When Did Growth Begin?

New Estimates of Productivity Growth in England from 1250 to 1860

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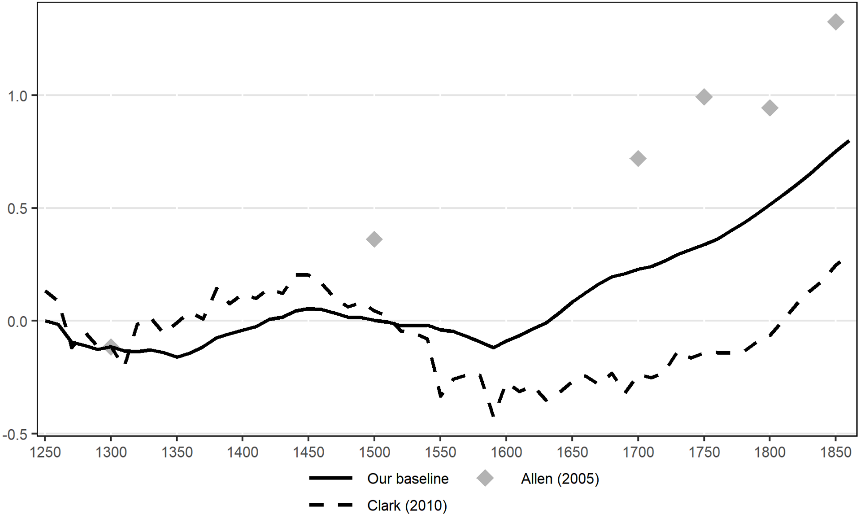
When Did Growth Begin?

- Industrial Revolution (i.e., around 1800)
  - No trend in real wages before 1800 Real Wages
- First Great Divergence after 1500
  - Divergence of urbanization rates after 1500 (e.g., Acemoglu, Johnson, and Robinson, 2005) Urbanization
  - Revisionist view of Industrial Revolution (Crafts, 1983, 1985; Harley 1982; Crafts and Harley, 1992)
- Way Back When
  - GDP per capita has grown since 1270 (Broadberry et al., 2015) GDP per capita
  - World population growth back to 1 million BC (Kremer, 1993)

Productivity Growth

- Our focus: Productivity growth
- Our main findings:
  - No productivity growth before 1600
  - Productivity growth began in 1600 at a modest rate of 3% per decade
  - Slight increase of productivity growth in 1760 to 4% per decade
  - Falling land share allowed economy to grow faster after 1760
- Findings shed light on **why** growth began:
  - Productivity growth began almost 100 years prior to Glorious Revolution
  - Indicates that economic change preceded 17th century political reforms in England (and may have contributed to causing them)

Comparison with Earlier Estimates



Allen (2005): TFP in agriculture (primal approach). Clark (2010): TFP for whole economy (dual approach).

Estimating Productivity Growth

- Primal approach (Solow, 1957; Allen, 2005):

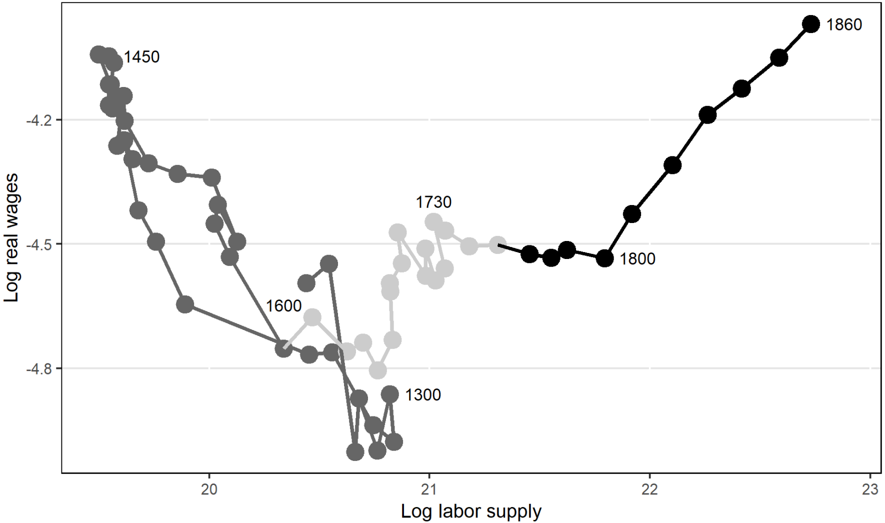
$$\Delta a_t = \Delta y_t - (1 - \alpha - \beta)\Delta l_t - \beta \Delta k_t$$

- Dual approach (Hsieh, 2002; Clark, 2010):

$$\Delta a_t = (1 - \alpha - \beta)\Delta w_t + \alpha \Delta s_t + \beta \Delta r_t$$

- Our approach different from both of these

Real Wages and Labor Supply



Alternative Approach

- Labor demand:

$$W_t = (1-\alpha)A_t\left(\frac{Z}{L_t}\right)^{\alpha}$$

  Or in logs:

$$w_t = \phi - \alpha l_t + a_t$$

- Empirical challenge:
  - a_t and l_t are correlated because of Malthusian forces
  - Higher a_t induces population growth in a Malthusian world

MALTHUSIAN MODEL

- We estimate a Malthusian model of economy
- In other words: We model the endogeneity of population dynamics

Outcomes:

- New series for productivity in England from 1250-1860
- Estimates of the strength of Malthusian forces
  in pre-industrial and early industrial England

A Malthusian Model of the Economy

Production, Labor and Capital Demand

- Production function:

$$Y_t = A_t Z^\alpha K_t^\beta L_t^{1-\alpha-\beta}$$

where Z is land, K_t is capital, L_t is labor, and A_t is productivity

- Labor demand:

$$W_t = (1-\alpha-\beta) A_t Z^\alpha K^\beta L_t^{-\alpha-\beta}$$

- Capital demand:

$$r_t + \delta = \beta A_t Z^\alpha K_t^{\beta-1} L_t^{1-\alpha-\beta},$$

Labor Supply and Population Dynamics

- Labor supply is given by:

$$L_t = D_t N_t$$

where D_t is days worked per year and N_t is population

- Population growth increasing in income:

$$\frac{N_t}{N_{t-1}} = \Omega(W_{t-1} D_{t-1})^{\gamma} \Xi_t$$

  - γ: Elasticity of population growth with respect to income
  - Ω: Normalizing constant
  - Ξ_t: Population shock

Early Industrial Economy

- Production function:

$$Y_t = A_t Z^{\alpha_t} K_t^{\beta_t} L_t^{1-\alpha_t-\beta_t}$$

  allows for changing factor shares: α_t and β_t

- Demand for land:

$$S_t = \alpha_t A_t Z^{\alpha_t - 1} K^{\beta_t} L_t^{1-\alpha_t-\beta_t}$$

- Use data on land rents (S_t) and capital to pin down evolution of α_t and β_t

- We date onset of Industrial Revolution at 1760 (when we have data on capital from Feinstein 88)

Productivity

Pre-Industrial Economy:

- Standard measure of productivity: A_t

Early Industrial Economy:

- With α_t, β_t changing, A_t no longer good measure of productivity Why
- Malmquist index:

$$M_t = \sqrt{\frac{F_t(Z, K_t, L_t)F_t(Z, K_0, L_0)}{F_0(Z, K_t, L_t)F_0(Z, K_0, L_0)}}.$$

- With constant α and β: $M_t = A_t / A_0$

Productivity

- Productivity has two components:

$$m_t = \tilde{m}_t + \epsilon_{2t}$$

where $m_t = \log M_t$

1. Permanent component:

$$\tilde{m}_t = \mu + \tilde{m}_{t-1} + \epsilon_{1t}$$

$$\epsilon_{1t} \sim \mathcal{N}(0, \sigma^2_{\epsilon_1})$$

2. Transitory component:

$$\epsilon_{2t} \sim \mathcal{N}(0, \sigma^2_{\epsilon_2})$$

POPULATION SHOCKS

- Two types of population shocks:

$$\xi_t = \xi_{1t} + \xi_{2t}$$

where $\xi_t = \log \Xi_t$

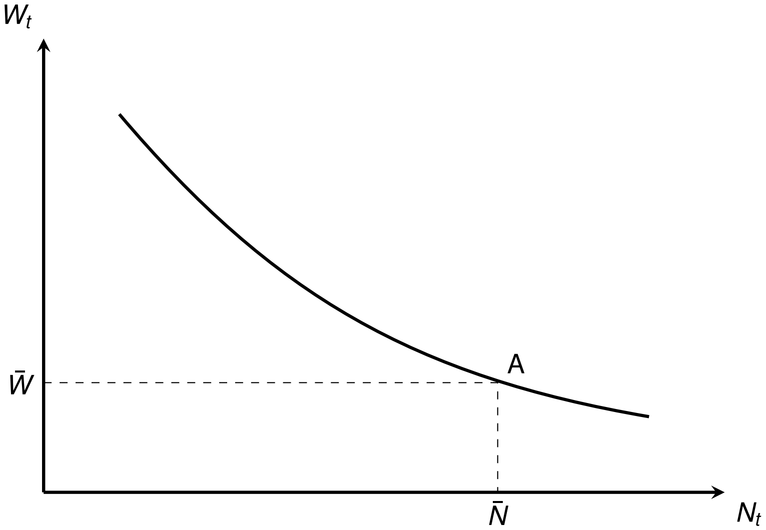
1. Plague shocks:

$$\exp(\xi_{1t}) \sim \begin{cases} \beta(\beta_1, \beta_2), & \text{with probability } \pi \\ 1, & \text{with probability } 1 - \pi \end{cases}$$

