based system may not be as effective in creating such incentives. A reference rule defined in terms of a single instrument may be of limited value during extended periods at the ZLB as it does not provide any guidance to policy makers when the instrument value implied by the rule is unachievable. If the reference rule called for a negative interest rate, the central bank might seek to close the gap between i_t and i_t^r by directly focusing on the variables that affect i_t^r in

---

[32]I adopt the standard practice of referring to a zero lower bound for nominal interest rates, but the recent experience with negative nominal interest rates in Denmark, Sweden, and the eurozone suggests the effective lower bound may be below zero.

an attempt to raise i_t^r above zero. In this case, either type of performance measure could promote policy innovations. However, because the rule-based measure is defined in terms of a specific policy instrument, and because it offers no guidance for how performance should be measured if that instrument is constrained, it may prove less likely to lead to the types of unconventional policies implemented by the Federal Reserve, the Bank of England, the Bank of Japan and the ECB during the past several years.

The focus in this paper has been on assessing policy performance in the presence of inefficient shifts in the central bank's objectives that potentially distort policy. Deviations of inflation from target or the policy interest rate from the recommendation of a Taylor rule were used as performance measures, creating incentives for the central bank to trade-off minimizing these deviations against achieving other objectives. This is not the only role deviations from the Taylor rule can play. In the face of model uncertainty, Ilbas et al. (2012) show how appending deviations from the Taylor rule to the central bank's (nondistorted) loss function can contribute to policy robustness. In addition, the distortions considered in the present analysis do not affect the economy's steady-state equilibrium. Thus, policy objectives that create steady-state inefficiencies are ignored. Rogoff (1985) showed how placing additional weight on an inflation target could help overcome a systematic inflation bias under discretionary policy; a rule-based performance measure might play a similar role in addressing any systematic policy bias that affects steady-state inflation.

Finally, I have only considered traditional monetary policy objectives associated with controlling inflation and stabilizing an appropriate measure of real economic activity. As a consequence of the global financial crisis, central banks are now frequently tasked with responsibilities for macroprudential policies. An interesting question is whether a goal-based performance measure or a rule-based measure would best serve to promote accountability and good macroprudential outcomes. One significant difficulty in designing a goal-based performance measure in the case of macroprudential policies is the absence of a clear measure of the ultimate goal of policy. Inflation is both an ultimate goal of macroeconomic policy and an indicator that can be measured frequently to provide an ongoing assessment of policy. Achieving financial stability may also be an ultimate goal of policy, but there is no agreed upon way to measure it. An index such as the ratio of credit to GDP may be a useful measure in this context, but it corresponds to an intermediate target. Assessing policy on the basis of movements in the credit to GDP ratio is much like using a monetary growth rate to assess the central bank's inflation performance. The usefulness of intermediate targets suffers if the link between the intermediate variable and the ultimate objective of policy is either uncertain or not well understood. While it may be difficult to develop a goal-based performance measure for macroprudential policy, difficulties also arise in defining a rule-based measure. Macroprudential policies may involve the use of multiple instruments. In this case, basing accountability on how one particular instrument is used can easily distort policy by causing undue attention to that one instrument at the neglect of others. And even when attention is restricted to a single instrument – the setting of capital buffer requirements for example – the state of research is that there is no benchmark rule that

has been extensively studied, is well understood, and that could serve as a reference policy rule. The lack of the equivalent to a Taylor rule for macroprudential policy instruments is a severe limitation on the usefulness of a rule-based performance measure in the context of macroprudential policies.

References

Alesina, A., Summers, L. 1993. Central Bank Independence and Macroeconomic Performance. Journal of Money, Credit, and Banking, 25, 157–162.

Athey, S. V. V. C., Atkeson, A., Kehoe, P. J. 2005. The Optimal Degree of Discretion in Monetary Policy. Econometrica, 73, 1431–1475.

Bade, R., Parkin, M. 1984. Central bank laws and monetary policy. University of Western Ontario.

Baker, G., Gibbons, R., Murphy, K. 1994. Subjective performance measures in optimal incentive contracts. Quarterly Journal of Economics, 109, 1125–1156.

Baker, G. P. 1992. Incentive Contracts and Performance Measurement.

Ball, L., Sheridan, N. 2004. Does Inflation Targeting Matter?. In B. S. Bernanke, M. Woodford eds. The Inflation-Targeting Debate, I, University of Chicago Press, 249–276.

Barro, R. J., Gordon, D. B. 1983. A Positive Theory of Monetary Policy in a Natural-Rate Model. Journal of Political Economy, 91, 589–610.

Benhabib J., S. S.-G., Uribe, M., Benhabib, J., Schmitt-Grohé, S. 2001. The Perils of Taylor Rules. Journal of Economic Theory, 96, 40–69.

Billi, R. M. 2013. Nominal GDP Targeting and the Zero Lower Bound: Should We Abandon Inflation Targeting?.

Blinder, A. S., Ehrmann, M., Fratzscher, M., De Haan, J., Jansen, D.-J. 2008. Central Bank Communication and Monetary Policy: A Survey of Theory and Evidence. Journal of Economic Literature, 46, 910–945.

