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On The Role of Trademarks: From Micro Evidence to Macro Outcomes

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Abstract

What are the effects of trademarks on the U.S. economy? Evidence from comprehensive firm-level data on trademark registrations and outcomes suggests that trademarks protect firm value and are associated with higher firm growth and marketing activity. Motivated by this evidence, trademarks are introduced in a general equilibrium framework to quantify their aggregate effects. In the model, firms invest in product quality and marketing to build a customer base subject to depreciation. Firms can register trademarks to protect their customer base and reduce the cost of informing consumers. The model's predictions on the incidence and timing of trademark registrations, as well as firm growth and advertising expenditures, are consistent with the empirical evidence. Analysis of the calibrated model indicates that the U.S. economy with trademarks generates higher product variety, quality, and welfare, along with higher concentration, compared to the counterfactual economy with no trademarks.

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1 Introduction

Nearly all firms strive to build a reputation and aim to be easily recognized by consumers. Trademark ownership helps this recognition by formally associating words, marks, or pictures with a firm's goods or services, and by legally protecting the firm against imitation and infringement. The damage to commerce from infringing activity is indeed large. For instance, in 2016 the global trade in infringing products amounted to about 3.3% of world trade, and the businesses most affected were primarily based in OECD countries, including the United States.[1] The strength of institutions and enforceability matter for the extent of this damage. A U.S. International Trade Commission survey found that U.S. firms doing business abroad report significant losses in sales and profits due to intellectual property infringement, including damage to brand names and reputation[2]. Trademark infringement was the most common form of infringement cited by firms, with nearly 92% of firms in the consumer goods manufacturing sector reporting material losses attributable to such infringement.

In the United States, trademark protections were first introduced by The Lanham (Trademark) Act of 1946. The Act prohibits activities such as trademark infringement and dilution, and false advertising. Trademark registrations in the United States and other countries have been rising in recent decades (Figure 1) [3]. U.S. firms' marketing activities, measured by advertising expenditures, have also grown substantially (Figure 2). At the same time, reallocation towards productive and large firms has resulted in increasing concentration of economic activity[4]. These trends, when taken together, suggest a connection between firms' marketing and brand protection efforts on one hand, and consumer reallocation and concentration, on the other. What role do trademarks play in this connection?

This paper studies the effects of federally registered trademarks, starting at the micro level with empirical evidence from firms and building up towards the macro implications in a general equilibrium framework. The model and its analysis are motivated by data on firm trademark registrations and outcomes. This paper is the first to analyze the incidence and timing of the first trademark registration over the firm life-cycle using data for all employer businesses in the United States. The analysis uses the matched trademark-firm data set introduced in Dinlersoz et al. (2018). The data set contains information on trademark registrations from the USPTO's Trademark Case File Database (TCFD), combined with firm-level information from the Longitudinal Business Database (LBD), allowing the observation of key events in a firm's life cycle, such as firm birth, subsequent growth, first trademark registration, and firm death. Indicators of firm innovation (first patent) and broader market expansion (first exporting activity) are also added. Furthermore, information on firm-level advertising expenditures available from the U.S. Census Bureau's economic censuses and surveys is incorporated to provide evidence on the connection between marketing activity and trademark registrations.

Empirical findings suggest a high degree of selection into trademark registrations. Firms that are born larger, innovate early, grow more rapidly and expand geographically are more likely to register trademarks, and tend to do so earlier in their life cycles. When compared with observationally similar

[1]See the 2019 OECD-EUIPO report, Trends in Trade in Counterfeit and Pirated Goods, https://euipo.europa.eu/ohimportal/en/web/observatory/trends-in-trade-in-counterfeit-and-pirated-goods.

[2]For instance, see the 2011 USITC report China: Effects of Intellectual Property Infringement and Indigenous Innovation Policies on the U.S. Economy, https://www.usitc.gov/publications/332/pub4226.pdf.

[3]See also Dinlersoz et al. (2018) and Graham et al. (2018) for recent trends in trademark activity.

[4]See, e.g., Autor et al. (2020).

firms without trademarks, firms with trademarks grow more rapidly after registering their trademarks. Furthermore, firms with trademarks also engage in more intense marketing. They have, on average, higher advertising expenditures, and these expenditures grow after trademark registration.

The empirical analysis also offers new evidence on how much registered trademarks matter in protecting a firm's value. Measuring the protective role of trademarks is challenging. To further explore the link between the protective role of trademarks and firm outcomes, the analysis relies on a trade shock that increases foreign competition for domestic firms and the likelihood of depreciation in their customer base. The entrance of China into the World Trade Organization (WTO) and the resulting rise in U.S. imports from China had an adverse effect on U.S. firms, especially on those with a higher degree of exposure to import competition—some of which may be imitative and infringing. The findings indicate that firms with trademarks registered prior to the shock (pre-existing protection) fare better relative to those without trademarks, and more so in the case of firms that have a higher degree of exposure to the shock.

Motivated by the empirical evidence, a model is proposed where consumer information about firms and their products is central. Firms invest in product development and gradually grow by building a customer base through marketing. However, firms also experience customer depreciation for a variety of reasons, including imitative or infringing competition. At any point in their life-cycles, firms can choose to register a trademark and mitigate customer depreciation. Building on the insights by Landes and Posner (Landes and Posner 1987, 1988, 2003), trademarks play two fundamental roles in the model: (i) help consumers identify a firm and its product, and reduce informational frictions between consumers and firms, making it easier for firms to reach and attract consumers, and (ii) protect a firm's customer base from depreciation through imitation and infringement. The first role is only possible because of the second, since it is the protection that prevents consumer confusion and allows unique identification of a firm or product. These roles are critical for the provision of quality and marketing. By protecting a firm's customer base, trademarks enhance firm value and encourage investment in quality and marketing. At the same time, trademarks distinguish firms and generate broader consumer awareness about a firm, all of which help a firm more effectively disseminate information about itself and increase the efficiency of marketing.

The model has predictions on which types of firms register trademarks, and when they do so in their life cycles, as well as the effect of trademarks on firm growth and marketing before and after a registration. Firm-level data on trademark registrations and firm outcomes are used to calibrate the model. To assess the effects of trademarks, the calibrated economy with trademarks is compared to a counterfactual economy with no trademarks. The economy with trademarks exhibits a broader range of quality and higher average quality. It also generates more marketing activity. In particular, high-quality firms with trademarks engage in more intense marketing, acquire more consumers, and can better retain their customers due to the protection provided by trademarks. As a result, they grow faster and achieve larger sizes. Concentration is higher in the economy with trademarks, as more consumers, and inputs of production, are allocated towards higher quality, larger firms with trademarks. Overall, there are significant welfare gains from the trademark system in the United States due to the more efficient allocation of production and consumption towards higher-quality firms. Further analysis also indicates that trademark protection substantially mitigates the adverse effects of an exogenous increase in imitative and infringing activity on product quality and welfare.

There is a growing empirical literature on the connection between trademarks and firm outcomes. Due to a lack of comprehensive data linking firm outcomes with trademark ownership in the United States, a large part of the empirical literature has developed around the data for other countries. In particular, research with European and Australian data has documented a number of empirical regularities[5]. Prior empirical work using U.S. data has been mainly limited to publicly-traded firms[6]. Systematic evidence on the timing and incidence of trademark registrations over the firm life-cycle and on the connection between firms' marketing activity and trademark ownership has been lacking for the broader set of U.S. firms. In addition, the implications of trademark protection for the aggregate economy have not been studied in a general equilibrium context. This paper fills these gaps by using a macroeconomic framework with firm dynamics informed by micro data moments from the matched trademark-firm data for all employer businesses in the United States.

The modeling approach is influenced by early work on the economics of trademarks, notably Landes and Posner 1987, 1988, 2003 and Economides 1988, 1998. These studies emphasize the role of trademarks in reducing consumer search costs and protecting a firm against imitation and infringement, thereby enhancing the provision of product quality. Nevertheless, this earlier body of work does not study firm dynamics, nor do they assess the macroeconomic implications of trademarks. The process of building a customer base through marketing is related to recent work on the role of customer capital in firm and industry dynamics[7]. The current model highlights the specific role of trademarks in the building and protection of a firm's customer base. The role of marketing in reducing informational frictions between consumers and firms is similar to that in Dinlersoz and Yorukoglu (2012), which emphasizes the informative role of advertising[8].

The rest of the paper is organized as follows. Empirical analysis is presented in Section 2. The model is introduced in Section 3, followed by the definition of the model's stationary equilibrium in Section 4. Section 5 studies the properties of the stationary equilibrium. Section 6 discusses calibration and the properties of the benchmark economy. The significance of the U.S. trademark system based on the calibrated model is quantified in Section 7. Section 8 concludes.

[5]Schautschick and Greenhalgh (2016) survey provides an excellent review of this literature. In the case of China, Alfaro et al. (2022) analyze various effects of the China's Trademark Law of 1923 from a historical perspective.

[6]For instance, Heath and Mace (2020) use the introduction of the 2006 Federal Trademark Dilution Act (FTDA) and find that the additional protection offered by FTDA to publicly-traded firms with famous trademarks have economically significant effects for these firms. Again using publicly-traded firms, Kost et al. (2019) provide some evidence on the effects of trademark acquisitions and sales on firm markups and profits. Similarly, Pearce and Wu (2022) use a match of the trademark data to brands in RMS NielsenIQ scanner data for a subset of products and firms in the economy to study brand reallocation among firms and implications for growth. Basker and Simcoe (2021) provide evidence on how UPC adoption leads to increased trademark activity.

[7]See, e.g., Dinlersoz and Yorukoglu 2012; Gourio and Rudanko 2014; Roldan and Gilbukh 2018. Ignaszak and Sedláček (2022) analyze economic growth implications of innovation and gradual customer acquisition by firms.

[8]See, for instance, Butters 1977; Grossman and Shapiro 1984; Stegeman 1991 for foundational analyses of informative advertising. See also Dinlersoz and Yorukoglu (2008) for an analysis of informative advertising in a competitive industry where firms differ in production efficiency. In a more recent paper Greenwood et al. 2021 develop a similar information-based model where traditional and digital advertising finance the provision of free media goods and affect price competition.