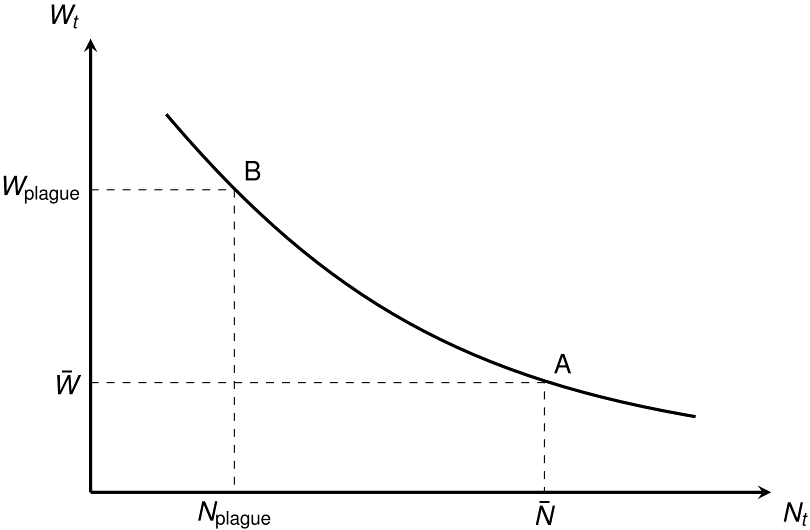
2. Symmetric shocks:

$$\xi_{2t} \sim \mathcal{N}(0, \sigma^2_{\xi_2})$$

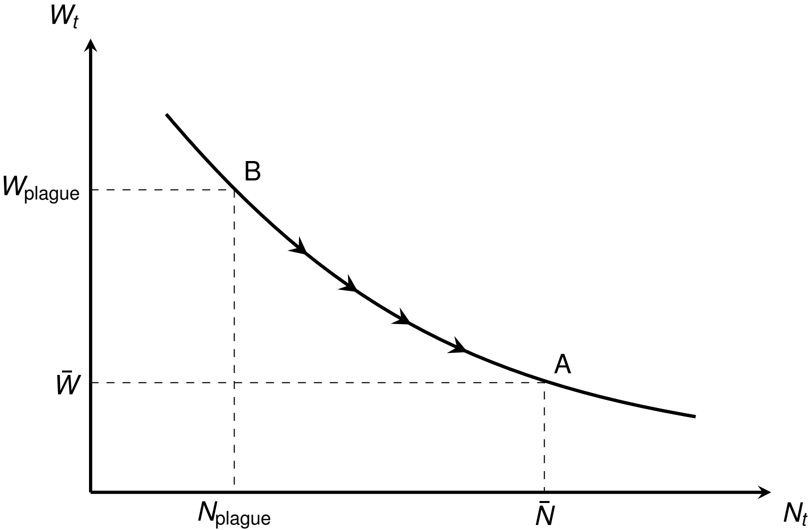
Plague Shock



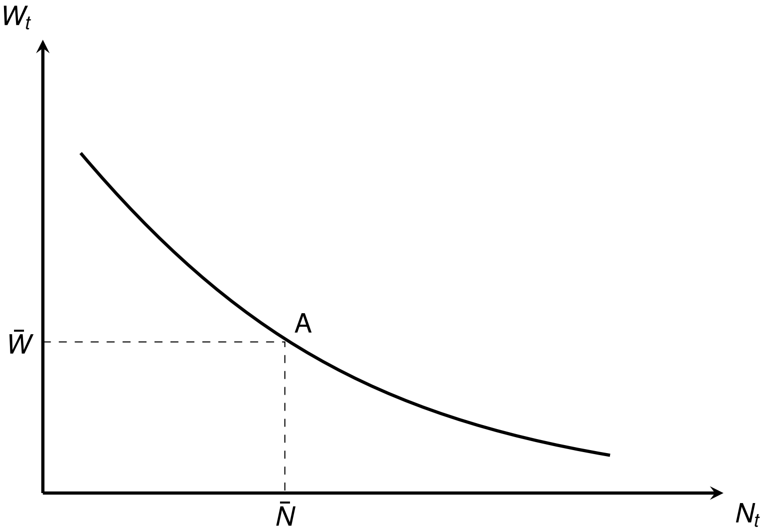
Plague Shock



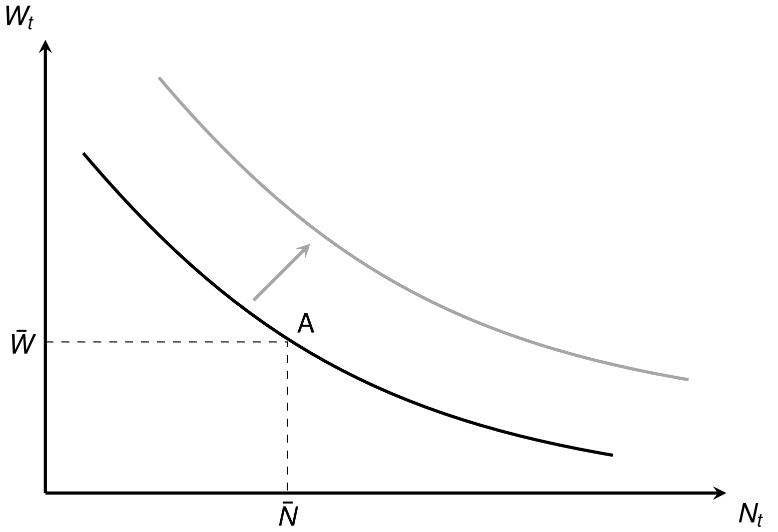
Plague Shock



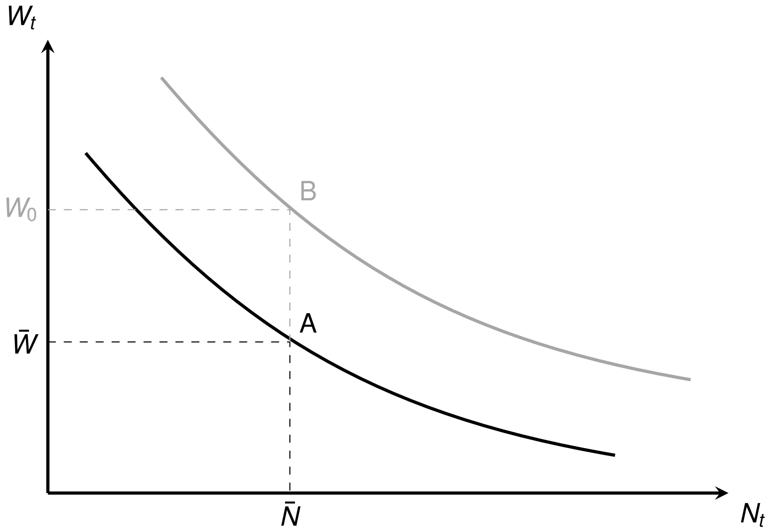
Productivity Shock



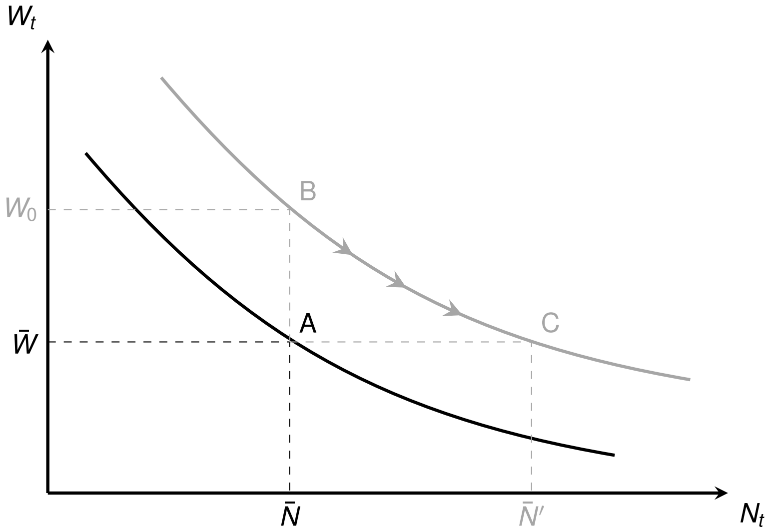
Productivity Shock



Productivity Shock



Productivity Shock



Data and Estimation

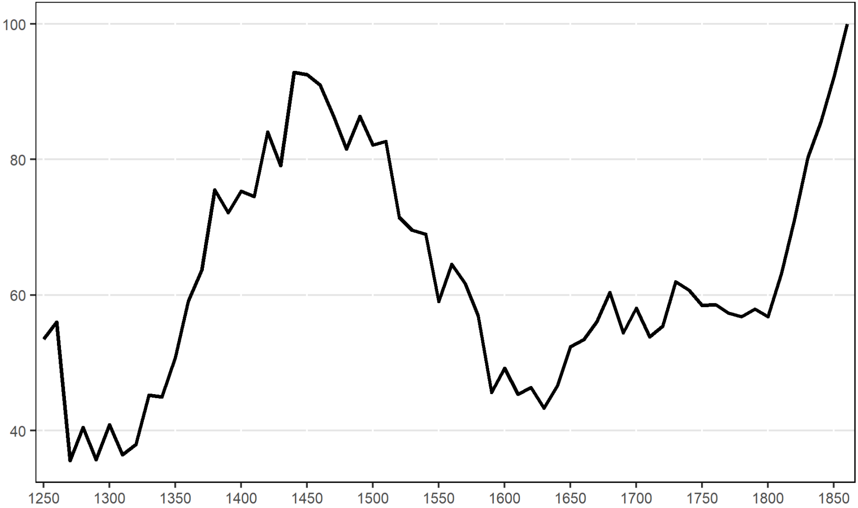
ESTIMATION

- We estimate the model using the following data:
  - Real wages of unskilled builders from Clark (2010) Wages
  - Population from Clark (2007b) Population
  - Days worked from Humphries and Weisdorf (2019) Days
  - Rates of return on land and rent charges from Clark (2002, 2010) R
  - Capital after 1760 from Feinstein (1988) Capital
  - Land rents after 1760 from Clark (2002, 2010) Land Rents