Canzoneri, M. B. 1985. Monetary Policy Games and the Role of Private Information. American Economic Review, 75, 1056–1070.

Carlstrom, C. T., Fuerst, T. S. 2009. Central Bank Independence and Inflation: a Note. Economic Inquiry, 47, 182–186.

Casares, M., Moreno, A., Vázquez, J. 2011. Wage stickiness and unemployment fluctuations: an alternative approach. SERIEs, 3, 395–422.

Chen, H., Curdia, V., Ferrero, A. 2012. The macroeconomic effects of large-scale asset pruchase programmes. Economic Journal, 122, F289–F315.

Clarida, R., Galí, J., Gertler, M. 1999. The Science of Monetary Policy: A New Keynesian Perspective. Journal of Economic Perspectives, 37, 1661–1707.

Crowe, C., Meade, E. E. 2007. The Evolution of Central Bank Governance around the World. Journal of Economic Perspectives, 21, 69–90.

Cukierman, A. 1992. Central Bank Strategies, Credibility and Independence. MIT Press.

Cukierman, A. 2008. Central bank independence and monetary policymaking institutions – Past, present and future. European Journal of Political Economy, 24, 722–736.

Cukierman, A., Web, S. B., Neyapti, B. 1992. Measuring the independence of central banks and its effect on policy outcomes. World Bank Economic Review, 6, 353–398.

Debelle, G., Fischer, S. 1994. How independent should a central bank be?. Carnegie Rochester Conference Series on Public Policy, 38, 195–225.

Dennis, R. 2007. Optimal Policy in Rational Expectations Models: New Solution Algorithms. Macroeconomic Dynamics, 11, 31–55.

Dincer, N. N., Eichengreen, B. 2014. Central bank transparency and independence: updates and new measures. International Journal of Central Banking, 10, 189–253.

Drazen, A. 2000. The Political Economy of Macroeconomics. Princeton, NJ, Princeton University Press, 792.

Erceg, C., Henderson, D., Levin, A. 2000. Optimal monetary policy with staggered wage and price contracts. Journal of monetary Economics, 46, 281–313.

Evjen, S., Kloster, T. B. 2012. Norges Bank's new monetary policy loss function – further discussion.

Frankel, A. 2014. Aligned Delegation. American Economic Review, 104, 66–83.

Galí, J. 2008. Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework. Princeton, Princeton University Press.

Galí, J. 2013. Notes for a New Guide To Keynes (I): Wages, Aggregate Demand, and Employment. Journal of the European Economic Association, 11, 973–1003.

Geraats, P. M. 2009. Trends in Monetary Policy Transparency. International Finance, 12, 235–268.

Ilbas, P., Roisland, O., Sveen, T. 2013. The influence of the Taylor rule on US monetary policy. Norges Bank Research Working Paper, 04, p. 34.

Ilbas, P., Roisland, O., Sveen, T. 2012. Robustifing optimal monetary policy using simple rules as cross-checks. Norges Bank Research Working Paper, 22, p. 30.

Kahn, G. A. 2012. The Taylor Rule and the Practice of Central Banking. In E. F. Koenig, R. Leeson, G. A. Kahn eds. The Taylor Rule and the Transformation of Monetary Policy, Stanford, CA, Hoover Institution, 63–102.

Kydland, F. E., Prescott, E. C. 1977. Rules Rather than Discretion: The Inconsistency of Optimal Plans.

Levin, A. T., Wieland, V., Williams, J. C. 1999. Robustness of Simple Monetary Policy Rules under Model Uncertainty. In J. B. Taylor ed. Monetary Policy Rules, Chicago, Chicago University Press.

Levin, A. T., Williams, J. C. 2003. Robust monetary policy with competing reference models. Journal of Monetary Economics, 50, 945–975.

Lund, K., Robstad, O. r. 2012. Effects of a new monetary policy loss function.

Poole, W. 1970. Optimal Choice of Monetary Policy Instrument in a Simple Stochastic Macro Model. Quarterly Journal of Economics, 84, 197–216.

Posen, A. 1993. Why Central Bank Independence Does Not Cause Low Inflation: There is no Institutional Fix for Politics. Finance and the International Economy, 7, 40–65.

Roger, S. 2010. Inflation targeting turns 20. Finance & Development, 46–49.

Rogoff, K. 1985. The Optimal Degree of Commitment to an Intermediate Monetary Target. The Quarterly Journal of Economics, 100, 1169–1189.

Rose, A. K. 2013. Recent Monetary Regimes of Small Economies.

Rudebusch, G. D. 2002. Term Structure Evidence on Interest Rate Smoothing and Monetary Policy Inertia. Journal of Monetary Economics, 49, 1161–1187.

Ruge-Murcia, F. J. 2014. Do Inflation-Targeting Central Banks Implicitly Target the Price Level?. International Journal of Central Banking, 10, 301–326.