2 Firm-level evidence on the effects of trademark registrations

This section presents evidence on the connection between registered trademarks and various firm-level outcomes. As in the case of many other firm-level decisions and outcomes, there is a high degree of selection into trademark registrations. Only a small fraction of firms choose to register trademarks, and these firms differ from others on a number of measurable dimensions. Firms also tend to exhibit different dynamics after trademark registration, suggesting the presence of effects associated with trademark ownership.

2.1 Data

The analysis utilizes the trademark-firm database introduced in Dinlersoz et al. (2018). The database pertains to the period 1976-2014. It contains information on trademarks from the USPTO Trademark Case Files Dataset (TCFD), which covers all federal trademark applications and registrations in the U.S. since 1870. [9] The information on trademarks includes dates of application and registration, trademark classification, and other events associated with a trademark. Dinlersoz et al. (2018) link the TCFD with the U.S. Census Bureau's Business Register (BR), administrative data for the universe of non-farm employer businesses in the United States. The TCFD and BR were matched at the firm-level using name and address matching techniques. About 83% of the trademarks match with firms for the sample period, most with a high degree of precision – see Dinlersoz et al. (2018) and the Appendix therein for further details on the match.

Information from the Longitudinal Business Database (LBD) was added to track firm outcomes and trademark registrations over the firm life-cycle. The LBD provides a longitudinally-linked version of the BR at the establishment level starting in 1976, and enables identification of entry and exit of firms and their establishments (Chow et al., 2021). Key firm-level variables that come from the LBD are employment, payroll, and revenue, as well as geography and industry classifications.

The trademark-firm data allows tracking of key events in a firm's life-cycle: firm birth and growth, first trademark application and registration, firm geographic expansion, and firm death—for details of data construction, see Dinlersoz et al. (2018). This data is further supplemented with additional information on firm innovation and expansion: first patenting activity from USPTO, and first exporting activity from Longitudinal Firm Trade Transactions Database (LFTTD).

The analysis is restricted to U.S. firms and their trademark activity.[10] The focus is on the critical event of the first trademark registration by a firm at the federal-level[11]. By simply engaging in commerce, firms in the United States can gain common law protection for their names and brands, which applies to a local geographic area. In addition, firms can register trademarks at the state level, which provides protection in a given state. A federal registration offers much stronger and broader protection and benefits. Among the benefits are nationwide protection, validity and eventual incontestability of a trademark (5 years after

[9] See Graham et al. (2013) for the construction and details of TCFD.

[10] Trademark filings by foreign entities in the United States are excluded. There is no detailed information available about the characteristics and dynamics of these entities.

[11] In contrast, Dinlersoz et al. (2018) focus only on trademark applications, which signal an intent to register, but may not turn into actual registrations for reasons such as the failure to successfully launch a brand or a product and the associated lack of commercial activity. The analysis here considers the trademarks that are granted federal registration, for which the proof of the presence of commercial activity associated with the applied-for trademark is a necessary condition for registration.

registration), protection against infringing imports, ability to defend the trademark in federal courts, and enhanced remedies in case of infringement. The overall effect of these key benefits is the main concern of this paper[12].

Filing an application for a U.S. trademark registration entails a relatively small upfront fee[13]. This fee, however, reflects only a fraction of the underlying costs associated with registering, maintaining and defending a trademark. For registration, an extensive search by an attorney may be required to ensure that the proposed trademark is sufficiently distinguished from the existing registered marks, and in the case of foreign applications, a U.S. attorney must be retained. In addition, the expected future legal costs of defending a trademark against infringement can be very large.

Not all applications for a trademark registration reach the registration stage. An application can become a registered trademark only if the associated good or service is already in commercial use, or if there is a bona fide intent to use the mark in commerce – these "intent-to-use applications" can only be registered after the trademark is used in commerce and evidence supporting such use is provided. The USPTO determines whether the trademark in consideration is legally protectable and if there is a "likelihood of confusion" with a previously registered trademark. If registerable, the USPTO publishes the trademark for a limited opposition period, during which time third parties may file a formal opposition to the registration[14]. Unopposed applications for trademarks already in-use are issued a registration[15].

The trademark-firm data is supplemented with data on firm-level advertising expenditures (cost of advertising) reported in the U.S. Census Bureau's Economic Census and Annual Survey of Manufacturers (ASM). While the data on advertising expenditures mainly pertain to manufacturing firms, it offers a rare opportunity to compare the advertising expenditures by firms that have registered trademarks versus those without, and the intensity of marketing activity before and after a firm registers its first trademark. In addition, data on firm-level exposure to trade is used to analyze the protective role of trademarks against an exogenous change in the competition faced by firms. Industry-level import penetration measures are combined with firm-level employment shares by industry to calculate measures of trade exposure.

2.2 Trademark registrations and firm life-cycle

Table 1 contains some descriptive statistics about firms with and without registered trademarks in 2014[16]. While only about 4% of firms had a registered trademark, these firms accounted for nearly half

[12] Unlike patents, trademarks do not expire but need to be renewed periodically to be valid. Renewals, trademark registrations after the first one, and transactions involving the buying, selling, or transfer of trademarks are also recorded in the data. While these dimensions of a trademark life-cycle are interesting, they are not explored in this paper as we primarily focus on the extensive margin of first registration. The trademark activity after the first registration is not independent of the initial registration and it is more challenging to assess the separate contributions of this subsequent activity to firm dynamics. The effects explored here could thus be thought of as the average effect of all trademark activity by a firm following the first registration.

[13] For most of the time period for which the matched data cover, the per class filing fee ranged from $175 to $375 for a paper filing and $325 to $335 for an electronic filing.

[14] However, oppositions rarely arise; less than 3% of published applications are opposed—see Graham et al. (2013).

[15] A trademark registration can be renewed as long as the trademark is used on the associated goods or services. The owner must provide proof of continued use and pay fees six years after registration and at each ten-year renewal event to maintain the trademark. Otherwise, the registration is canceled.

[16] The final year of analysis is set to 2014 in order to be able to use the years 2015-2018 in the LBD to study early firm dynamics and its relation to trademark activity.

of the non-farm private sector employment. The average size of firms with a registered trademark was about 285 employees, 23 times the average size (12) for firms without trademarks. These differences in size are even more stark at the right tails of the firm size distributions. The top 1% of employment distribution for firms with a trademark had an average size of nearly 20,000 employees, nearly 50 times of that for firms without trademarks (427).

While firms are more likely to obtain a trademark early in their life-cycle, among newborn firms, trademark activity is still a rare event. About 0.9% of firms born in 2014 registered trademarks in their year of birth, and they accounted for 1.5% of the total employment of newborn firms in 2014. Firms that own a trademark as of 2014 registered their first trademark about 6 to 7 years after birth on average, but the distribution of age at first trademark is highly skewed towards the first few years of a firm. Overall, trademark registration is uncommon in the firm population. Still, it is much more prevalent than other key firm outcomes, such as R&D, patenting, or exporting – less than 1% of U.S. firms export or have patents, while around 2.5% perform R&D (see Foster et al. (2020)).

Recent literature on firm growth suggests that early characteristics of firms are highly relevant for subsequent firm dynamics (Guzman and Stern (2020), Sterk et al. (2021)). For instance, initial firm size is a good predictor of firm survival[17]. If firm trademark activity reflects underlying superior firm characteristics (e.g., better entrepreneurial ability, higher product quality, or a higher potential for growth in general), trademark registrations should be positively correlated with these early indicators of firm success. Observable dimensions of this selection are explored in Table 2. The first column presents estimates from a linear probability model that relates the indicator of a trademark registration within the first five years of a firm to initial firm characteristics[18]. Firms that are larger at birth and grow faster within their first year are more likely to register a trademark. Being in the top 5 percent of firm initial size and first-year growth distribution is associated with a 2.2 and 2.8 percentage point increase in trademark registration likelihood, respectively. Firms that export, have patents, or operate in multiple-states in their first year are also more likely to register. Notably, having a patent upon birth is associated with a 16 percentage points higher likelihood of trademark registration. Exporting is associated with a 3 percentage points increase, whereas being a multi-state firm is related to a 0.6 percentage point increase. All of these estimates are economically significant, given the small overall likelihood of firm trademark ownership (4%) and the likelihood of trademark registrations at birth ($< 1\%$).

The second column in Table 2 uses the duration (in years) between firm birth and first trademark registration as the dependent variable. The estimated coefficients indicate that firms born larger and grow faster in their first year register their first trademark earlier. Similarly, new firms with patents and those that expand beyond local boundaries (multi-state and exporting firms) in their first year tend to register earlier. Overall, the patterns in Table 2 indicate selection into trademark registration early in a firm's life-cycle. A federal trademark registration is more likely to benefit firms that grow fast, innovate, and expand beyond their local markets – firms that are more likely to be exposed to infringement and imitation, and need the broader protection provided by federal registration.

Early growth patterns are markedly different for firms that have a trademark registration at some point

[17] For recent analysis of the connection between early characteristics of firms and subsequent firm growth, see, e.g., Bayard et al. 2018; Sedláček and Sterk 2017.

[18] This analysis excludes firms born before 1976 as their birth year is not known and their initial conditions cannot be measured. This estimation also makes no claim to causality and simply identifies some early firm characteristics associated with early trademark behavior.

in their life-cycle. Figure 3 contains the average Davis-Haltiwanger-Schuh (DHS) employment growth rates for the first five years of a firm[19]. Conditional on survival (Panel (a)), firms with trademarks grow twice as fast in their first year with the gap between growth rates increasing over time. The growth rate of trademark firms in the fifth year is about 4.2 times that of non-trademark firms. Large differences are also observed in unconditional growth rates (Panel (b)). The average first year growth rate of trademark firms is about 14 times that of non-trademark firms, compared with only 2 times in the conditional case—driven by the higher failure rates of non-trademark firms. The unconditional growth rates of non-trademark firms is negative for years two through five, whereas it remains positive for trademark firms for all years but year five.

Beyond the first five years, the size gap between the two types of firms continues to expand — see Table 3. At birth (age zero), firms with no trademark have an average size of 5.6 employees, compared to 15.2 employees for firms with a trademark, implying a relative average size of 2.7. The relative size grows over time, reaching about 20 by age 30, at which point the average size of trademarking firms is about 320 employees, versus 16 for others. Firms with a trademark achieve a much larger average size over their life-cycles.