- Sample period: 1250-1860

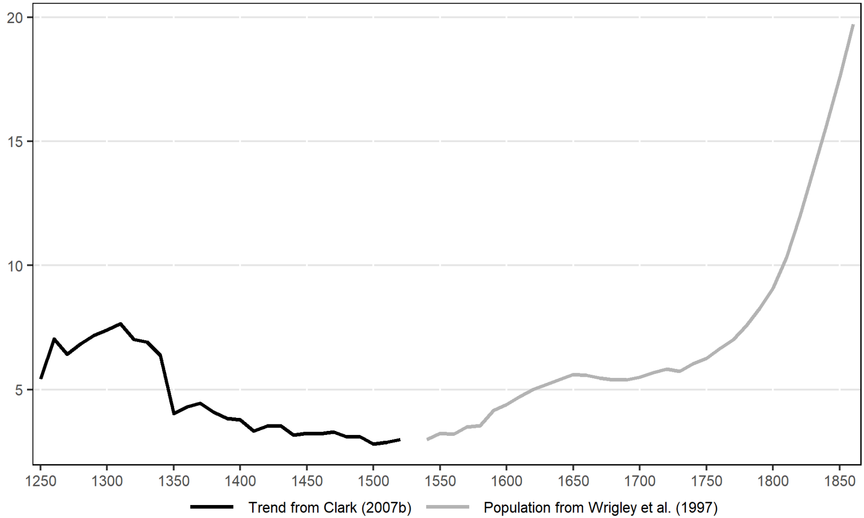
Full Model

Real Wages in England, 1250-1860



Source: Clark (2010) ◂ Back Other Wage Series

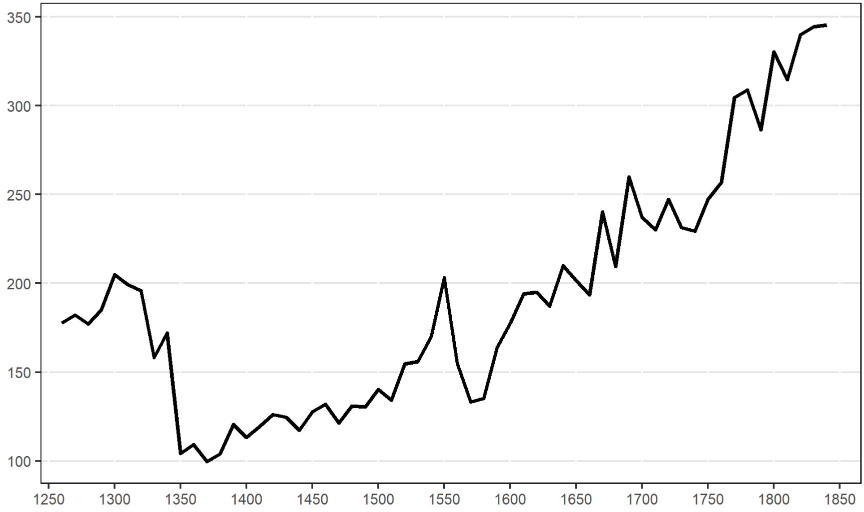
Population Data



Source: Clark (2007b)

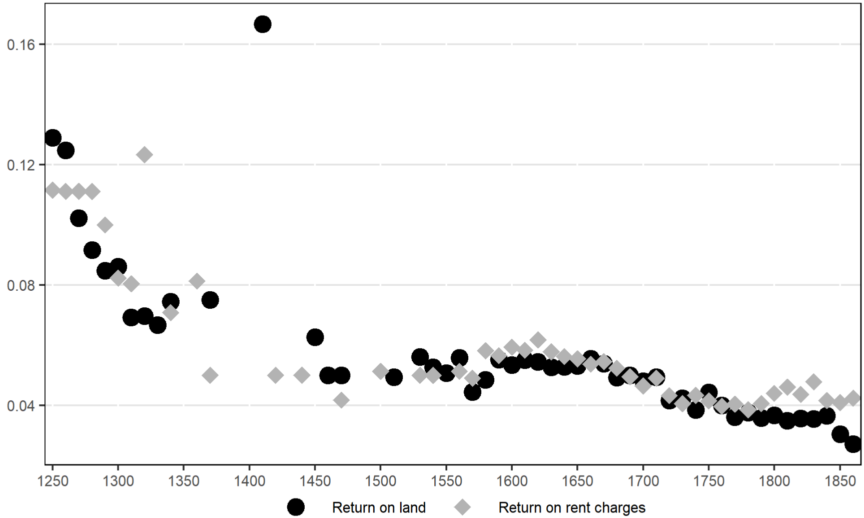
Back Population Details Other Population Series

Days Worked Per Year



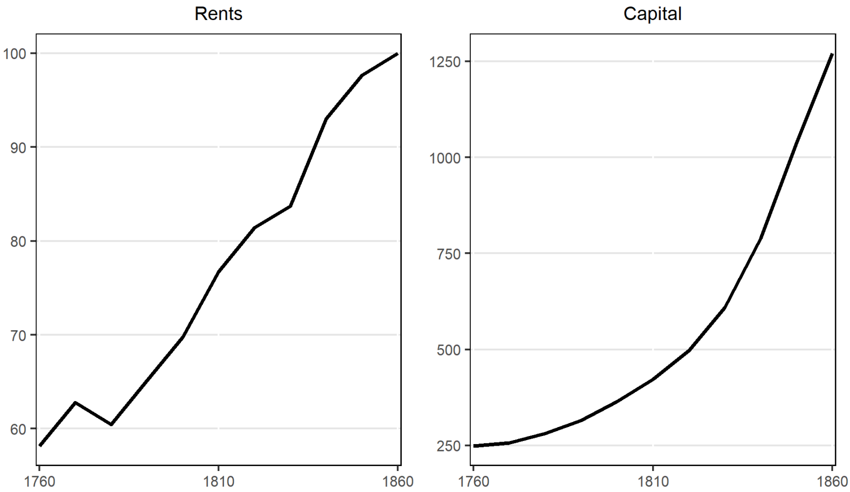
Source: Humphries and Weisdorf (2019) Back Days Worked Details

Rates of Return



Source: Clark (2002,2010) Back Rate of Return Details

Land Rents and Capital after 1760



Source: Clark (2002,2010), Feinstein (1988) Back Details

Estimation Method

- We estimate the model using Bayesian methods
  - Hamiltonian Monte Carlo sampling (HMC)
  - Implemented using software package called Stan
- We choose highly dispersed priors for all parameters ◂ Priors
- We allow for measurement error
- Allow for two breaks in productivity parameters μ, σ_1, σ_2
  - Break in 1760 (Industrial Revolution / capital data available)
  - We allow for one additional break

Method for Selecting Break Date

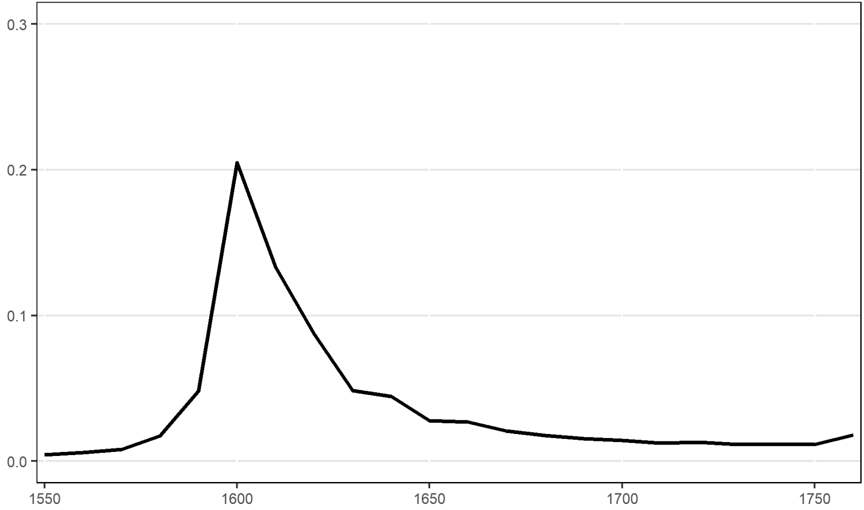
- We estimate a mixture model
- Three regimes for μ, σ_1, σ_2:
  - 1250-1540: $\mu = \mu(1)$
  - 1550-1760: $\mu = (1 - I)\mu(1) + I\mu(2)$
  - 1770-1860: $\mu = \mu(3)$

  (same for σ_1, σ_2)
- I is an indicator variable:
  - Switches from 0 to 1 when 1st break occurs
  - Multinomial distribution
  - Dirichlet prior with concentration vector $0.001 \times (1, ..., 1)$

    (i.e., draws tend to be close to a corner)

When Did Productivity Growth Begin?