Sherwin, M. 1999. The origins of New ZealandâĂŹs inflation targeting regime and its evolution over the past decade. Speech to the NZ Association of Economists Conference, Rotorua, NA, July 1, 1999, 1–13.

Svensson, L. E. O. 1997. Optimal Inflation Targets, "Conservative" Central Banks, and Linear Inflation Contracts. American Economic Review, 87, 98–114.

Svensson, L. E. O. 2003. What Is Wrong with Taylor Rules? Using Judgment in Monetary Policy through Targeting Rules. Journal of Economic Literature, 41, 426–477.

Taylor, J. B. 1993. Discretion versus policy rules in practice. Carnegie Rochester Conference Series on Public Policy, 39, 195–214.

Taylor, J. B. 2011. The Rules-Discretion Cycle in Monetary and Fiscal Policy. Finnish Economic Papers, 24, 78–86.

Taylor, J. B. 2012. Monetary Policy Rules Work and Discretion Doesn't: A Tale of Two Eras. Journal of Money, Credit and Banking, 44, 1017–1032.

Taylor, J. B. 2013. The Effectiveness of Central Bank Independence vs. Policy Rules. Business Economics, 1–21.

Taylor, J. B., Williams, J. C. 2010. Simple and robust rules for monetary policy. In J. B. Taylor, M. Woodford eds. Handbook of Monetary Economics, 3, Elsevier, 829–859.

Tillmann, P. 2012. Cross-Checking Optimal Monetary Policy with Information from the Taylor Rule. Economic Letters, 117, 204–207.

Tirole, J. 1994. The Internal Organization of Government. Oxford Economic Papers, 46, 1–29.

Vestin, D. 2006. Price-Level Targeting versus Inflation Targeting. Journal of Monetary Economics, 53, 1361–1376.

Walsh, C. E. 1986. In Defense of Base Drift. American Economic Review, 76, 692–700.

Walsh, C. E. 1987. Monetary Targets and Inflation: 1976-1984. Federal Reserve Bank of San Francisco Economic Review.

Walsh, C. E. 1995a. Optimal contracts for central bankers. The American Economic Review, 85, 150–167.

Walsh, C. E. 1995b. Is New Zealand's Reserve Bank Act of 1989 an Optimal Central Bank Contract?. Journal of Money, Credit, and Banking, 27, 1179–1191.

Walsh, C. E. 2003a. Accountability, Transparency, and Inflation Targeting. Journal of Money, Credit and Banking, 35, 829–849.

Walsh, C. E. 2003b. Speed limit policies: the output gap and optimal monetary policy. American Economic Review, 93, 265–278.

Walsh, C. E. 2010. Monetary Theory and Policy, Third Edition. The MIT Press, 640.

Woodford, M. 2005. Central Bank Communication and Policy Effectiveness. NBER Working Papers, 11898.

Woodford, M. 2010. Financial intermediation and macroeconomic analysis. The Journal of Economic Perspectives, 24, 21–44.

Woodford, M. 2013. Forward Guidance by Inflation-Targeting Central Banks. In Two Decades of Inflation Targeting, Sveriges Riksbank.

6 Appendix

6.1 Equilibrium in the simple model

The first order conditions for the central bank maximizing the loss function (3) leads to the following standard targeting criterion:

$$\kappa \left(\hat{\pi}_t - \phi_t \right) + \lambda \left(x_t - u_t \right) = 0. \tag{26}$$

Substituting (26) into (4) yields

$$\hat{\pi}_t = \beta \mathrm{E}_t \hat{\pi}_{t+1} + \kappa \left[u_t - \frac{\kappa}{\lambda} \left(\hat{\pi}_t - \phi_t \right) \right] + e_t.$$

When the shocks are $i.i.d.$, $\mathrm{E}_t \hat{\pi}_{t+1} = 0$. Hence,

$$\hat{\pi}_t = \kappa \left[u_t - \frac{\kappa}{\lambda} \left(\hat{\pi}_t - \phi_t \right) \right] + e_t = \left(\frac{1}{\lambda + \kappa^2} \right) \left(\lambda \kappa u_t + \kappa^2 \phi_t + \lambda e_t \right).$$

From (26),

$$x_t = u_t - \left(\frac{\kappa}{\lambda} \right) \left(\hat{\pi}_t - \phi_t \right),$$

or

$$x_t = - \left(\frac{1}{\lambda + \kappa^2} \right) \left(\kappa e_t + \lambda \kappa u_t + \lambda \kappa^2 \phi_t \right).$$

Social loss in this equilibrium is

$$L_t^s = \frac{1}{2} \mathrm{E}_0 \sum \beta^i \left(\hat{\pi}_{t+i}^2 + \lambda x_{t+i}^2 \right) = \frac{1}{2} \left(\frac{1}{1 - \beta} \right) \left(\sigma_\pi^2 + \lambda \sigma_x^2 \right).$$