To further assess the evolution of firm size for firms before and after their first trademark registration, firms with trademark registrations are closely matched with firms with no trademark registrations in their life-cycle to account for selection based on observed characteristics. An event study is then carried out to compare the outcomes of the treated and control groups before and after registration. We then study the effect of an exogenous shock to entry and competition to provide additional evidence on the connection between trademark registrations and protection against customer depreciation in the face of heightened competition. The shock is the entry of China to the World Trade Organization (WTO) and the resulting rise in foreign competition, some of which is potentially imitating and infringing. Trademarks are a credible defense mechanism against potentially infringing foreign competition, as the owner of a trademark can prevent or stop the importation of goods that are substantially similar in name or brand. The underlying hypothesis is that existing trademarks registered before the shock (and thus not induced by the shock itself) provide protection against increased import competition and prevent the erosion of firm value. The outcomes of firms with trademarks registered before the shock are compared with those of the firms without registered trademarks. The analysis also recognizes the non-random nature of firms' exposure to the trade shock and uses an instrumental variables approach to measure the protective role of trademarks.

2.3 An event study

In order to explore the relationship between trademark registration and firm outcomes, an event study is implemented around the year of a firm's first trademark registration. Firms with a trademark registration are matched with firms without any trademarks based on key observable characteristics in the year of registration, including industry (4-digit NAICS), dominant location (state), firm age, legal form of organization, multi-unit status, and employment measured in terms of the deciles of firm employment distribution within a 4-digit NAICS industry. Then, a single control firm is selected for each trademarking firm by minimizing the Euclidean distance in employment between trademark and non-trademark firms

[19]The analysis of growth rates is restricted to firms born after 1976.

for the entire pre-registration period[20].

Figure 4a shows the time profiles for the average employment of the treated and control firms before and after the year of the first trademark registration—normalized to $t = 0$. Despite differences in levels (trademark firms are larger), the two groups exhibit broadly similar trends before registration and the gap between the two groups is relatively stable[21]. However, trends start to diverge after trademark registration. Firms with a registration grow by about 76% over the 5 years following registration. The control group, in contrast, grows only by about 21%[22].

A similar pattern is observed for revenue. Figure 4b contains the average revenue profile for the two groups. For average revenue, the trends are similar when $t \leq 0$ —despite the fact that the matching process does not select control firms based on revenue. The trends again start to diverge after $t = 0$. Firms with trademarks exhibit higher growth, roughly doubling in size after five years, whereas the control group grows by about 40% during the same period.

Next, a series of event study regressions are considered

$$y_{it} = \alpha + \sum_{j \neq 0} \beta^{TM} TM_i + \sum_{j \neq 0} \beta^j d_j + \sum_{j \neq 0} \gamma^j (TM_i \times d_j) + \varepsilon_{it}, \tag{1}$$

where y_{it} is the firm employment or revenue in log, dummies d_j ($j \in \{-5, -4, ..., 0, ..., 5\}$) for each time period before and after the first registration event are included, together with treatment indicator TM_i and its interactions with period dummies, $TM_i \times d_j$. Since outcomes are in natural logs, this specification conditions on survival.

The estimates for γ^j based on specification (1) are in Figure 5[23]. As shown in Figure 5a, there is a slight increase in employment in the pre-period for trademark firms relative to the control group, but trademark firms reach much higher employment levels following the first trademark registration. Based on the estimated semi-elasticities γ^j, five years after the first registration the trademark firms' employment is about 25% higher. The estimates for revenue in Figure 5b indicate no significant pre-trend differences despite revenue not being used in matching. However, the treated firms' average revenue grows by 27% five years after the registration event relative to the control group. To account for time-invariant firm

[20]For each candidate treated-control pair, (i, j), the pre-treatment employment distance is computed $EmpDiff_{i,j} = \sum_{k=-5}^{0} (Emp_{i,k} - Emp_{j,k})^2$,where $k = 0$ denotes the year of registration and values of $k < 0$ denote years before registration (up to 5 years). After matching exactly on the industry, location, age, legal form, multi-unit status, and employment decile in the treatment year, the control firm with the minimal pre-treatment difference in size is selected. Ties are broken randomly. Once a control firm is selected for a given trademark firm, it cannot be selected as a control for any other trademark firm.

[21]Despite the tight matching on industry size percentiles and the Euclidian distance minimization to find the match that is most similar to the treated firm in terms of employment in the pre-registration period, size differences between treated and control firms remain. The level difference in the pre-period between treated and control firms reflect the fact that trademark firms are much larger - especially within the top decile - and it is hard to find a control firm that has the exact size as the treated firm and possesses no trademarks at the same time.

[22]In Figure 4a, both groups have a relatively flat average size profile for $t \leq 0$, and the average size of the control group exhibits a slight decline between $t = -1$ and $t = 0$. The decline at $t = 0$ is more pronounced for revenue in Figure 4b. The reason for this decline is that the age composition varies during the pre-period as age zero firms are added. For instance, the firms in $t = -5$ consist of those that register their first trademark at age 5 or older, and their controls. At $t = -4$, firms that register their first trademark at age 4 are added, and these firms, which are new-born (age zero) in $t = -4$, tend to be smaller on average than the firms that were present at $t = -5$, which are now a year older. In other words, the changing age composition of firms between $t = -5$ and $t = 0$ puts downward pressure on average employment for both groups, counteracting the growing average firm size of surviving firms from the previous period.

[23]For the set full set of estimates from specification (1), see Appendix Table A1.

characteristics, (1) is also estimated with firm-specific fixed effects[24]. The five-year growth rates associated from having registered a trademark are 24% and 18%, for employment and revenue respectively. These growth rates, though lower, are comparable to the ones based on (1) without firm fixed effects.

Overall, the evidence from the event study suggests a persistent positive firm growth effect associated with trademark registrations in the post-period. The next section uses an exogenous shock to competition to assess the benefits to firms from trademarks registered before the shock.

2.4 The protective role of trademarks: Evidence from a trade shock

Do trademarks protect firm value in the presence of rising competition that can increase customer depreciation? Quantifying the causal and fundamental role of trademarks in preserving firm value is not straightforward. However, an exogenous shock to the business environment, such as a removal of entry barriers, or a change in trademark law unrelated to the firms' incentives to register trademarks can help further isolate the impact of trademarks on firm outcomes[25]. The analysis here considers a trade shock, China's entrance to WTO in 2002, which has been used to assess the effects of an exogenous shock in competition for U.S. firms (See Acemoglu et al. 2016; Autor et al. 2013, 2016). Some of the competition associated with this shock is potentially imitating and infringing in nature. A key benefit of a federally registered trademark is the ability to record the registration with U.S. Customs and Border Protection and the International Trademark Commission to deter or stop importation of infringing foreign goods. In 2020 alone, the U.S. authorities seized 26,503 shipments of goods infringing intellectual property—the value of the goods exceeded $1.3 B. The credible threat of protection against infringing imports provided by trademarks can help maintain firm value in the presence of increased import competition. The analysis focuses on trademarks by firms registered before the shock and investigates whether this pre-determined protection was significant in preventing the erosion of firm value[26].

Using a difference in difference framework, the post-shock performance of firms with trademarks registered before the shock is compared with that of the firms with no such trademark registrations. The main econometric specification is

$$y_{it} = \alpha + \beta^{C} \times IC_{it} + \beta^{TP} \times TM_i \times Post_t + \beta^{CP} \times IC_{it} \times Post_t + \beta^{TCP} \times TM_i \times IC_{it} \times Post_t + \tau_t + f_i + \varepsilon_{it}, \quad (2)$$

where y_{it} is firm i's outcome measure in year t, TM_i is an indicator of trademark ownership by firm i before the shock ($t < 2001$), IC_{it} is a firm-level import competition measure, and $Post_t$ is an indicator of years after the shock ($t \geq 2001$). Year and firm fixed effects, τ_t and f_i, are also included. The import competition measure IC_{it} is defined as

[24]Note that the the trademark firm indicator, TM_i, is necessarily excluded in this specification as it is collinear with firm fixed effects.

[25]See Heath and Mace (2020) for the latter type of analysis based on the Federal Trademark Dilution Act (FDTA). They study the effects of FDTA that provided additional protection against the dilution of famous trademarks owned by large publicly traded firms.

[26]The shock has also changed incentives for both Chinese and domestic firms to register trademarks in the United States. After the China's entrance to WTO, there has been a rise in U.S. trademark filings by Chinese entities. Domestic firms may have also responded with more intense trademark activity during the post-shock period to ensure that both existing and future products that may be prone to Chinese competition are properly protected. However, trademark filings by U.S. firms in the post-shock period are an endogenous response by U.S. firms. Dinlersoz et al. (2019) provide an analysis of the U.S. firms' trademark activity in response to increasing competition from Chinese firms.

$$IC_{it} = \sum_j s_{ijT} \frac{m_{jt}}{o_{jt} + x_{jt} - m_{jt}}, \tag{3}$$

where s_{ijt} is firm i's employment share in industry j in pre-shock year $T = 2000$, m_{jt} is Chinese imports in industry j in year t, o_{jt} is domestic output in industry j in year t, and x_{jt} is exports in industry j in year t. In other words, IC_{it} is a firm-level weighted import exposure measure. The sample is restricted to firms that have non-zero value for IC_{it} to exclude firms that are not subject to the shock – these firms are largely outside of manufacturing.

To account for the potential endogeneity associated with the import penetration measure, an instrumental variables approach is used. The instrument, IV_{it}, for IC_{it} is identical to the one used in Autor et al. 2013, 2016 and is obtained by replacing m_{jt} in (3) with Chinese imports in other developed countries, $\widetilde{m}_{jt}$.[27] Table 4 presents the second-stage results from the estimation of the model (2) using a two-stage least squares IV approach[28]. Three different firm outcomes are used as the dependent variable: employment, revenue, and labor productivity (revenue per worker), all in inverse hyperbolic sine transformation to account for any cases with zero employment or revenue[29]. Three versions of (2) are estimated for each outcome. The first version (I) in Table 4 measures existing registered trademark ownership as of 2001. The second version (II) instead uses 1999 as the cutoff year for this measurement, as a robustness check – in case some trademarks were registered close to 2001 in anticipation of the shock. Finally, a third version (III) includes firm deaths, by allowing firms that exit to reduce their employment to zero – instead of conditioning on survival as in versions I and II.