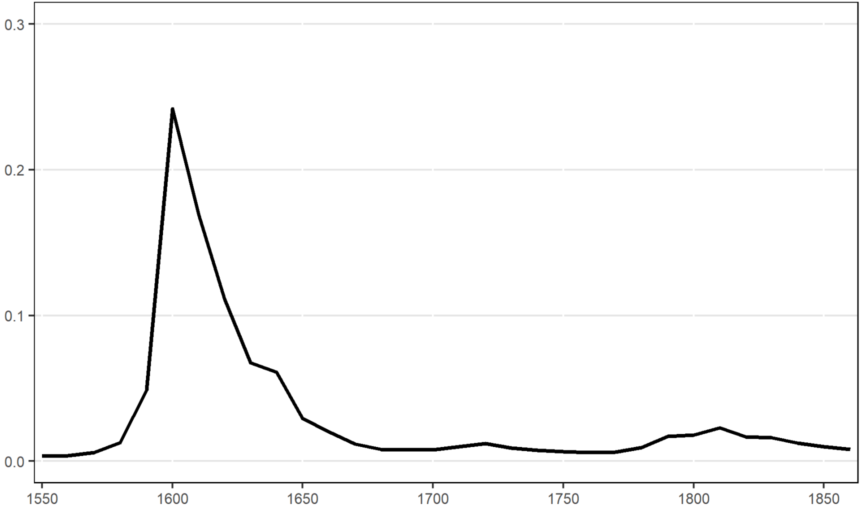
Posterior Probability of Break Date



Prob break before 1640: 56% (68% for 1680). Probability of no break before 1770: 20%.

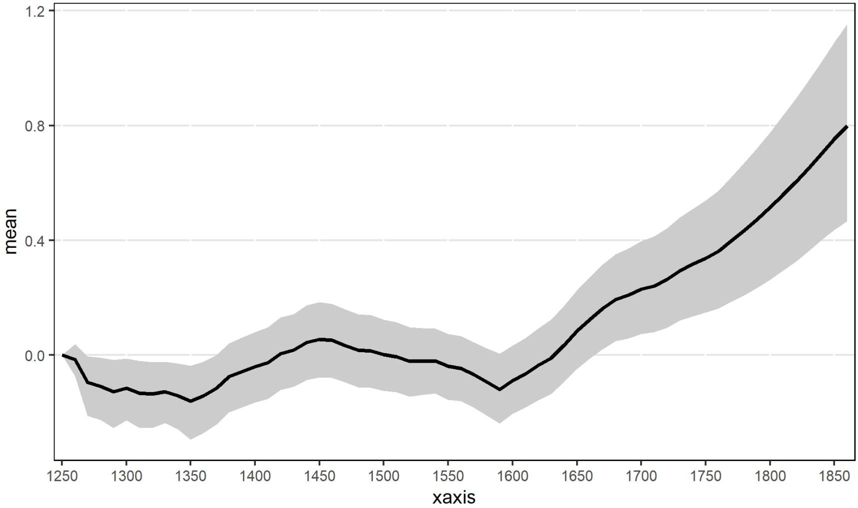
Single Break Break Robustness

Posterior Probability of Single Break



Prob break before 1640: 66% (79% for 1680). Back

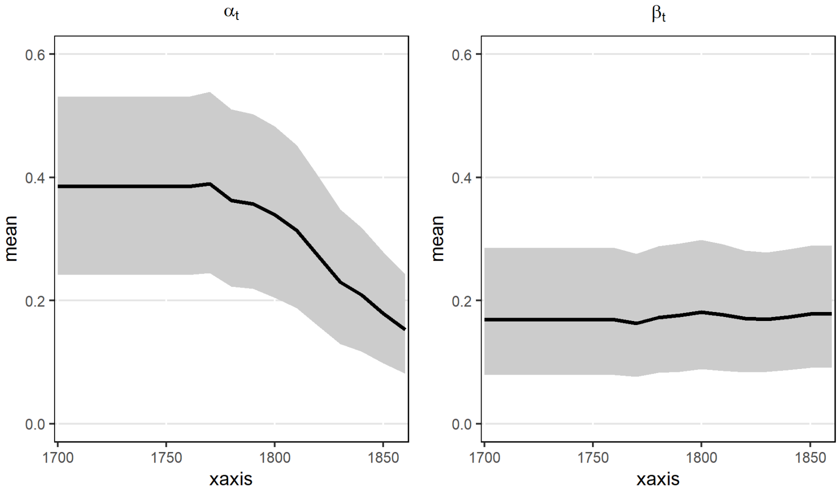
PRODUCTIVITY ($\tilde{m}_t$)



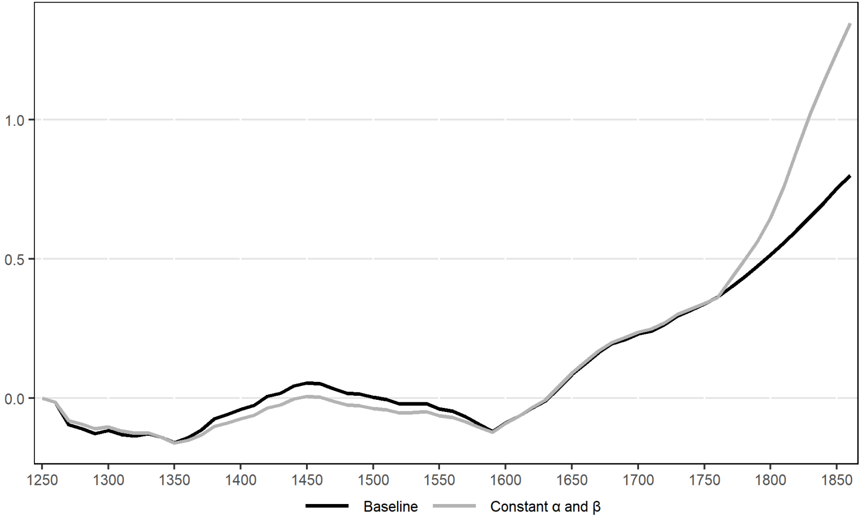
The black line is the posterior mean. The grey area is the 90% credible interval.
$\mu_{<1600} = 0.00$, $\mu_{1600-1760} = 0.03$, $\mu_{>1760} = 0.04$. Parameter Estimates

Productivity and Real Wages

Falling Land Share after 1760



Productivity and Falling Land Share



Real Wages and Labor | Capital and Land Rents

ROBUSTNESS

- Constant days worked Results
- Other real wage series Results
- Broadberry et al. (2015) population estimates Results
- Different priors Results

Strength of Malthusian Population Force

Malthusian Population Force

- After a population shocks (e.g., plague)
  - Wages rise
  - Increase in wages induces population growth
  - Increase in population reduces wages
- Population dynamics:

$$n_{t+1} = \left(1 - \frac{\gamma\alpha}{1-\beta}\right) n_t + \text{constant}.$$

- Key parameters:
  - Response of population growth to real wages (γ)
  - Slope of labor demand curve ($\alpha/(1-\beta)$)

Malthus Parameters

	Mean	St Dev	2.5%	97.5%
γ	0.09	0.02	0.05	0.13

- Small γ implies that the Malthusian force was relatively weak
- Doubling of real income increased population growth rate by only 6% per decade
- Half-life of plague induced drops in the population pre-1760 was roughly 150 years
- Half-life rose as α_t fell after 1760, to 420 years by 1860

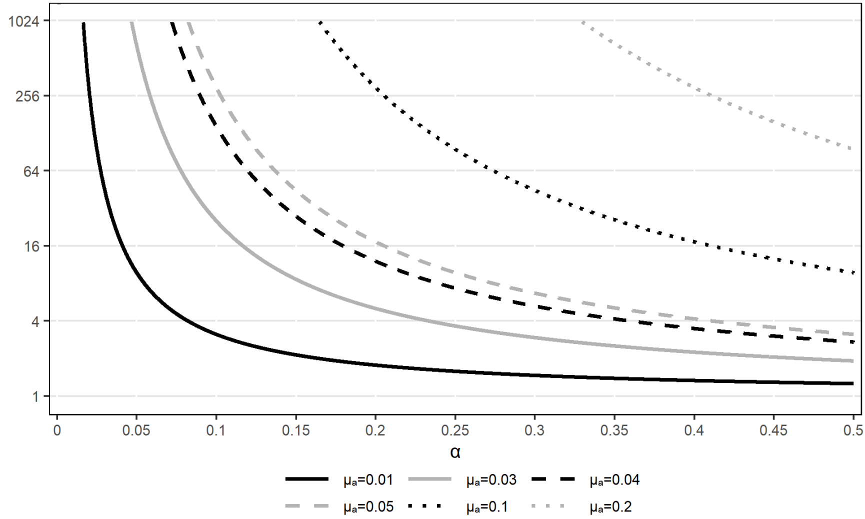
Gamma Dist | Population Parameters | Population Shocks | Demographic Transition