Using the results for equilibrium inflation and the output gap,

$$
\begin{aligned}
L_t^s &= \frac{1}{2} \left(\frac{1}{1 - \beta} \right) \left[\left(\frac{\lambda}{\lambda + \kappa^2} \right)^2 \sigma_e^2 + \left(\frac{\kappa}{\lambda + \kappa^2} \right)^2 \sigma_v^2 \right] \\
&\quad + \frac{1}{2} \left(\frac{1}{1 - \beta} \right) \lambda \left[\left(\frac{\kappa}{\lambda + \kappa^2} \right)^2 \sigma_e^2 + \left(\frac{\lambda}{\lambda + \kappa^2} \right)^2 \sigma_v^2 \right] \\
&= \frac{1}{2} \left(\frac{1}{1 - \beta} \right) \left[\begin{array}{c} \left(\frac{\lambda}{\lambda + \kappa^2} \right) \sigma_e^2 \\ + \left(\kappa^2 + \lambda^3 \right) \left(\frac{1}{\lambda + \kappa^2} \right)^2 \sigma_v^2 \end{array} \right]
\end{aligned}
$$

where $\sigma_v^2 \equiv \lambda^2 \sigma_u^2 + \kappa^2 \sigma_\phi^2$.

In the absence of political distortions ($\sigma_v^2 \equiv 0$), social loss is

$$\frac{1}{2} \left(\frac{1}{1-\beta} \right) \left(\frac{\lambda}{\lambda + \kappa^2} \right) \sigma_e^2 \leq L_t^s.$$

In the absence of political distortions (and maintaining the assumption of i.i.d. shocks), social loss would be

$$\frac{1}{2} \left(\frac{1}{1-\beta} \right) \left(\frac{\lambda}{\lambda + \kappa^2} \right) \sigma_e^2.$$

6.1.1   Delegation

Suppose the central bank's objective is modified by the assignment of additional weight on achieving inflation stability and on not deviating from an instrument rule. In this case, the central bank aims to minimize

$$L_t^{pol} = \frac{1}{2} E_t^{cb} \sum \beta^i \left[\left(\hat{\pi}_{t+i} - \phi_{t+i} \right)^2 + \tau \hat{\pi}_{t+i}^2 + \lambda \left(x_{t+i} - x_{t+i}^* \right)^2 + \delta \left(i_{t+i} - i_{t+i}^r \right)^2 \right].$$

Policy continues to be set under discretion.

6.1.2   Goal-based

With $\delta = 0$ but τ is potentially non-zero, the central bank's problem under discretion is

$$\min_{\hat{\pi}_t, x_t, \hat{i}_t} \frac{1}{2} \left(1 + \tau \right) \hat{\pi}_t^2 - \phi_t \pi_t + \frac{1}{2} \lambda x_t^2 - \lambda u_t x_t$$

subject to

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa x_t + e_t$$

and

$$x_t = E_t x_{t+1} - \left(\frac{1}{\sigma} \right) \left(\hat{i}_t - E_t \hat{\pi}_{t+1} - \varphi_t \right).$$

Actual inflation and the output gap are given by

$$\hat{\pi}_t = \left[\frac{\kappa}{\lambda + \kappa^2 \left(1 + \tau \right)} \right] \left(\lambda u_t + \kappa \phi_t \right) + \left[\frac{\lambda}{\lambda + \kappa^2 \left(1 + \tau \right)} \right] e_t$$

$$x_t = \left[\frac{\lambda}{\lambda + \kappa^2 \left(1 + \tau \right)} \right] u_t + \left[\frac{\kappa}{\lambda + \kappa^2 \left(1 + \tau \right)} \right] \phi_t$$
$$- \left[\frac{\kappa \left(1 + \tau \right)}{\lambda + \kappa^2 \left(1 + \tau \right)} \right] e_t,$$

where each shock is assumed to be i.i.d.

The central-bank-design problem is to pick τ to minimize the unconditional expectation

of social loss. That is, τ minimizes

$$\mathcal{L} = \frac{1}{2}\frac{1}{1-\beta}\left(\sigma_{\hat{\pi}}^2 + \lambda\sigma_x^2\right).$$

Using the equilibrium solutions for inflation and the output gap,

$$
\begin{aligned}
\mathcal{L} \;=\; &\frac{1}{2}\left\{\left[\frac{\lambda\kappa}{\lambda\left(1-\rho_u\beta\right)+\kappa^2\left(1+\tau\right)}\right]^2\sigma_u^2 + \left[\frac{\kappa^2}{\lambda\left(1-\rho_\phi\beta\right)+\kappa^2\left(1+\tau\right)}\right]^2\sigma_\phi^2\right.\\
&+\left[\frac{\lambda}{\lambda\left(1-\rho_e\beta\right)+\kappa^2\left(1+\tau\right)}\right]^2\left(\sigma_e^2-\sigma_{\tilde{e}}^2\right)+\lambda\left[\frac{\kappa\left(1+\tau\right)}{\lambda\left(1-\rho_e\beta\right)+\kappa^2\left(1+\tau\right)}\right]^2\left(\sigma_e^2-\sigma_{\tilde{e}}^2\right)\\
&+\lambda\left[\frac{\lambda\left(1-\rho_u\beta\right)}{\lambda\left(1-\rho_u\beta\right)+\kappa^2\left(1+\tau\right)}\right]^2\sigma_u^2 + \lambda\left[\frac{\kappa\left(1-\rho_\phi\beta\right)}{\lambda\left(1-\rho_\phi\beta\right)+\kappa^2\left(1+\tau\right)}\right]^2\sigma_\phi^2\\
&\left.+\lambda\left(\frac{1}{\sigma}\right)^2\left(1+\kappa^2\right)\sigma_{\tilde{\varphi}}^2 + \sigma_{\tilde{e}}^2\right\}.
\end{aligned}
$$