In Table 4, the coefficient β^{TP} is always positive and significant, indicating that firms with trademarks fared better in the post period compared to firms without trademarks. Note also that β^{CP} is negative in all specifications – a higher import penetration in the post period implies worse outcomes for firms without a trademark. However, the positive and significant estimate for β^{TCP} suggests that trademark ownership mitigated the adverse effects of import competition: firms with a trademark experience less negative outcomes as the degree of import penetration increases in the post period. A one standard deviation increase in the import competition measure is associated with a reduction of 2.3% in employment, 2.7% in revenue, and 0.5% in labor productivity on average in the post-period for non-trademark firms, but a lower reduction in all three outcomes for trademark firms. [30] Overall, the estimated effects suggest an important role for pre-existing registered trademarks in protecting firm revenue and employment in the wake of an exogenous shock to the competitive environment.

The estimates provided in this section should be interpreted with caution, as they do not measure the full benefits from registered trademarks that accrue to firms over their life-cycles. Registered trademarks offer more benefits than just protection against import competition. Moreover, due to the scope of the

[27]The instrument IV_{it} is used to construct instruments for all three endogenous variables involving IC_{it}. That is, the instruments are IV_{it}, $IV_{it} \times Post_t$, and $TM_i \times IV_{it} \times Post_t$.

[28]See Appendix Table A2 for statistics on the first stage, which suggest that the instruments do indeed well in predicting the three endogenous variables that involve (3) in the model (2).

[29]Because the LBD measures firm employment as of March 12 in each year, some firms that have positive employment in the rest of the year but not as of March 12 report zero employees. Similarly, a revenue of zero is observed for some firms in certain years. All of our results are robust to using a log transformation which excludes zeros.

[30]These calculations assume that the coefficient estimates measure semi-elasticity and are an approximation to the actual IHS-based elasticities. The approximation works well if the average value of the dependent variable is larger than 10 – see Bellemare and Wichman (2019). This condition is satisfied in our samples.

trade shock, the estimates also pertain primarily to manufacturing firms, and may not be informative about other sectors such as services and retail. Furthermore, the estimates do not capture various general equilibrium effects of trademark registrations. For example, firms with trademarks may engage in more intense marketing, which imposes negative externalities on firms without trademarks and can curtail their growth. In general, the effects of consumer reallocation in the economy induced by trademark registrations cannot be fully measured by the estimates at the firm level only. The model developed below quantifies the broader effects of trademarks by incorporating key general equilibrium channels at work.

2.5 Trademark registrations and advertising

U.S. aggregate advertising spending increased substantially over time (Figure 2). However, advertising's share of GDP moved in a narrow range, hovering slightly above 2% for much of the post-WWII period. This relatively stable aggregate pattern likely masks significant heterogeneity at the firm level. A key benefit of trademarks is to increase the appropriability of a firm's marketing activities. Because registered trademarks can help firms build a customer base and prevent customer depreciation, returns to advertising are likely larger for firms with trademarks.

Table 5 contains statistics on advertising intensity (advertising expenditures as a percentage of revenue) for firms in the Census of Manufactures (CMF) 2007 and 2012[31]. Advertising expenditures are likely interpreted by respondents to CMF as formal outlays to media and advertising agencies. It is not surprising that many firms do not formally incur such expenses, and hence, report zero advertising. However, nearly all firms undertake some form of passive or active informal advertising. For instance, a firm's name posted on its premises, or its website can serve as advertising. Word-of-mouth advertising by consumers works in a similar way. Much of this informal or indirect advertising is unmeasured. Thus, the reported advertising expenditures likely underestimate the true extent of information dissemination by firms to consumers.

In Table 5, average advertising expenditures and advertising intensity are broken down by trademark registration status[32]. When zero advertising instances are included, average advertising expenditure for trademark firms is about 12 times that for non-trademark firms. Average advertising intensity for firms without a trademark registration is about 0.48%, compared to 0.65% for firms with a registered trademark. When only nonzero advertising cases are considered, this difference diminishes to some extent since zero advertising cases are more concentrated in non-trademark firms – consistent with the fact that trademark firms engage in more advertising activity in general. Advertising intensity for non-trademark firms is now about 0.56%, as opposed to 0.70% for trademark firms.

Because trademark firms tend to be larger and older, it is important to examine the differences in advertising activity between the two types of firms after controlling for differences in life-cycle stage. Table

[31] The 2007 and 2012 surveys contain much higher fractions of firms with non-missing information on advertising expenditures compared to other years. Therefore, these two surveys are used to provide some broad evidence on advertising intensity by trademark versus non-trademark firms in the manufacturing sector. Available data on firm-level advertising pertains to manufacturing firms, and leaves out firms in other sectors, particularly retail and services, which may engage in higher advertising activity.

[32] Because advertising intensity is a ratio, some misreported values of advertising expenditures especially for small firms can lead to outlier observations. To mitigate the influence of potential major outliers, advertising intensity is winsorized at the top 0.1%.

6 examines the variation in advertising intensity using regression analysis. The sample is an unbalanced annual panel of firms from the Annual Survey of Manufactures (ASM) and CMF over the period 2007-2014. In all specifications, only the observations with nonzero advertising expenditures are considered[33]. Measures of life-cycle stage (firm size and age) are included as controls, in addition to firm, industry, and year fixed effects, depending on the specification. Some specifications also include a "registration" dummy that indicates firms with a trademark registration during the sample period, and a "post-registration" dummy that is turned on for the periods after the registration of the firm's first trademark. The goal of this exercise is not to assign causality, but simply to measure how advertising behavior is associated with trademarking activity after controlling for observable firm characteristics.

Pooled cross-section specification (I) indicates that advertising intensity is negatively related to firm size. Having a trademark registration, on the other hand, is associated with a higher advertising intensity. Furthermore, firms in their post-registration period have an even higher advertising intensity. The fixed effects specification (IV) suggests that a 1% increase in firm size (revenue) is associated with a 0.063 percentage point decline in advertising intensity. Again based on specification (IV), after a firm registers its first trademark, advertising intensity is higher on average by about 0.066 percentage points, equivalent to about 9.4% of the pre-registration mean advertising intensity of firms that register a trademark (see Table 7)[34].

One concern with specifications I and IV of Table 6 is division bias, since the revenue variable used in the denominator of the independent variable (advertising intensity) also appears as an explanatory variable. To mitigate this potential bias, two approaches are considered. First, firm employment is used as an instrument for revenue. Employment and revenue are highly correlated at the firm level, and to the extent their measurement errors are uncorrelated, the bias should be lower. Columns 2 and 5 present the instrumental variable estimation results. The estimated coefficients for revenue attenuate, but retain their statistical and economic significance. The fixed effects specification V implies that a 1% increase in firm size is associated with a 0.023 percentage point decline in advertising intensity. The second approach directly replaces revenue with employment as an explanatory variable (columns III and VI). This approach also results in attenuated estimates for size compared to columns I and IV, but still indicates that advertising intensity declines as firm size increases. Overall, the findings in Table 6 point to a declining advertising intensity as a firm grows.

Finally, Table 7 provides average advertising intensity measures for the firms included in the regression samples in Table 6. Firms with no trademark registration have an average advertising intensity of about 0.56%. On the other hand, pre-registration average advertising intensity for firms with trademarks is about 0.70%. These figures are consistent with selection documented in Table 2. Would-be registrants have better growth prospects, which can lead to higher advertising intensity even before registration, compared with firms that never register a trademark. Advertising intensity of trademark firms increase to 0.79% in the post-registration period, consistent with trademark registrations boosting the incentives to advertise.

The evidence overall suggests that firms with trademarks engage in more advertising activity both

[33]The focus on nonzero advertising is driven by the fact that in the model studied all firms advertise every period. Thus, the model's implications mainly apply to firms that advertise.

[34]The estimates for firm age dummies (not reported) also indicate a negative relationship between age and advertising intensity.

before and after their first trademark registration, compared to firms without trademarks. However, the relationship between advertising intensity and trademark activity is not necessarily causal. Both advertising intensity and trademark registrations can be driven by unobserved firm characteristics. Consistent with this possibility, the model below features unobserved firm product quality as the fundamental driver of both trademark registration and marketing. In the model, trademarks also affect the incentives to advertise, as they reduce informational frictions between firms and consumers.

3 The model

The model builds on the industry dynamics framework in Dinlersoz and Yorukoglu (2012), where firms gradually build a customer base through advertising. The framework is extended to general equilibrium. Product quality choice and trademark decisions by firms are introduced, while some other dimensions, such as the determination of prices, are simplified.

3.1 Preliminaries

Consider an economy with a continuum of symmetric industries (product varieties) indexed by $i \in [0, 1]$. In each industry there is a continuum of firms offering the same product with different quality levels, q. Time, t, is discrete. For every period, there is a large mass (continuum) of potential entrants, and entry is free. Upon paying a sunk entry cost, κ, an entrant learns its efficiency in product development, e, a random variable with *c.d.f.* $H(e)$ and density function $h(e)$. This efficiency is time-invariant and determines the product development cost for a given level of quality. After the realization of e, a firm can exit at no cost, without developing its product. If the firm stays, it incurs a one-time product development cost and chooses its quality, which remains constant for the rest of the firm's life[35]. In any period, a firm exits only if it receives an exogenous exit shock (with probability χ).

There is a unit mass of households (consumers), who live for several periods. In every period, a fraction $\lambda \in (0, 1)$ of consumers exit, and are replaced by new ones – representing exogenous consumer turnover. Consumers are not aware of the full set of firms selling a given product variety. To grow, a firm must inform consumers about itself, and build a customer base over time via marketing. Firms send ads to consumers randomly, who then make their purchases based on the set of ads they receive. Owning a registered a trademark helps reduce informational frictions between firms and consumers, and allows firms with a trademark to increase consumer awareness. A trademark firm can more easily attract consumers, in part because consumers recognize its trademark and uniquely associate it with the firm without confusion, since the trademark is registered and legally protected against infringement. This key benefit of trademarks is captured by a more efficient marketing technology for trademark firms.