Overwhelming Malthus

- Simplistic View: Wages always return to subsistence in the long run in a Malthusian world
- Not true if productivity growth is positive
  - Productivity growth constantly pushing wages up
  - Malthusian population force constantly pushing wages down
- Steady state wage depends on strength of two opposing forces:

$$\bar{w} = \frac{\mu}{\alpha\gamma} + \text{constant},$$

Steady State Real Wage for different μ and α



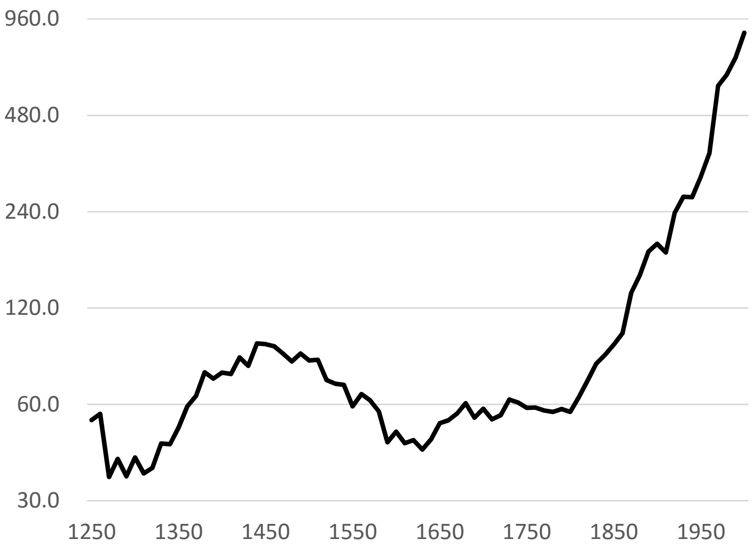
The figure has α on the x-axis and the steady state log real wage $\bar{w}$ on the y-axis. Each line is for a different level of productivity growth.

Conclusion

- New estimates of productivity for England from 1250-1860
  - Backed out from shifts in labor demand curve
  - Estimated using a Malthus model
- Main results:
  - Zero productivity growth before 1600
  - Productivity growth began in 1600: 3% per decade
  - Modest speed-up after 1760: 4% per decade
  - Falling land share allowed economy to grow faster after 1760
  - Weak Malthusian forces: doubling of wages increases population growth by 6% per decade

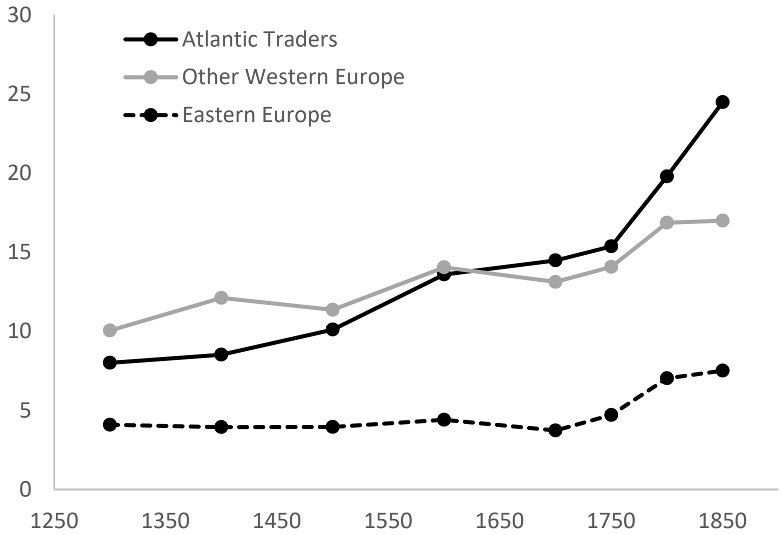
Appendix

Real Wages in England, 1250-2000



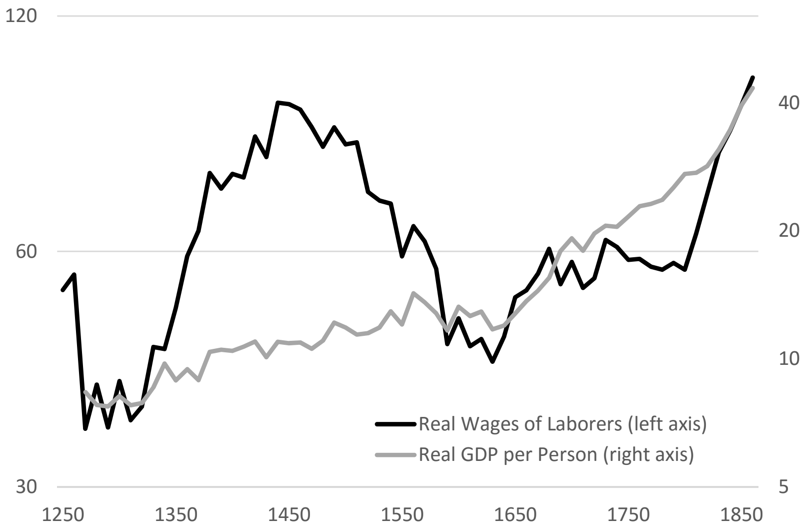
Source: Clark (2005, 2010) ◂ Back

Urbanization Rates, 1300-1850



Source: Acemoglu, Johnson, and Robinson (2005) Back

Real Wages and GDP per Person



Source: Clark (2010) and Broadberry et al. (2015) Back

Productivity

- Changing production function:

$$Y_t = A_t Z^{\alpha_t} K^{\beta_t} L_t^{1-\alpha_t-\beta_t},$$

- Consider change of units for labor: $\tilde{L}_t \equiv \psi L_t$
- Then production function becomes:

$$Y_t = A_t Z^{\alpha_t} K_t^{\beta_t} L_t^{1-\alpha_t-\beta_t} = \frac{A_t}{\psi^{1-\alpha_t-\beta_t}} Z^{\alpha_t} K_t^{\beta_t} \tilde{L}_t^{1-\alpha_t-\beta_t}.$$

- Is productivity A_t or $A_t/\psi^{1-\alpha_t-\beta_t}$?

Back

Full Model

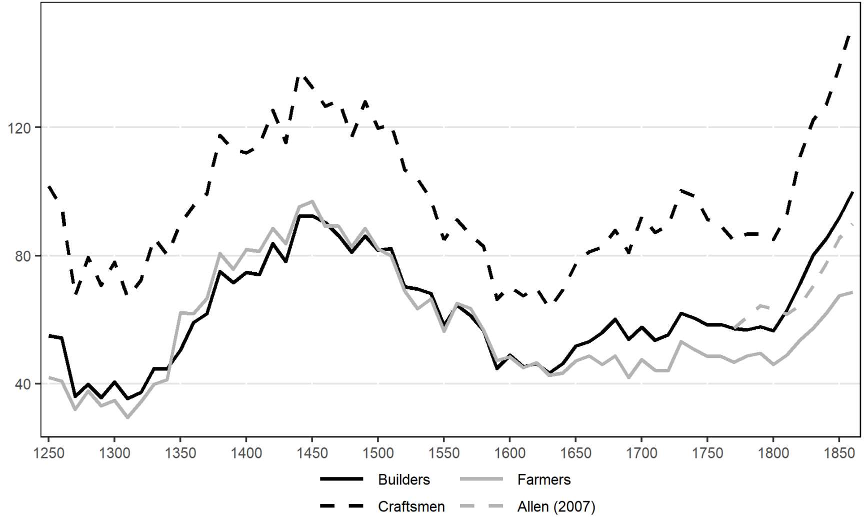
$$
\begin{aligned}
w_t &= \phi_t + \frac{1}{1-\beta_t} a_t - \frac{\alpha_t}{1-\beta_t}(d_t + n_t) - \frac{\beta_t}{1-\beta_t} \log(r_t + \delta) \\
\phi_t &= \log \beta_t + \log(1 - \alpha_t - \beta_t) + \frac{\alpha_t}{1-\beta_t} z - (\alpha_t + \beta_t)\lambda \\
s_t &= w_t + n_t + d_t - z + \log \alpha_t - \log(1 - \alpha_t - \beta_t) \\
k_t &= w_t + l_t - \log(r_t + \delta) + \log \beta_t - \log(1 - \alpha_t - \beta_t) \\
n_t &= n_{t-1} + \omega + \gamma(w_{t-1} + d_{t-1}) + \xi_{1t} + \xi_{2t} \\
m_t &= \hat{a}_t + \hat{\alpha}_t z + \hat{\beta}_t \bar{k}_t - \left(\hat{\alpha}_t + \hat{\beta}_t\right)(\bar{d}_t + \bar{n}_t) \\
m_t &= \tilde{m}_t + \epsilon_{2t} \\
\tilde{m}_t &= \mu + \tilde{m}_{t-1} + \epsilon_{1t}
\end{aligned}
$$

$$
\exp(\xi_{1t}) \sim \begin{cases} \beta(\beta_1, \beta_2), & \text{with probability } \pi \\ 1, & \text{with probability } 1 - \pi \end{cases}
$$

$$
\epsilon_{1t} \sim \mathcal{N}(0, \sigma^2_{\epsilon_1}), \quad \epsilon_{2t} \sim \mathcal{N}(0, \sigma^2_{\epsilon_2}), \quad \xi_{2t} \sim \mathcal{N}(0, \sigma^2_{\xi_2})
$$

Back

Alternative Real Wage Series



Back

Methodology for Population

- Data on Population:
  - 1540-1860: Wrigley et al. (1997)
  - 1250-1520: Clark (2007): Panel of village and manor population estimates
- Clark (2007) constructs population series for 1250-1540
  - We cannot directly use this series
  - It embeds assumptions about evolution of productivity
- Use time fixed effects Clark estimates from village/manor data
- Allow for measurement error