The first order condition for the value of τ that minimizes $\mathcal{L}$ implies

$$
\begin{aligned}
\frac{\partial\mathcal{L}}{\partial\tau} \;=\; &-\kappa^2\left(\lambda+\kappa^2\right)\left[\frac{1}{\lambda+\kappa^2\left(1+\tau\right)}\right]^3\left(\lambda^2\sigma_u^2+\kappa^2\sigma_\phi^2\right)\\
&+\tau\lambda^2\kappa^2\left[\frac{1}{\lambda+\kappa^2\left(1+\tau\right)}\right]^3\sigma_e^2\\
\;=\; &0.
\end{aligned}
$$

Solving for τ one obtains

$$\tau^* = \left(\frac{\lambda+\kappa^2}{\lambda^2}\right)\left(\frac{\lambda^2\sigma_u^2+\kappa^2\sigma_\phi^2}{\sigma_e^2}\right) \geq 0,$$

which is equation (8).

6.1.3   Rule-based

Now suppose $\tau = 0$ but δ may be non-zero. The central bank's problem takes the form

$$\min_{\hat{\pi},x,\hat{\imath}}\left[\frac{1}{2}\hat{\pi}_t^2 - \phi_t\pi_t + \frac{1}{2}\lambda x_t^2 - \lambda u_t x_t + \frac{1}{2}\delta\left(\hat{\imath}_t - \psi_\pi\pi_t - \psi_x x_t\right)^2\right]$$

subject to

$$\hat{\pi}_t = \beta\hat{\pi}_{t+1} + \kappa x_t + e_t$$

$$x_t = x_{t+1} - \left(\frac{1}{\sigma}\right)\left(\hat{\imath}_t - \mathrm{E}_t\hat{\pi}_{t+1} - \varphi_t\right)$$

where

$$\hat{\imath}_t \equiv i_t - i.$$

Because the central bank is judged in part on how it sets its instrument, the expectational IS equation becomes relevant.

Let the Lagrangian multipliers on the two constraints be θ and χ, respectively. The first order conditions are

$$\hat{\pi}_t - \phi_t - \psi_\pi \delta \left(\hat{\imath}_t - \psi_\pi \pi_t - \psi_x x_t\right) + \theta_t = 0$$

$$\lambda x_t - \lambda u_t - \psi_x \delta \left(\hat{\imath}_t - \psi_\pi \pi_t - \psi_x x_t\right) - \kappa \theta_t + \chi_t = 0$$

$$\delta \left(\hat{\imath}_t - \psi_\pi \pi_t - \psi_x x_t\right) + \chi_t \left(\frac{1}{\sigma}\right) = 0.$$

Eliminating the Lagrangian multipliers yields a relationship between the variables appearing in the central bank's loss function that can be written as

$$\hat{\imath}_t = \frac{1}{a\delta}\left[(\kappa + a\delta\psi_\pi)\,\hat{\pi}_t + (\lambda + a\delta\psi_x)\,x_t - \kappa\phi_t - \lambda u_t\right],$$

where $a \equiv \sigma + \psi_x + \kappa\psi_\pi$.

With $i.i.d.$ shocks, equilibrium is obtained by jointly solving

$$\hat{\pi}_t = \kappa x_t + e_t$$

$$x_t = \left(\frac{1}{\sigma}\right)\varphi_t - \left(\frac{1}{\sigma}\right)\hat{\imath}_t$$

$$\alpha\delta\hat{\imath}_t = (\kappa + a\delta\psi_\pi)\,\hat{\pi}_t + (\lambda + a\delta\psi_x)\,x_t - \kappa\phi_t - \lambda u_t.$$

Doing so yields

$$\hat{\pi}_t = \left[\frac{\kappa\alpha\delta\varphi_t + \kappa\lambda u_t + \kappa^2\phi_t}{\lambda + \kappa^2 + a^2\delta}\right] + \left[\frac{\lambda + a\delta\,(\sigma + \psi_x)}{\lambda + \kappa^2 + a^2\delta}\right]e_t + \left(\frac{\kappa}{\sigma}\right)\tilde{\varphi}_t + \tilde{e}_t$$

$$x_t = \frac{\alpha\delta\varphi_t + \lambda u_t + \kappa\phi_t - (\kappa + a\delta\psi_\pi)\,e_t}{\lambda + \kappa^2 + a^2\delta} + \left(\frac{1}{\sigma}\right)\tilde{\varphi}_t.$$