Competition in marketing implies that a firm can lose customers to others that provide a higher surplus. A firm's customer base can also depreciate for other reasons. For example, customers can be lured away to other firms through imitation and infringement. The rate of this *customer diversion* is given by $\delta \in (0, 1)$ — each period a firm loses a fraction δ of its customers as a result of diversion. This type of depreciation in customer base can be prevented only by registering a trademark. The assumption that

[35]For simplicity, quality upgrading is not considered. It is possible to introduce quality upgrading by allowing, for instance, firms to make investments over time to improve their product development efficiency.

firms with trademarks are not exposed to diversion captures, in a stark form, the key role of registered trademarks in credibly deterring imitation and infringement[36]. A firm can register at most one trademark, and owning a trademark trademark entails a one-time fixed cost, c_T.

The timing of events within a period is given in Figure 6. Each period new entrants and incumbents without an existing trademark decide whether to register a trademark. After trademark registration decisions are made, firms decide on their marketing expenditures, prices, and employment.

3.2 Households

Households receive utility from consuming all product varieties $i \in [0,1]$

$$U = \int_0^1 u(q_i, x_i) di$$

where $u(q_i, x_i)$ is the utility from consuming x_i amount of good i at quality level q_i. The utility function is given by

$$u(q_i, x) = q_i^{\xi} x_i^{\theta}, \quad \xi, \theta \in (0,1).$$

Each agent supplies one unit of labor inelastically at wage, w_t.

3.2.1 Consumer information

Firm information relevant for consumer decision making is conveyed through ads. Consumers receive ads randomly and independently. Within a period, each consumer is equally likely to receive an ad, no consumer receives more than one ad from the same firm, and each ad is received by some consumer[37]. Let I_t^i denote a consumer's period- t information about firms selling variety i. The set I_t^i is either empty ($\varnothing$), implying that the consumer has no information, or contains elements corresponding to the firms the consumer has received ads from. Each element ι_{jt}^i of I_t^i is a vector of variables that describe firm j's attributes that help a consumer identify a firm and evaluate the surplus from patronizing it. Because consumers live for several periods, I_t^i may contain information about the set of firms the consumer was informed of in the past. To simplify this dependence on history, it is assumed that the consumer only knows the current period payoff-relevant information of the firm that provided the highest surplus in the past — the firm the consumer purchased from in the previous period (unless that firm exits).

Next, consider the contents of $\iota_{jt}^i \in I_t^i$. An ad truthfully reveals the quality of the product[38]. Hence, a firm's product quality, q_j^i, is included in ι_{jt}^i. A firm's price, on the other hand, is not included in an ad: prices are flexible enough that firms cannot commit to them. However, consumers know the distribution of prices.

In a dynamic environment where consumers can purchase from a firm repeatedly, the firm attributes that matter for firm survival may also be payoff relevant. One such attribute is firm size, m_{jt}^i—measured

[36] In general, both firms with and without trademark protection are subject to customer depreciation. However, it is plausible that firms with trademarks experience less depreciation given the legal protection afforded by federal trademark registrations.

[37] These assumptions are compatible with the fact that there is a very large number (continuum) of consumers.

[38] The analysis abstracts from imperfect information about quality. While perfect information is a strong assumption, it simplifies the analysis substantially by ruling out potential multiple equilibria when quality is unknown and firms can adopt strategies that may hide their true quality.

by the number of customers a firm has at the beginning of period t. Current-period information about the firm consumer purchased from in the previous period is in I_t^i unless that firm exits. Hence, if exit likelihood depends on firm size, firm size matters for the expected surplus from a firm. For simplicity, it is assumed that a firm's size is known by the consumers—it can either be revealed in the ad or it may be public information. Finally, the information in an ad also includes whether the firm has a trademark or not, denoted by $\Gamma_{jt}^i = T$ if the firm has trademark and $\Gamma_{jt}^i = N$, otherwise. To sum up, consumer information about a firm consists of $\iota_{jt}^i = \{\Gamma_{jt}^i, q_j^i, m_{jt}^i\}$. As detailed below, the dimension of ι_{jt}^i will be further reduced given the additional structure imposed on the model below.

3.2.2 Household's problem

If the consumer has no information about firms selling variety i (i.e. $I_t^i = \varnothing$), the consumer does not purchase variety i in period t. Otherwise, the consumer purchases from the firm in I_t^i that provides the highest net expected lifetime surplus. Let $W_t^{\Gamma_{jt}^i}(q_j^i, m_{jt}^i)$ be the surplus from purchasing variety i from firm j with type $(\Gamma_{jt}^i, q_j^i, m_{jt}^i)$.

Consider now a household with information $\mathbf{I}_t = \{I_t^i\}_{i\in[0,1]}$ and assets a_t. Suppose that the next period's assets, a_{t+1}, is given. The household then solves a static problem that determines for each variety the optimal quality and quantity to consume

$$\max_{\{x_{ij(i),t}\}_{j(i)\in\ I_t^i}} \int_0^1 u(q_{ij(i)}, x_{ij(i),t})di \tag{4}$$

s.t.

$$\begin{aligned} j(i) &= \arg\max_j \left\{ W_t^{\Gamma_{jt}^i}(q_j^i, m_{jt}^i) \right\}_{j\in I_t^i}, \text{ for all } i \in [0,1], \\ \int_0^1 p_{ij(i),t} x_{ij(i),t} di &= w_t + a_t(1+r_t) - a_{t+1}, \quad x_{ij(i),t} \geq 0 \text{ for all } (i, j(i)),\ a_{t+1} \text{ given.} \end{aligned}$$

The first constraint states that for each variety i, the household chooses the firm $j(i) \in I_t^i$ that provides the highest net lifetime surplus among the firms the household is informed of. The second is the budget constraint, given a_{t+1}. Note that the quantities $x_{ij(i),t}$ are determined only when the quality levels are chosen.

Denote the maximized value in problem (4) as a function of a_{t+1} and $\mathbf{I}_t$ by $\mathfrak{U}(a_{t+1}, \mathbf{I}_t)$. The household then solves the following dynamic problem

$$U(a_t, \mathbf{I}_t) = \max_{a_{t+1}} \{\mathfrak{U}(a_{t+1}, \mathbf{I}_t) + \beta U(a_{t+1}, \mathbf{I}_{t+1})\} \tag{5}$$

s.t.

$$\int_0^1 p_{ij(i),t} x_{ij(i),t} di + a_{t+1} = w_t + a_t(1+r_t) \quad \text{for all } t,$$

In (5), the household picks the optimal amount of total expenditure (or equivalently, asset level, a_{t+1}) to be allocated to the current period. The information about firms the household carries over to the next period is included in $\mathbf{I}_{t+1}$ and consists of the relevant information for all the firms from which a purchase was made in the current period.

Next, turn to the consumer surplus from each firm type, $W_t^\Gamma(q,m)$, where the indices i, j, and t are dropped from q and m for notational ease. Let $W_t^\varnothing$ be the expected surplus of a consumer with no information, i.e. $I_t^i = \varnothing$. Denote by Λ_t^k the probability that a consumer receives $k \geq 0$ ads for variety i. The consumer surplus from a trademark firm ($\Gamma = T$) is

$$\begin{aligned} W_t^T(q,m) \;=\; & \widetilde{u}(q) \\ & +\beta(1-\lambda)\{\Lambda_{t+1}^0\left((1-\chi)W_{t+1}^T(q,m') + \chi W_{t+1}^\varnothing\right) \\ & +\sum_{k=1}^\infty \Lambda_{t+1}^k\left[(1-\chi)\int \max\{W, W_{t+1}^T(q,m')\}dZ_{t+1}^k(W) + \chi\int W dZ_{t+1}^k(W)\right]\}. \end{aligned} \tag{6}$$

where $\widetilde{u}(q) = \max_x[u(q,x) - \upsilon px]$ is the current period surplus from a firm with quality q that charges price p, and υ represents the Lagrange multiplier associated with the consumer's static problem (4). The next period size of the firm is denoted by m'. The second line on the right hand side of (6) is the expected discounted future surplus if no ad is received next period for that specific variety. It has two components: the future surplus if the consumer's current firm stays in the market, and the future surplus if that firm exits (with probability χ), leaving the consumer with a surplus of

$$W_{t+1}^\varnothing = \beta(1-\lambda)\{\Lambda_{t+2}^0 W_{t+2}^\varnothing + \sum_{k=1}^\infty \Lambda_{t+2}^k \int W dZ_{t+2}^k(W)\}. \tag{7}$$

In this case, the surplus in period $t+1$ is zero, and the following period's surplus depends on the number of ads received. If no ad is received, the surplus is $W_{t+2}^\varnothing$, otherwise the consumer chooses the firm that yields the highest surplus. Z_t^k is the period-t cumulative distribution function for consumer surplus if $k > 0$ ads are received[39]. The third line in (6) gives the expected future surplus from receiving one or more ads for the variety. If the firm stays in the market, the consumer is aware of $k+1$ firms corresponding to k new ads, plus the most recently visited firm. If the firm exits, the consumer has k new ads. In either case, the consumer chooses the firm with the highest surplus.

The consumer surplus from a non-trademark firm ($\Gamma = N$) reads

$$\begin{aligned} W_t^N(q,m) \;=\; & \widetilde{u}(q) \\ & +\beta(1-\lambda)\left\{\Lambda_{t+1}^0\left((1-\chi)(1-\delta)W_{t+1}^N(q,m') + (1-\chi)\delta W_{t+1}^D(q^D,m^D) + \chi W_{t+1}^\varnothing\right)\right. \\ & +\sum_{i=1}^\infty \Lambda_{t+1}^i[(1-\chi)(1-\delta)\int \max\{W, W_{t+1}^N(q,m')\}dZ_{t+1}^i(W) \\ & +(1-\chi)\delta W_{t+1}^D(q,m') + \chi\int W dZ_{t+1}^i(W)]\}. \end{aligned} \tag{8}$$

In (8), the first line gives the current surplus. The second, third, and fourth lines give the expected future surplus for the cases when the consumer receives no ad and some ads. In both cases, the consumer can be diverted with probability δ by another firm. The consumer's surplus when diverted by another firm is denoted by $W_{t+1}^D(q^D, m^D)$. The consumer is assumed to know the type of the diverting firm (Γ^D, q^D, m^D).

[39]Because ads from firms arrive independently, one can write $Z_t^k(W) = [Z_t(W)]^k$, where $Z_t(W)$ is the probability that an ad is from a firm that offers a surplus of at most W.