Back

Methodology for Population

We assume:

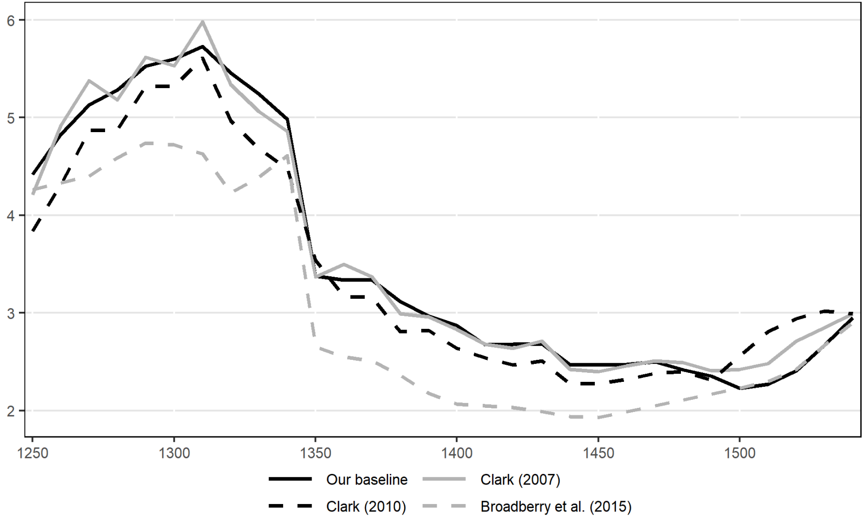
$$n_t = \psi + \hat{n}_t + \iota_t^n$$

where

- n_t: True population
- $\hat{n}_t$: Measured population
  - 1250-1520: Clark's time fixed effects
  - 1540-1860: Wrigley et al. (1997) series
- ψ: Normalization constant
  - Zero after 1540
  - Estimate for pre-1540 period
- $\iota_t^n \sim t_{\nu_n}(0, \sigma_n^2)$ is measurement error

Back

Alternative Population Series



Back

Days Worked Per Year

- Humphries and Weisdorf (2019):
  - New series on income of workers on annual contracts
  - Infer days worked by dividing annual contract payments by Clark's day wages
- Issue: Data is missing for the 1250, 1850 and 1860
- Infer missing data assuming that days worked follow a random walk

$$d_t = d_{t-1} + \eta_t \quad \text{where} \quad \eta_t \sim \mathcal{N}(0, \sigma_d^2)$$

- Allow for measurement error in days worked

$$d_t = \tilde{d}_t + \iota_t^d \quad \text{where} \quad \iota_t^d \sim t_{\nu_d}(0, \tilde{\sigma}_d^2)$$

where d_t are true days worked and $\tilde{d}_t$ are measured days worked

Back

Rates of Return Details

- Rate of return on agricultural land: R/P

  R: rental payment, P: Price of land

- Rate of return on "rent charges": R/P

  R: annual payment, P: Price of rent charge

- Rent Charge: Perpetual nominal obligation secured by land or houses

- Each noisy measure of rate of return on capita:

$$r_t = \tilde{r}_{it} + \iota^r_{it} \qquad \text{where} \qquad \iota^r_{it} \sim t_{\nu_{ir}}(0, \tilde{\sigma}^2_{ir})$$

  where r_t is true rate of return on capita, $\tilde{r}_{it}$ are noisy measures

- When return is missing: $r_t \sim \mathcal{N}_{(0,.2)}(r_{t-1}, 0.01^2)$

Back

Land Rents and Capital Details

- Capital only available after 1760 (Feinstein, 1988)
- We assume both series are measured with error:

$$s_t = \tilde{s}_t + \iota_t^s \qquad \text{where} \qquad \iota_t^s \sim t_{\nu_s}(0, \tilde{\sigma}_s^2)$$

$$k_t = \tilde{k}_t + \iota_t^k \qquad \text{where} \qquad \iota_t^k \sim t_{\nu_k}(0, \tilde{\sigma}_k^2)$$

where s_t and k_t are true land rent and capital stock,
$\tilde{s}_t$ and $\tilde{k}_t$ are noisy measures of land rents and the capital stock

Back

Priors

Parameter	Prior	Parameter	Prior
α	$\mathcal{U}(0,2)$	γ	$\mathcal{U}(-2,2)$
φ^x	$\mathcal{N}(0,100^2)$	ψ	$\mathcal{N}(10.86,0.07^2)$
ω	$\mathcal{N}(0,1)$	μ	$\mathcal{N}(0,1)$
μ_{ξ_1}	$\mathcal{U}(0.5,0.9)$	ν_{ξ_1}	$\mathcal{P}_I(0.1,1.5)$
π	$\mathcal{U}(0,0.5)$	δ	$\mathcal{N}_{(0,0.2)}(0.1,0.05^2)$
$\sigma^2_{\epsilon_1}$	$I\Gamma(3,0.001)$	$\sigma^2_{\epsilon_2}$	$I\Gamma(3,0.005)$
$\sigma^2_{\xi_2}$	$I\Gamma(3,0.005)$	σ^2_n	$I\Gamma(3,0.005)$
σ^2_d	$I\Gamma(3,0.005)$	$\tilde{\sigma}^2_s$	$I\Gamma(3,0.005)$
$\tilde{\sigma}^2_k$	$I\Gamma(3,0.005)$	$\tilde{\sigma}^2_{ir}$	$I\Gamma(3,0.005)$
ν_n^{-1}	$\mathcal{U}(0,1)$	ν_d^{-1}	$\mathcal{U}(0,1)$
ν_s^{-1}	$\mathcal{U}(0,1)$	ν_k^{-1}	$\mathcal{U}(0,1)$
ν_{ir}^{-1}	$\mathcal{U}(0,1)$		

Re-parametrization of beta Densities for Shocks

Prior for ψ implies that pre-Black Death population was between 4.5 and 6 million with 95% prior probability.

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Re-parametrization of a Beta Distribution

- The plague shocks follow the distribution:

$$\xi_{1t} \sim \begin{cases} \log \beta(\beta_1, \beta_2), & \text{with probability } \pi_t \\ 1, & \text{with probability } 1 - \pi_t \end{cases}$$

- The mean μ_{ξ_1} and pseudo sample size ν_{ξ_1} of the beta distribution are defined as:

$$\mu_{\xi_1} = \frac{\beta_1}{\beta_1 + \beta_2}$$

$$\nu_{\xi_1} = \beta_1 + \beta_2$$

- As a flat prior, Gelman et al. (2013, p. 110) recommend:

$$\mu_{\xi_1} \sim \mathcal{U}(0, 1)$$

$$\nu_{\xi_1} \sim \mathcal{P}_I(.1, 1.5)$$

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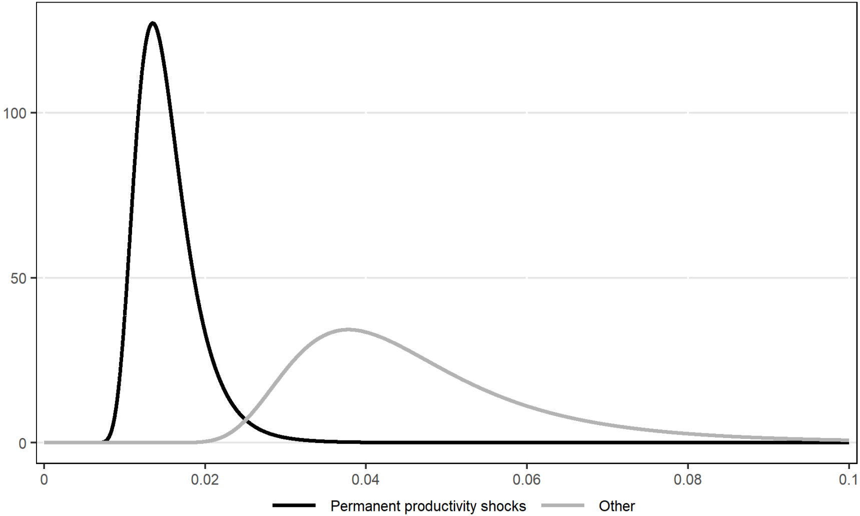
Prior Densities for Standard Deviations



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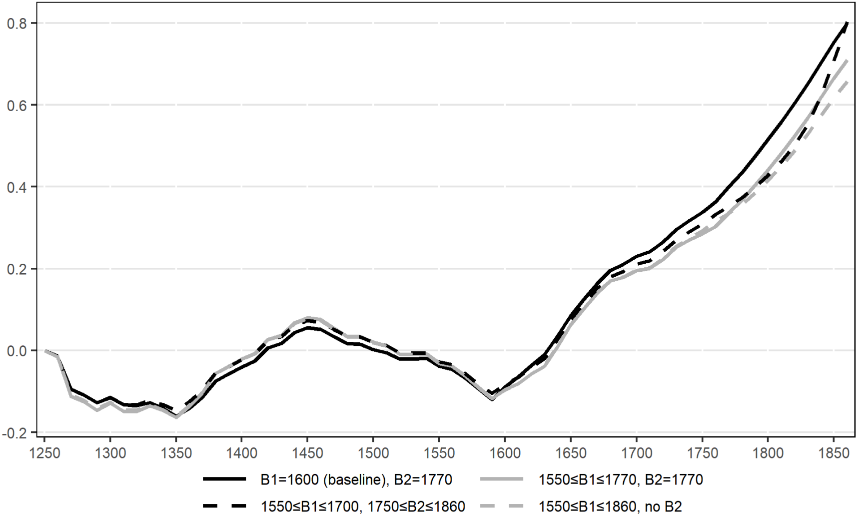
Posterior Probability of Breaks