Using these expressions, social loss is

$$
\begin{aligned}
\mathcal{L} \;=\; & \frac{1}{2}a^2\left(\lambda + \kappa^2\right)\left[\frac{\delta}{\lambda + \kappa^2 + a^2\delta}\right]^2\sigma_\varphi^2 \\
& +\frac{1}{2}\lambda^2\left(\lambda + \kappa^2\right)\left[\frac{1}{\lambda + \kappa^2 + a^2\delta}\right]^2\sigma_u^2 \\
& +\frac{1}{2}\kappa^2\left(\lambda + \kappa^2\right)\left[\frac{1}{\lambda + \kappa^2 + a^2\delta}\right]^2\sigma_\phi^2 \\
& +\frac{1}{2}\left[\frac{\lambda + a\delta\,(\sigma + \psi_x)}{\lambda + \kappa^2 + a^2\delta}\right]^2\sigma_e^2 + \frac{1}{2}\lambda\left[\frac{\kappa + a\delta\psi_x}{\lambda + \kappa^2 + a^2\delta}\right]^2\sigma_e^2 \\
& +\frac{1}{2}\lambda\left(\frac{1}{\sigma}\right)^2\left(1 + \kappa^2\right)\sigma_{\tilde{\varphi}}^2 + \frac{1}{2}\sigma_{\tilde{e}}^2,
\end{aligned}
$$

and the first order condition for the optimal δ is

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \delta} =\ & a^2 \delta \left(\lambda + \kappa^2\right)^2 \left[\frac{1}{\lambda + \kappa^2 + a^2 \delta}\right]^3 \sigma_\varphi^2 \\
& -a^2 \lambda^2 \left(\lambda + \kappa^2\right) \left[\frac{1}{\lambda + \kappa^2 + a^2 \delta}\right]^3 \sigma_u^2 - a^2 \kappa^2 \left(\lambda + \kappa^2\right) \left[\frac{1}{\lambda + \kappa^2 + a^2 \delta}\right]^3 \sigma_\phi^2 \\
& +a \left[\frac{1}{\lambda + \kappa^2 + a^2 \delta}\right]^3 \left\{\begin{array}{c} \left[\lambda + a\delta\left(\sigma + \psi_x\right)\right]\left[\left(\sigma + \psi_x\right)\left(\lambda + \kappa^2\right) - a\lambda\right] \\ +\lambda\left(\kappa + a\delta\psi_x\right)\left[\psi_x\left(\lambda + \kappa^2\right) - a\kappa\right] \end{array}\right\} \sigma_e^2 = 0.
\end{aligned}
$$

Solving for δ, noting that $a \equiv \sigma + \psi_x + \kappa\psi_\pi$,

$$
\delta^* = \frac{\left(\lambda + \kappa^2\right)\left(\lambda^2 \sigma_u^2 + \kappa^2 \sigma_\phi^2\right)}{\left(\lambda + \kappa^2\right)^2 \sigma_\varphi^2 + \left\{\begin{array}{c} \left(\sigma + \psi_x\right)\left[\left(\sigma + \psi_x\right)\kappa^2 - \kappa\psi_\pi\lambda\right] \\ +\lambda\psi_x\psi_x\left(\lambda + \kappa^2\right) - \left(\sigma + \psi_x + \kappa\psi_\pi\right)\lambda\psi_x\kappa \end{array}\right\} \sigma_e^2},
$$

which is equation (11).

6.2   Optimal policy in the estimated model

The results reported in section 4 were obtained using the solution method for optimal discretionary policy of Dennis (2007). The equilibrium depends on the form of the loss function assigned to the central bank. Dennis (2007) does not allow for interaction terms in the loss function of the policy maker between endogenous variables and policy instruments. Such terms arise in the rule-based regimes as the squared deviations from the instrument rule, $\left(i_t - i_t^{tr}\right)^2 = i_t^2 - 2i_t i_t^{tr} + \left(i_t^{tr}\right)^2$, involves $i_t i_t^{tr}$ and so includes such interaction terms. Given a specification of social loss an the central bank's objective function, the model is solved over a grid of values for τ and δ; τ^* and δ^* are the values that result in the smallest value of social loss.

Dennis's method involves writing the model in the form

$$
A_0 y_t = A_1 y_{t-1} + A_2 E_t y_{t+1} + A_3 x_t + A_4 E_t x_{t+1} + A_5 v_t \tag{27}
$$

where y is a vector of endogenous variables, x is a vector of controls, and

$$
v_t = i.i.d.\left[0, \Sigma\right].
$$

The policy maker is assumed to minimizes a loss function given by

$$
Loss\left(0, \infty\right) = E_0 \sum_{t=0}^{\infty} \beta^t \left[y_t' W y_t + 2y_t' U s_t + x_t' Q x_t\right].
$$