How is $W_{t+1}^D(q^D, m^D)$ determined? Only non-trademark firms can lose customers to diversion. The diversion process can be specified in many ways. For instance, diversion can be random, or targeted (e.g., low-quality firms can imitate high-quality firms and divert their customers). However, the possibility of diversion to firms with lower quality or surplus contradicts consumer rationality in the current model, since consumers know the surplus from the firms they are informed of. Why would a consumer be lured away by a firm that provides lower surplus, given that the consumer has perfect information about the diverting firm? Maintaining consumer rationality requires that only the firms that offer identical surplus can divert customers from each other. In this case, the expected surplus of a customer diverted from a type (N, q, m) firm is $W_{t+1}^D(q^D, m^D) = W_{t+1}^N(q, m')$. Under this assumption, the consumer problem becomes

$$W_t^N(q,m) = \widetilde{u}(q) + \beta(1-\lambda)\left\{\Lambda_{t+1}^0\left((1-\chi)W_{t+1}^N(q,m') + \chi W_{t+1}^{\varnothing}\right)\right.$$
$$\left. + \sum_{k=1}^{\infty}\Lambda_{t+1}^k\left((1-\chi)\int \max\{W, W_{t+1}^N(q,m')\}dZ_{t+1}^k(W) + \chi W dZ_{t+1}^k(W)\right)\right\}.$$

Consider now the probability that a consumer informed of firm type (Γ, q, m) purchases from this firm. Each such consumer can be of two types: one who purchased from the firm in the previous period, and the other who did not. Let the probability of purchase for these two types of consumers be $z_t^\Gamma(q,m)$ and $\omega_t^\Gamma(q,m)$, respectively. The function $z_t^\Gamma(q,m)$ can be written as

$$z_t^\Gamma(q,m) = \sum_{k=0}^{\infty}\Lambda_t^k Z_t^k(W_t^\Gamma(q,m)), \tag{9}$$

where the kth term is the probability of sale to a consumer who has received k ads from firms other than firm (Γ, q, m). Furthermore,

$$\omega_t^\Gamma(q,m) = [\eta_t + (1-\eta_t)\psi_t^\Gamma(q,m)]z_t^\Gamma(q,m). \tag{10}$$

where η_t is the fraction of consumers with no information from any other firms, and $\psi_t^\Gamma(q,m)$ is the probability that the firm a consumer purchased from in period $t-1$ provides more surplus than firm (Γ, q, m)[40].

Consumer surplus has the following properties. First, consumers have nonzero expenditures, or in other words, they always prefer to patronize some firm compared to not consuming a product at all, i.e. $W_t^\Gamma(q,m) > W_{t+1}^{\varnothing}$ for $\Gamma \in \{T, N\}$. Second, because exit probability is the same across firms, firm size does not matter for consumer surplus, i.e. $W_t^\Gamma(q,m_i) = W_t^\Gamma(q,m_j)$ for any i,j. Third, because customers of a non-trademark firm can only be diverted by a firm which provides identical surplus, a firm's trademark status does not affect consumer surplus directly, conditional on quality[41]. Therefore, $W_t^T(q,m) = W_t^N(q,m)$. Given these properties, quality is the only consumer surplus-relevant attribute

[40]Because $\eta_t, \psi_t^\Gamma(q,m) \leq 1$, it follows that $z_t^\Gamma(q,m) \geq \omega_t^\Gamma(q,m)$, i.e. firm (Γ, q, m) faces more demand from its customers in $t-1$ than consumers it reaches through advertising in t.

[41]This is not to say that a firm's brand or reputation does not matter. Consuming a famous brand can enhance utility (not modelled here), but not simply because that brand has a registered trademark.

of a firm. The consumer surplus then simplifies to

$$W_t(q) = \widetilde{u}(q) + \beta(1-\lambda) \left\{ \begin{array}{c} \Lambda^0_{t+1} \left[(1-\chi) W_{t+1}(q) + \chi W^{\varnothing}_{t+1} \right] \\ + \sum_{i=1}^{\infty} \Lambda^k_{t+1} \left[(1-\chi) \int \max\{W, W_{t+1}(q)\} dZ^k_{t+1}(W) + \chi \int W dZ^k_{t+1}(W) \right] \end{array} \right\},$$

and the probabilities of sale in (9) and (10) can be written $z_t(q)$ and $\omega_t(q)$.

3.3 Firms

For an entrant, the cost of developing quality q with efficiency e is

$$C(q;e) = B\left(\frac{q}{e}\right)^{\eta}, \quad B > 0, \eta > 1.$$

C is strictly convex in q, and strictly decreasing in e – a higher quality product costs more to develop, but more efficient firms incur lower costs at any quality.

A firm's production technology is linear in the only input, labor n

$$F(n;q) = Anq^{-\sigma}, \quad A > 0, \sigma > 0. \tag{11}$$

F is decreasing in q – a higher quality product has a higher marginal cost of production.

3.3.1 Marketing technology

Firms disseminate information using a marketing (advertising) technology, $\Phi^{\Gamma}(n_a)$, $\Gamma \in \{T, N\}$, which gives the total number of new consumers informed by the firm (equivalently, the number of ads sent) as a function of marketing labor, n_a

$$\Phi^{\Gamma}(n_a) = A^{\Gamma} n_a^{\rho}, \quad A^{\Gamma} > 0, \rho \in [0,1).$$

Φ^{Γ} is strictly concave, implying diminishing returns to marketing labor[42]. In addition, there is no fixed cost of advertising, $\Phi^{\Gamma}(0) = 0$, and the marginal benefit from marketing labor is very high when the firm does not advertise. [43] These assumptions imply that all firms advertise every period.

The marketing technology differs between firms with and without trademarks. The relative productivity of trademark firms in marketing is given by $\varrho = A^T/A^N > 1$. For any given marketing labor, a trademark firm reaches more consumers. In other words, consumer awareness is higher for trademark firms, because it is easier for consumers to recognize a registered trademark that cannot be confused with others – consistent with the key role registered trademarks play in reducing the informational frictions between a firm and consumers.

[42]From an individual firm's perspective, there is a very large pool of consumers and each ad reaches a distinct consumer. If the marketing technology exhibits constant or increasing returns to scale, some firms can reach an arbitrarily large number of consumers and grow without bound. Decreasing returns to advertising is documented in Sutton (1991). Grossman and Shapiro (1984) point out that decreasing marginal productivity of advertising can stem from media saturation, overlapping media, or different predispositions to view ads.

[43]Note that $\lim_{n_a \to 0} \frac{\partial \Phi^{\Gamma}(n_a)}{\partial n_a} = +\infty$.

3.3.2 The cost of a trademark

There is a one-time cost, c_T, of obtaining a registered trademark. This cost includes not only the registration fees, but also any outlays on designing a trademark that is distinguished, searching the existing trademarks for potential similarity, and hiring legal professionals for the registration process. In addition, firms incur costs to maintain and protect their trademarks. For instance, trademarks have to be renewed periodically and be defended against infringement. In particular, the costs of protecting a trademark can be large, as firms need to be able to detect infringement, hire lawyers, and engage in litigation. For simplicity, c_T represents the present discounted value of all costs associated with owning a registered trademark.

3.3.3 Firm's dynamic problem

Let $\mathrm{T}_t(q,m)$ be an indicator of firm trademark status $\Gamma \in \{T, N\}$, such that $\mathrm{T}_t(q,m) = 1$ if $\Gamma = T$, and 0 otherwise. The period profit of a type (Γ, q, m) firm as a function of its price p, production labor n, and marketing labor n_a is given by

$$\Pi_t^\Gamma(p, n, n_a; q, m) = pm_t^\Gamma(n_a; q, m)x(p, q) - n - n_a, \tag{12}$$

where wage rate, w_t, is normalized to one. The production constraint is

$$F(n; q) = m_t^\Gamma(n_a; q, m)x(p, q), \tag{13}$$

where the right hand side is the total demand for a type Γ firm's product with quality q and price p, if the firm starts the period with m customers and utilizes n_a marketing labor. A customer's demand for a product with quality q and price p is $x(p, q)$, and m^Γ denotes the firm's end-of-period number of customers

$$m_t^\Gamma(n_a; q, m) = \underbrace{m(1-\lambda)(1-(1-\mathrm{T}_t(q,m))\delta)z_t(q)}_{\text{continuing customers}} + \underbrace{\Phi^\Gamma(n_a)\omega_t(q)}_{\text{new customers}} + \underbrace{d_t(q)}_{\text{diverted customers}}, \tag{14}$$

The first term in (14) represents customers from the previous period that continue to purchase from the firm. A firm starts the period with m customers loses λ fraction of its customers due to exogenous customer turnover. A non-trademark firm ($\mathrm{T}_t(q,m) = 0$) loses, in addition, a fraction δ of its customers to customer diversion. Of the remaining consumers, a fraction $z_t(q)$ continue to purchase from the firm. The second term gives the mass of new customers the firm gains through current marketing activity. The firm reaches $\Phi^\Gamma(n_a)$ new consumers, but only a fraction $\omega_t(q)$ of them choose to purchase from the firm.

The third term is the mass of new consumers, $d_t(q)$, diverted by a firm from non-trademark firms with the same quality q[44]. Each firm at quality q gets an equal share of the mass of consumers diverted from non-trademark firms, i.e. $d_t(q)$ is the same across all firms with quality q. In other words, the customer diversion rate δ can be viewed as a "tax" on non-trademark firms at a quality level, and $d_t(q)$ represents the equal redistribution of the gains from this tax across all firms at that quality. Because firms of quality

[44]Note that $d_t(q)$ is a function of quality, since only the firms providing identical surplus can divert consumers from each other.

q differ by customer size, m, the process of customer diversion implies that smaller firms are net gainers, while larger firms are net losers in terms of customer size. Thus, the process captures the notion that smaller and younger firms tend to benefit more from customer diversion due to imitating and infringing activity, compared to larger, more established firms.

The value of a type (T, q, m) firm is

$$V_t^T(q, m) = \max_{p,n,n_a} \left[\Pi_t^T(p, n_a, n; q, m) + ((1-\chi)/(1+r))V_{t+1}^T(q, m^T)\right]. \tag{15}$$

In (15), the firm exits with probability χ in the beginning of the next period, and the firm value upon exit is normalized to zero.