B1 and B2 stand for breaks 1 and 2. Back

Productivity Growth

	Mean	St Dev	2.5%	97.5%
$\mu_{a,t<1600}$	-0.00	0.01	-0.02	0.01
$\mu_{a,1600\leq t<1770}$	0.03	0.01	0.01	0.04
$\mu_{a,t\geq 1770}$	0.04	0.01	0.02	0.07
$\sigma_{\epsilon_1,t<1600}$	0.04	0.01	0.02	0.06
$\sigma_{\epsilon_1,1600\leq t<1770}$	0.02	0.01	0.01	0.04
$\sigma_{\epsilon_1,t\geq 1770}$	0.02	0.01	0.01	0.04
$\sigma_{\epsilon_2,t<1600}$	0.06	0.01	0.04	0.08
$\sigma_{\epsilon_2,1600\leq t<1770}$	0.04	0.01	0.02	0.05
$\sigma_{\epsilon_2,t\geq 1770}$	0.04	0.01	0.02	0.06

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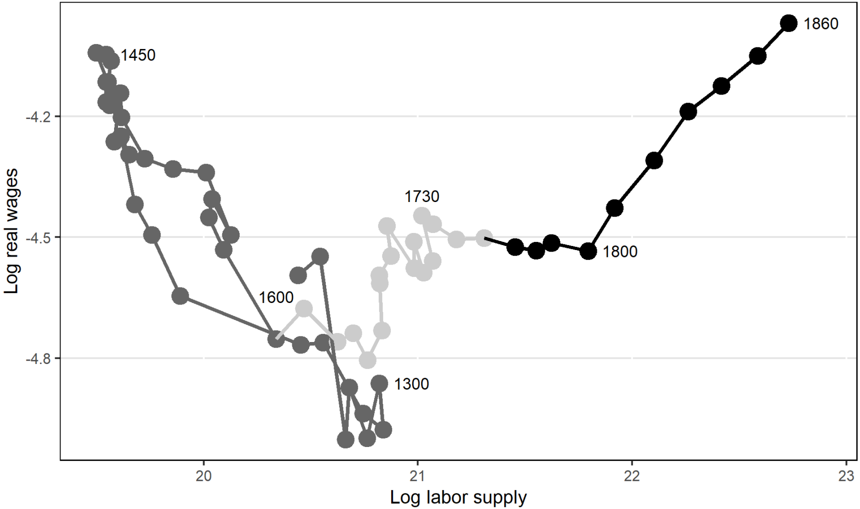
Real Wages and Labor Supply



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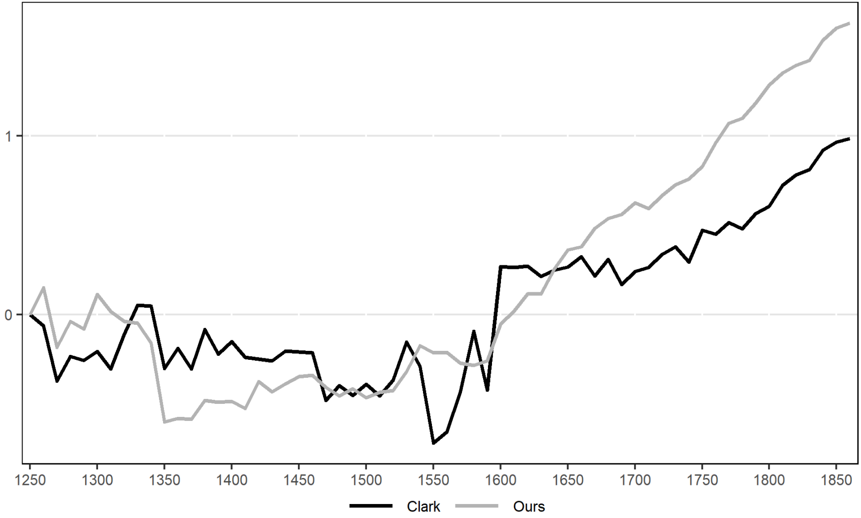
Land Rents



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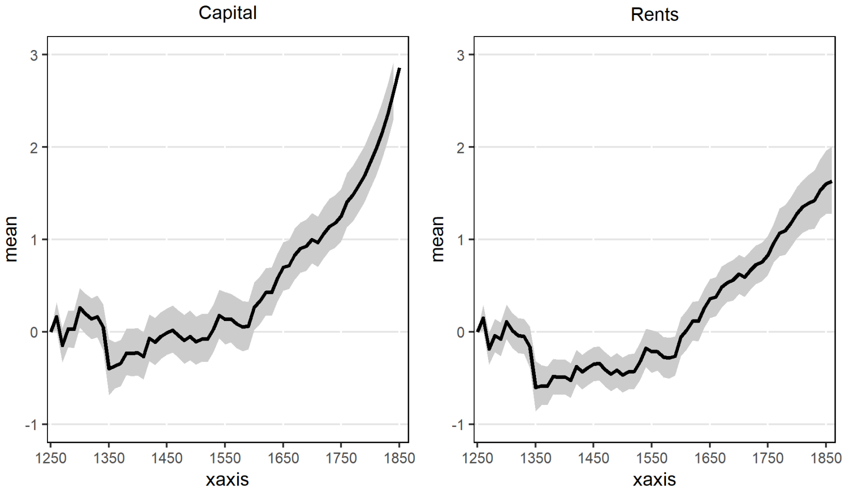
Capital and Land Rents



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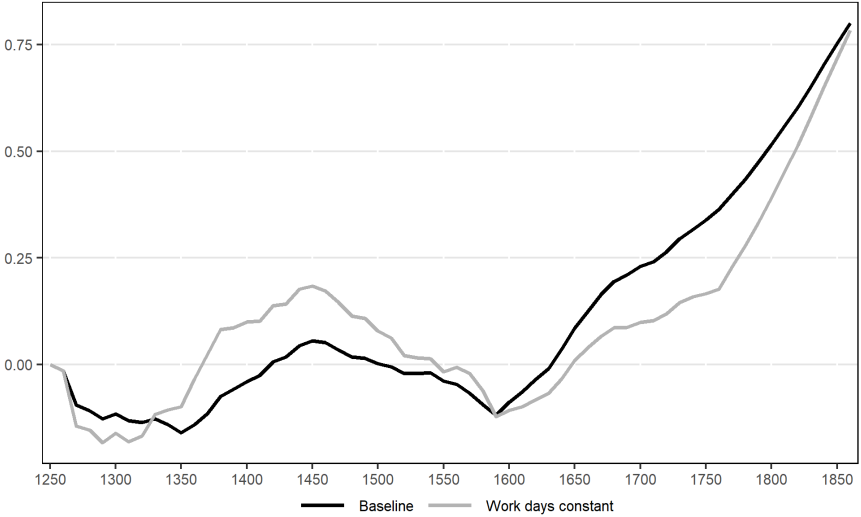
Clark Land Rents

Productivity: Constant Days Worked



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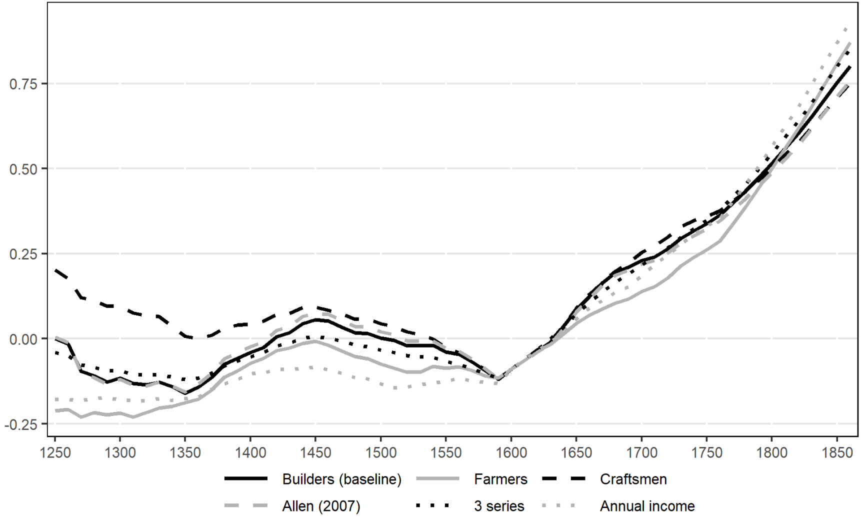
Productivity: Other Real Wage Series