This differs from Dennis (2007) who assumes $U = 0$. The solutions for y_t and x_t will be of the form

$$
y_t = H_1 y_{t-1} + H_2 v_t
$$

$$x_t = F_1 y_{t-1} + F_2 v_t.$$

Using these to form expectations of $t+1$ variables and substituting the results into (27) yields

$$y_t = (A_0 - A_2 H_1 - A_4 F_1)^{-1} (A_1 y_{t-1} + A_3 x_t + A_5 v_t)$$

or

$$y_t = D^{-1} (A_1 y_{t-1} + A_3 x_t + A_5 v_t). \tag{28}$$

Dennis provides the first order conditions for x_t under discretion when $U = 0$. When $U \neq 0$,

$$x_t = -\Phi^{-1} \left(A_3' D'^{-1} P D^{-1} + U' D^{-1} \right) [A_1 y_{t-1} + A_5 v_t]$$

where

$$\Phi \equiv \left[Q + A_3' D'^{-1} P D^{-1} A_3 + A_3' D'^{-1} U + U' D^{-1} A_3 \right],$$

which reduces to Dennis's equation (24), p. 38 when $U = 0$. This implies

$$F_1 = -\Phi^{-1} \left(A_3' D'^{-1} P D^{-1} + U' D^{-1} \right) A_1 \tag{29}$$

$$F_2 = -\Phi^{-1} \left(A_3' D'^{-1} P D^{-1} + U' D^{-1} \right) A_5 \tag{30}$$

$$H_1 = D^{-1} (A_1 + A_3 F_1) \tag{31}$$

$$H_2 = D^{-1} (A_5 + A_3 F_2). \tag{32}$$

The matrix P is defined by

$$P = W + \beta F_1' Q F_1 + \beta H_1' U F_1 + \beta H_1' P H_1.$$

The solution algorithm starts with initial values for H_1, H_2, F_1 and F_2. These are used to solve for D and P. These are then used in (29) - (32) to obtain updated values for H_1, H_2, F_1 and F_2. The process is repeated until convergence.
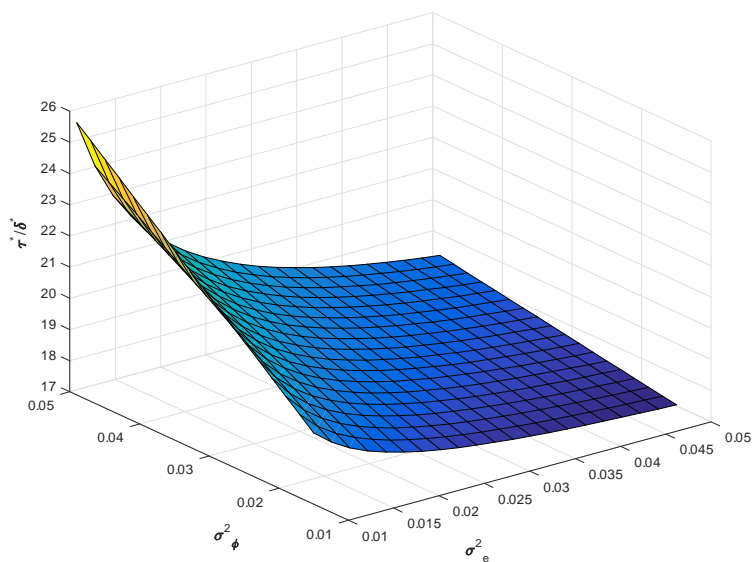
Figure 1: Ratio of optimal τ to optimal δ when jointly optimized as function of the variances of demand (σ_ϕ^2) and cost (σ_e^2) shocks.
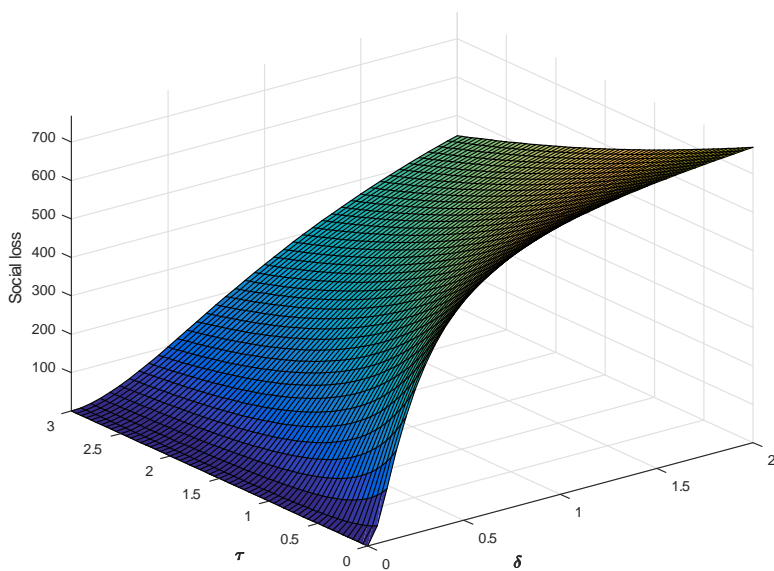