Similarly, the value of a type (N, q, m) is given by

$$V_t^N(q, m) = \max \left\{ \begin{array}{c} \max_{p,n,n_a} \left[-c_T + \Pi_t^T(p, n, n_a; q, m) + ((1-\chi)/(1+r))V_{t+1}^T(q, m^T)\right], \\ \max_{p,n,n_a} \left[\Pi_t^N(p, n, n_a; q, m) + ((1-\chi)/(1+r))V_{t+1}^N(q, m^N)\right] \end{array} \right\}, \tag{16}$$

In (16), the firm chooses between whether to register a trademark by paying c_T, or continue without a trademark.

3.3.4 Entry

Each period a mass of $M_t \geq 0$ new firms enter. If a firm decides to stay after entry, it starts with zero customers ($m = 0$) and no trademark, and has to develop its product first. An entrant with product development efficiency e chooses its quality to maximize its net value

$$\max_q \left[V_t^N(q, 0) - C(q; e)\right], \tag{17}$$

Let $q_t(e)$ solve (17). An entrant exits in its first period without developing a product if $V_t^N(q_t(e), 0) < C(q_t(e); e)$.

Free entry implies

$$\int \max\left\{0, \left[V_t^N(q_t(e), 0) - C(e, q_t(e))\right]\right\} h(e)de \leq \kappa, \tag{18}$$

which holds with equality when $M_t > 0$.

3.3.5 Trademark Decision

A firm registers a trademark in period t if

$$\begin{aligned} & -c_T + \max_{p,n,n_a} \left[\Pi_t^T(p, n, n_a; q, m) + ((1-\chi)/(1+r))V_{t+1}^T(q, m^T)\right] \\ \geq \; & \max_{p,n,n_a} \left[\Pi_t^N(p, n, n_a; q, m) + ((1-\chi)/(1+r))V_{t+1}^N(q, m^N)\right]. \end{aligned} \tag{19}$$

In other words, the net benefit from registering a trademark exceeds that from staying as a non-trademark firm.

4 Stationary equilibrium

A stationary equilibrium for the model economy is defined as follows.

Definition (Stationary Equilibrium). *A stationary equilibrium consists of product quality, employment, pricing, advertising, and trademark decisions,* $q(e)$, $n(\Gamma, q, m)$, $p(q, m)$, $n_a(\Gamma, q, m)$, *and* $T(q, m)$, *mass of customers gained from diversion,* $d(q)$, *a cumulative distribution of consumer surplus* (W) *across ads* $Z(W)$, *a measure* $\mu(\Gamma, q, m)$ *over firm types, and an entry mass* $M \geq 0$, *such that*

(i) Firms maximize their profits,

(ii) Consumers maximize their surplus,

(iii) M *satisfies the free entry condition (18),*

(iv) Markets clear,

(v) Z *and* μ *are consistent with conditions (i) through (iv).*

At any quality level q, the mass of customers gained by all firms from diversion equals the mass of customers lost by non-trademark firms to diversion

$$d(q) \int_m d\mu(\cdot, q, m) = \delta \int_m m d\mu(N, q, m).$$

The interest rate is given by $r = (1/\beta) - 1$.

5 Characterization of stationary equilibrium

Consider first the pricing and advertising policies of firms in stationary equilibrium.

5.1 Pricing policy

Because firms cannot commit to prices and prices are not advertised, consumers decide which firm to purchase from by correctly anticipating that once they commit to purchasing from a firm they will face monopoly pricing. Firms set prices taking this consumer behavior into account, and the resulting pricing policy follows a constant mark-up rule. For a consumer, the efficiency condition for optimal consumption of each variety yields[45]

$$\frac{x(p(q_i), q_i)}{x(p(q_j), q_j))} = \left[\frac{p(q_j) q_i^{\xi}}{p(q_i) q_j^{\xi}}\right]^{1/(1-\theta)},$$

for all i, j. Without loss of generality, designate the lowest quality $\underline{q}$ as the reference, and express the consumer demand for quality q as

$$x(p, q) = \left[\frac{q}{\underline{q}}\right]^{\frac{\xi}{1-\theta}} \left[\frac{p(\underline{q})}{p}\right]^{\frac{1}{1-\theta}} x(p(\underline{q}), \underline{q}). \tag{20}$$

[45] Consumer optimization yields $\frac{\frac{\partial u(q_i, x_i)}{\partial x_i}}{\frac{\partial u(q_j, x_j)}{\partial x_j}} = \frac{\theta q_i^{\xi} x_i^{\theta-1}}{\theta q_j^{\xi} x_j^{\theta-1}} = \frac{p(q_i)}{p(q_j)}$.

The consumer's total expenditure, y, is given by

$$y = (1-\omega_o) \int_{\underline{q}}^{\overline{q}} p(\mathfrak{q}) x(p(\mathfrak{q}), \mathfrak{q}) d\omega(\mathfrak{q}) \tag{21}$$

where ω_o is the fraction of varieties the consumer has no information of, and $\omega(\mathfrak{q})$ is the probability that the consumer purchases quality $\mathfrak{q}$ of any given variety – equivalent to the (unconditional) probability in (10) that the highest quality a consumer is informed of is $\mathfrak{q}$.

Using (20) and (21), the demand function can be written as

$$x(p, q) = \frac{\left(\underline{q}^{\xi}/p\right)^{\frac{1}{1-\theta}} y}{(1-\omega_o) \int_{\underline{q}}^{\overline{q}} p(\mathfrak{q})^{\frac{-\theta}{1-\theta}} \mathfrak{q}^{\xi} d\omega(\mathfrak{q})}.$$

Consider now a firm's pricing decision. The firm's demand in (13) is separable in customer size and individual consumer's demand. Therefore, the optimal price is the solution to

$$\max_{p} x(p, q) \left(p - (q^{\sigma}/A)\right), \tag{22}$$

where (q^{σ}/A) is the marginal cost of production, based on the production function in (11).

Proposition 1 *Price, p, depends only on q, and follows a constant mark-up rule, $p(q) = \frac{1}{\theta}\frac{q^{\sigma}}{A}$, which is increasing in q.*

Higher quality products provide consumers higher utility, but they are also more expensive (Proposition 1). Consumers will always prefer higher quality products under the condition $\xi > \sigma$, which is assumed hereafter. Consumer surplus and firm value are then also increasing in quality.

Proposition 2 *(i) $W(q)$, $z(q)$, and $\omega(q)$ are increasing in q, (ii) $V^{\Gamma}(q, m)$ is increasing in q and m.*

Therefore, in equilibrium, consumers purchase from the firm that offers the highest quality (or surplus) among the firms he is informed of, and firms' pricing follows a constant mark-up rule.

5.2 Quality policy

In stationary equilibrium, the optimal quality choice for an entrant with product development efficiency e solves

$$\max_{q} \left[V^{N}(q, 0) - C(q; e)\right]. \tag{23}$$

Assume that there exists a unique $q(e) \in (0, \infty)$ that solves (23). Firms with higher efficiency e choose higher quality levels.

Proposition 3 *$q(e)$ is increasing in e.*

In equilibrium, firms offer a range of quality $[\underline{q}, \overline{q}]$, and the distribution of quality across firms is driven by the distribution of efficiency e.

5.3 Advertising policy

A firm's advertising expenditure is equivalent to marketing labor, n_a. Let $\pi(q) = x(p(q), q)\,(p(q) - (q^\sigma/A))$ denote the variable profit per consumer that includes only the marginal cost of production. Consider a firm that is $\tau \in \mathbb{Z}$ periods away from its trademark registration, where $\tau = 0$ corresponds to the period of trademark registration, $\tau > 0$ to the periods after registration, and $\tau < 0$ to the periods before registration. The case of the firm never registering a trademark is $\tau = -\infty$. Denote by $n_a^{\Gamma(\tau)}(q,\tau)$ the optimal advertising policy, where $\Gamma(\tau) = N$ if $\tau < 0$ and $\Gamma(\tau) = T$, otherwise. The first order condition for $n_a^{\Gamma(\tau)}(q,\tau)$ is

$$\rho A^{\Gamma(\tau)}[n_a^{\Gamma(\tau)}(q,\tau)]^{\rho-1}\omega(q)\pi(q)L(q,\tau) = 1, \tag{24}$$

where

$$L(q,\tau) = \sum_{t=0}^{-\tau}\left(\tfrac{(1-\chi)l^N(q)}{1+r}\right)^t + \left[\left(\tfrac{(1-\chi)l^N(q)}{1+r}\right)^{-\tau I(\tau<0)}\sum_{t=1}^{\infty}\left(\tfrac{(1-\chi)l^T(q)}{1+r}\right)^t\right], \tag{25}$$

and $I(\tau < 0)$ is an indicator function.

The left hand side of (24) is the marginal benefit of increasing the marketing labor by one unit, and the right hand side is the marginal cost (normalized wage rate). The marginal benefit depends on the discounted flow of profit from a customer acquired through advertising. This customer continues to patronize the firm from one period to another with probability $l^{\Gamma(\tau)}(q)$, which is the effective customer loyalty rate

$$l^{\Gamma(\tau)}(q) = \begin{cases} z(q)(1-\lambda) & \text{if } \Gamma(\tau) = T, \\ z(q)(1-\lambda)(1-\delta) & \text{if } \Gamma(\tau) = N. \end{cases}$$

Hence, $L(q,\tau)$ is the (discounted) likelihood that an acquired customer stays with the firm. The solution to (24) is

$$n_a^{\Gamma(\tau)}(q,\tau) = [\rho\omega(q)\pi(q)L(q,\tau)]^{1/1-\rho}. \tag{26}$$

Two special cases are noteworthy. One is the case when the firm already has a trademark ($\tau \geq 0$), and the other is when the firm never registers a trademark ($\tau = -\infty$). For these two cases, optimal advertising policy is time-invariant and given by

$$n_a^{\Gamma(\tau)}(q,\tau) = \left[\rho A^{\Gamma(\tau)}\omega(q)\pi(q)\frac{1+r}{1+r-(1-\chi)l^{\Gamma(\tau)}(q)}\right]^{\frac{1}{1-\rho}}, \text{ for } \tau = -\infty \text{ or } \tau \geq 0. \tag{27}$$

What is the effect of a trademark on advertising? Consider a firm with a trademark ($\tau \geq 0$). Then, (27) implies

$$\frac{n_a^T(q,\tau)}{n_a^N(q,-\infty)} = \left[\varrho\left(\tfrac{1}{1-\delta}\right)\left(\tfrac{1+r-(1-\chi)l^N(q)}{1+r-(1-\chi)l^T(q)}\right)\right]^{\frac{1}{1-\rho}} > 1, \text{ for } \tau \geq 0, \tag{28}$$

because $\varrho > 1$, and $l^T(q) > l^N(q)$. In other words, the firm advertises more when it has a trademark compared to the case of not registering a trademark[46].