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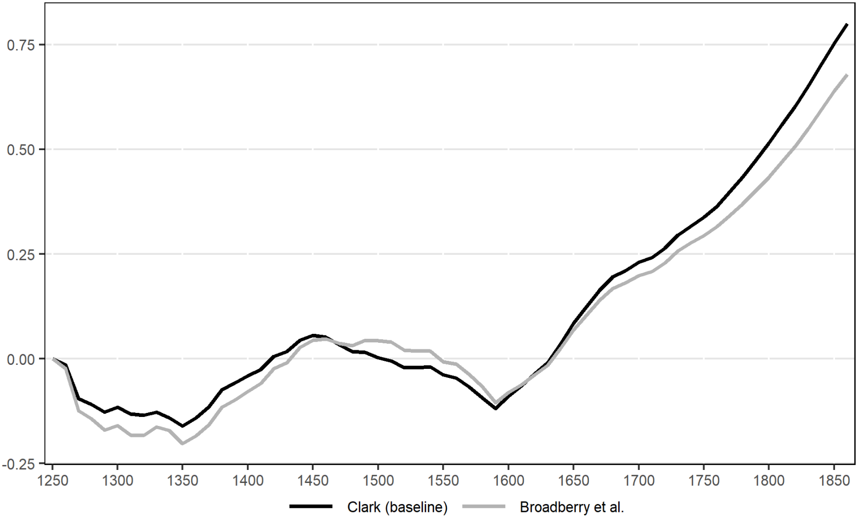
PRODUCTIVITY: ALTERNATIVE POPULATION SERIES



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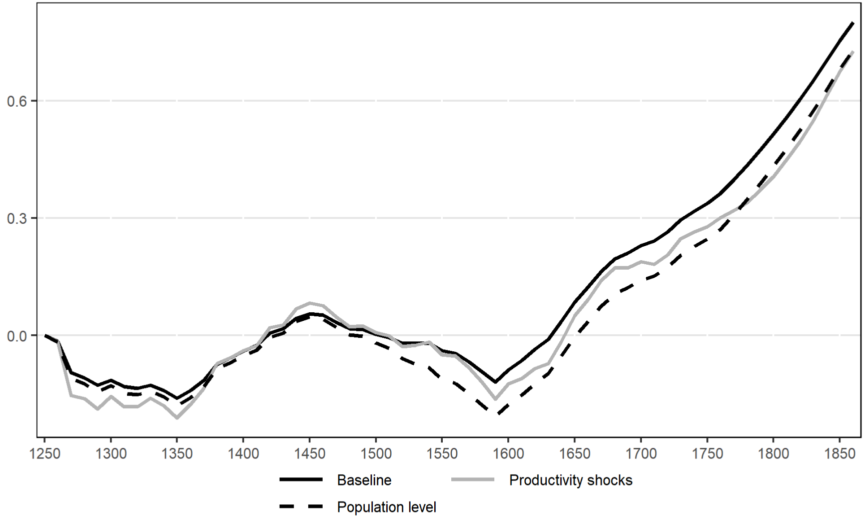
Productivity: Alternative Priors



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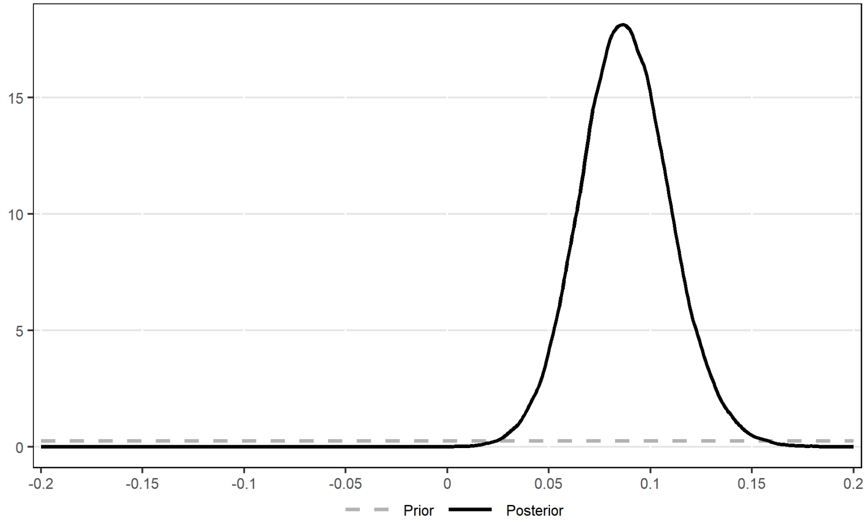
Prior and Posterior Densities for γ



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Population Parameters

	Mean	St Dev	2.5%	97.5%
Population Parameters				
$\pi_{t<1680}$	0.08	0.07	0.01	0.32
$\pi_{t\geq 1680}$	0.06	0.06	0.00	0.21
μ_{ξ_1}	0.64	0.11	0.50	0.87
ν_{ξ_1}	6.47	6.36	1.10	23.89
σ_{ξ_2}	0.06	0.01	0.04	0.07
Population Measurement Error Parameters				
$\sigma_{n,t<1540}$	0.04	0.01	0.02	0.06
$\sigma_{n,t\geq 1540}$	0.03	0.00	0.02	0.04
$\nu_{n,t<1540}$	9.30	27.59	1.16	64.21
$\nu_{n,t\geq 1540}$	90.38	954.32	2.17	318.29

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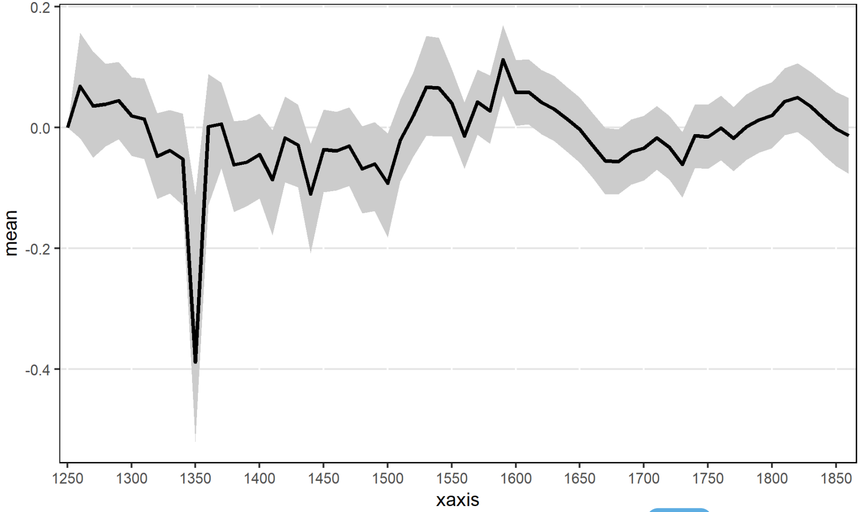
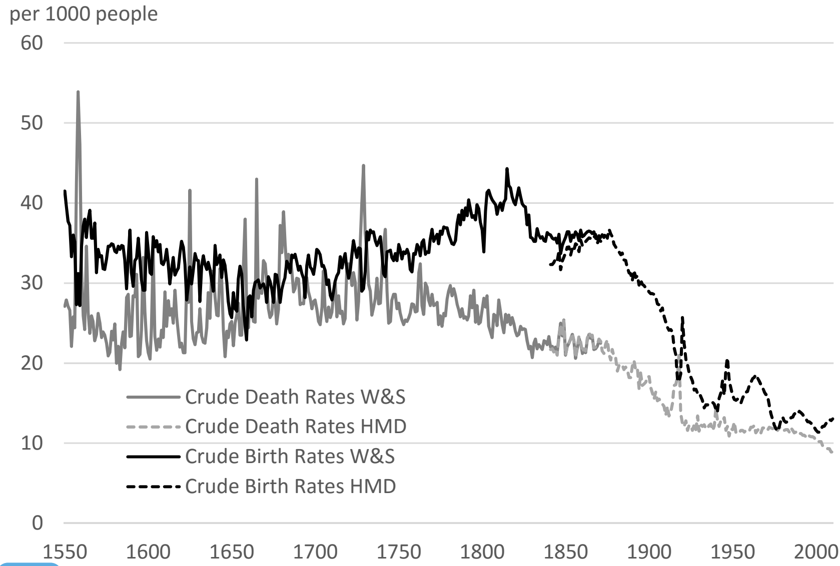
Population Shocks



The black line is the posterior mean. The grey area is the 90% credible interval ◂ Back

The Demographic Transition



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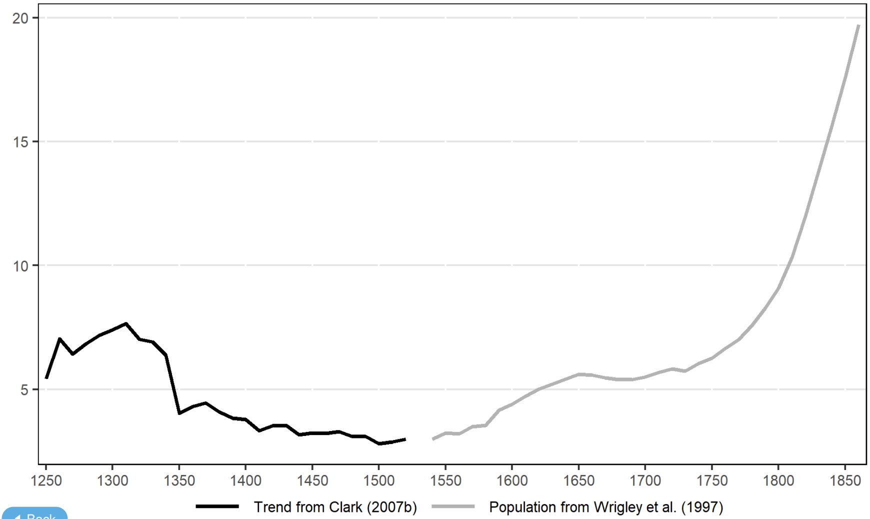
Population Explosion After 1750



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