Figure 2: Loss rises quickly with δ when the reference policy rule depends on y (social loss given by (21) and central bank loss by (25) distorted by presence of u shocks to output gap objective..
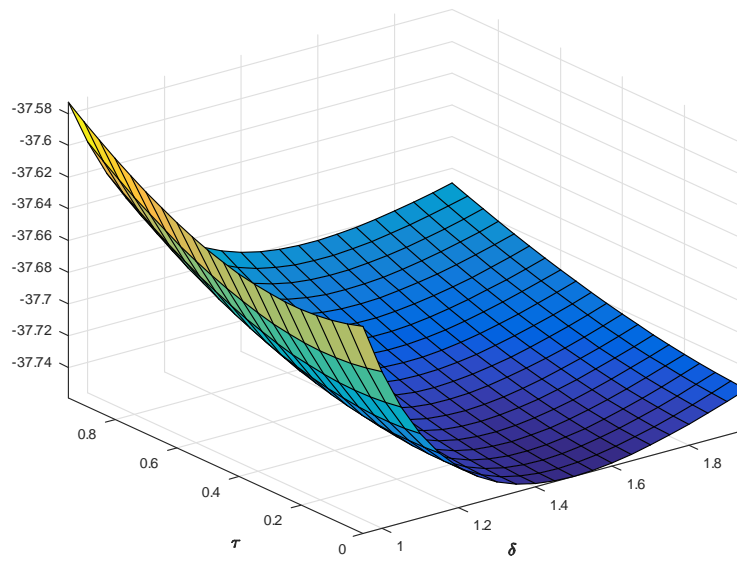
Figure 3: When the reference policy rule is based on $\hat{\pi}$ and x, social loss is given by (21) and the central bank's loss is (23), $\tau^* = 0$ and $\delta^* > 0$. (Compare with figure 3.)
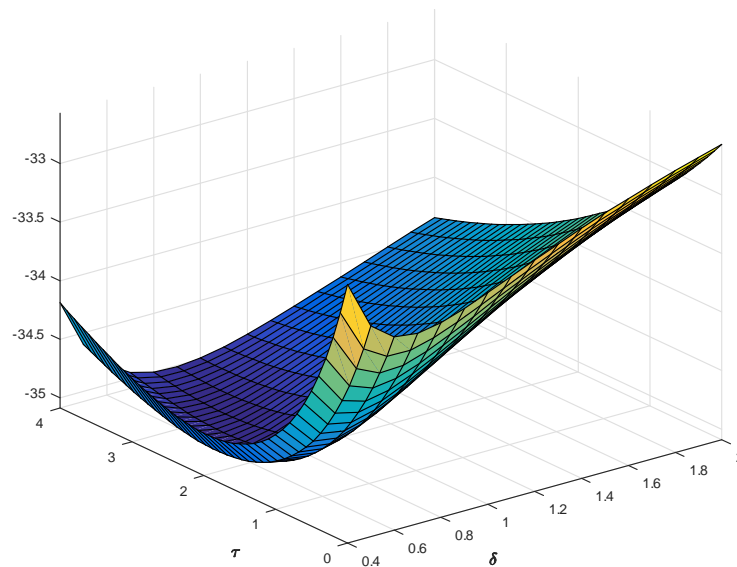

Figure 4: When the reference policy rule is based on $\hat{\pi}$ and x, social loss is given by (21) and the central bank's loss is (23), an increase in the volatility of aggregate demand shocks increases τ^* and reduces δ^*. (Compare with figure 3.)
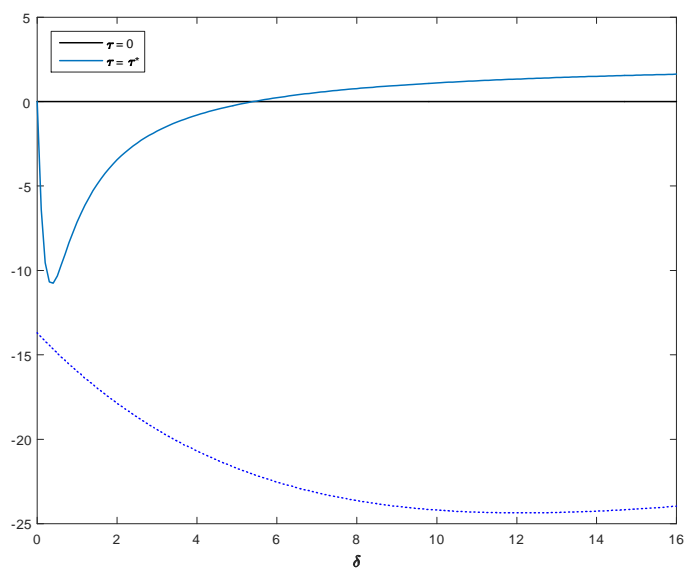
Figure 5: Percent change in social loss defined by (21) as a function of δ for $\tau = 0$ and for $\tau = \tau^* = 36.05$. Central bank's objective given by (25) distorted by presence of u shocks to output gap objective. Output measure in instrument rule is x.