Higher quality firms advertise more, ceteris paribus, as they make higher profit per consumer and have higher probability of attracting and retaining consumers. Moreover, for a firm that eventually registers a

[46]Note that in stationary equilibrium a given firm type (q, m) either registers a trademark or not – hence the comparison of advertising expenditures is a counterfactual exercise.

trademark, advertising expenditure increases as the firm approaches its trademark date. This effect arises from the anticipated increase in the effective loyalty rate, $l^{\Gamma(\tau)}(q)$, that occurs at trademark registration. The properties of the advertising policy is summarized below.

Proposition 4 *i)* $n_a^{\Gamma(\tau)}(q,\tau)$*, is increasing in* q*,*

ii) $n_a^{\Gamma(\tau)}(q,\tau) > n_a^{\Gamma(\tau)}(q,-\infty)$ *for* $\tau > -\infty$*,*

iii) For a firm that registers a trademark at some point in its life-cycle, $n_a^{\Gamma(\tau)}(q,\tau)$ *is increasing in* τ *before its trademark date* $(-\infty < \tau < 0)$*, and constant after registration (for* $\tau \geq 0$*); for firms that never register a trademark,* $n_a^{\Gamma(\tau)}(q,\tau)$ *is time-invariant.*

Proposition 4(iii) implies advertising expenditure is higher after trademark registration compared to any period before the registration. As discussed in Section 5.5, firms that register trademarks have higher quality than those that do not register trademarks. Hence, by Proposition 4(i) and (ii) firms with a trademark advertise more on average, both before and after registration, compared to those without trademarks. These implications are consistent with the empirical findings in Section (2.5) that suggest higher average advertising spending by trademark firms both pre and post-registration compared to firms with no trademarks, and an increase in advertising expenditure after registration for those firms that register trademarks.

How much of its revenue does a firm allocate to advertising? Advertising intensity, defined as the share of advertising expenditures in revenue, is given by

$$\mathfrak{a}^{\Gamma(\tau)}(q;\tau) = \frac{n_a^{\Gamma(\tau)}(q;\tau)}{m^{\Gamma(\tau)}(n_a^{\Gamma(\tau)};q,m)p(q)\widetilde{x}(q)}, \tag{29}$$

where $\widetilde{x}(q) = x(p(q),q)$ and m is the size at the beginning of the period. For firms with a trademark and those that never register one, advertising expenditures do not vary over time (Proposition 4) but revenue increases monotonically, implying that advertising intensity declines as the firm ages (or equivalently, as its size increases). For firms that register a trademark, advertising expenditures increase until the trademark date (Proposition 4), along with revenue. However, the change in advertising intensity during this period is ambiguous and depends on the parameters.

Overall, in a panel of firms, advertising intensity tends to decline over time (as a firm grows or ages) if the composition of firms is dominated by firms that never register a trademark or firms already in their post-trademark period. Consistent with this scenario, in the sample of firms studied in Section 2.5, advertising intensity declines over time as firms grow[47].

5.4 Firm size dynamics and heterogeneity

Using the law of motion in (14) and (27), the customer size of an age-A firm with quality q which registers its trademark at birth is

$$m_{\text{A}}^T(q) = \sum_{j=0}^{\text{A}} \left(\Phi^T(n_a^T(q,0))\omega(q) + d(q)\right)[l^T(q)]^j = [\Phi^T(n_a^T(q,0))\omega(q) + d(q)]\frac{1-[l^T(q)]^{\text{A}+1}}{1-l^T(q)}.$$

[47]In general, the average advertising intensity of a trademark firm in its post-trademark period can be higher or lower than that in the pre-trademark period, depending on the parameters. The analysis in Section 2.5 indicates that advertising intensity tends to be higher in the post-trademark period in the sample studied.

Similarly, the size of an age-A firm with quality q which never registers a trademark is

$$m_{\text{A}}^N(q) = \left[\Phi^N(n_a^N(q,-\infty))\omega(q) + d(q)\right] \frac{1-[l^N(q)]^{\text{A}+1}}{1-l^N(q)}.$$

The long-run size of a firm that registers a trademark at birth is related to its initial size (at age zero) as follows

$$\frac{m_\infty^T(q)}{m_0^T(q)} = \frac{\lim_{\text{A}\to\infty} m_{\text{A}}^T(q)}{m_0^T(q)} = \frac{1}{1-l^T(q)}.$$

In other words, the steady state size of this firm is $1/(1-l^T(q))$ times its initial size. Since $l^T(q)$ is strictly increasing, the ratio is larger for higher quality firms that trademark. For the highest quality firm with a trademark, $l^T(\overline{q}) = 1-\lambda$, and hence, $m_\infty^T(\overline{q})/m_0^T(\overline{q}) = \lambda^{-1}$.

Similarly, for a firm which never registers a trademark

$$\frac{m_\infty^N(q)}{m_0^N(q)} = \frac{1}{1-l^N(q)}.$$

Since $l^N(q) = z(q)(1-\lambda)(1-\delta)$, the growth potential for firms without a trademark depends critically on the customer diversion rate, δ. For the lowest quality firm that never trademarks, the life cycle growth potential is the lowest among all firms, since $z(\underline{q})$ is the lowest. Because the lowest quality firm can only sell to consumer that do not receive any other ads, $z(\underline{q})$ is lower in an economy with higher number of ads per consumer, implying a lower growth potential for this type of firm when there is more advertising activity in the aggregate.

The heterogeneity in firm size depends on the range of quality in equilibrium. In an economy where a positive mass of firms register trademarks, the support of the firm size distribution is $[m_0^N(\underline{q}), m_\infty^T(\overline{q})]$, such that

$$\begin{aligned}
\frac{m_\infty^T(\overline{q})}{m_0^N(\underline{q})} &= \frac{\Phi^T(n_a^T(\overline{q},0))\omega(\overline{q}) + d(\overline{q})}{\Phi^N(n_a^N(\underline{q},-\infty))\omega(\underline{q}) + d(\underline{q})} \frac{1}{1-l^T(\overline{q})}, \\
&= \frac{\Phi^T(n_a^T(\overline{q},0))}{\Phi^N(n_a^N(\underline{q},-\infty))\omega(\underline{q}) + d(\underline{q})} \frac{1}{\lambda},
\end{aligned}$$

where the last equality follows because $\omega(\overline{q}) = 1$, $l^T(\overline{q}) = 1-\lambda$ and $d(\overline{q}) = 0$ – since all firms at quality $\overline{q}$ have trademarks, no firm can divert consumers.

5.5 Trademark policy

Consider a type (q, m) firm without a trademark. Based on (16), the value of this firm is given by

$$V^N(q,m) = \max\{V^{NN}(q,m), V^T(q,m)\}.$$

Here, V^{NN} is the value from not registering a trademark in the current period and continuing as a non-trademark firm into the next period

$$V^{NN}(q,m) = \widetilde{\Pi}^N(q,m) + ((1-\chi)/(1+r))V^N(q,m^N),$$

where $\widetilde{\Pi}^N(q,m)$ is the firm's period profit. From (19), the difference between the value from registering a trademark versus not registering in the current period is

$$V^D(q,m) = V^T(q,m) - V^{NN}(q,m). \tag{30}$$

The firm registers a trademark in the current period if $V^D(q,m) > c_T$. Now, denote the value of this firm in terms of its quality q and age A by $V^{\Gamma}_{\text{A}}(q) \equiv V^{\Gamma}(q,m)$, $\Gamma = N, T,$or NN, where $m \equiv m(\text{A}, q)$. The following proposition shows that the difference, $V^D_{\text{A}}(q) = V^T_{\text{A}}(q) - V^{NN}_{\text{A}}(q)$, is monotonic in q and A (or m).

Proposition 5 $V^D_{\text{A}}(q)$ *is increasing in* q *and* A.

When does stationary equilibrium feature both firms that trademark and those that never trademark? The answer depends on how high the cost of registration, c_T, is relative to the benefits from registration. Suppose in equilibrium some firm with quality $\widetilde{q} \in [\underline{q}, \overline{q}]$ registers a trademark at some age A. This outcome is obtained if c_T is sufficiently low. Then, by Proposition (5), all firms with quality $q > \widetilde{q}$ also register a trademark at most by age A. Depending on the range of quality in equilibrium, there can also be a quality level $\widehat{q} > \widetilde{q}$ such that all firms with $q > \widehat{q}$ register a trademark at birth ($\text{A}= 0$). If the cost of registration is high enough, no firm will register a trademark.

Figure 7 summarizes the discussion above, and shows the trademark policy in the size-quality plane. In region I, a firm enters the industry, but the quality of the firm's product is so low that it never registers a trademark. In region II, firms register a trademark once they reach a threshold size. The last region (III) consists of very high quality firms which register a trademark at birth (upon entry). The solid curve in Figure 7 represents the maximum possible size of a trademark firm $m^T_{\infty}(q)$, whereas the dashed curve represents the maximum size for a non-trademark firm in that region, $m^N_{\infty}(q)$.

6 Calibration

The model is now taken to data. The parameters are calibrated so that several key features of the model economy match closely to those of the U.S. economy.[48] There are 17 model parameters – preference parameters: $\{\beta, \xi,$ and $\theta\}$; technology parameters: $\{A, A^N, B, \delta, \lambda, \eta, \rho, \chi, \sigma, \varrho\}$; cost parameters: $\{c_e, c_T\}$; and distributional parameters: $\{\underline{e}, \overline{e}\}$. The efficiency distribution, $H(e)$, is assumed to be uniform over $[\underline{e}, \overline{e}]$. Without loss of any economic representation, three parameters are normalized: $A, B, \underline{e} = 1$. The time preference parameter is set at $\beta = 0.95$. The remaining parameters are jointly determined so that a measure of distance between the targeted moments and their corresponding value in the data is minimized. Table 8 lists the values of the data moments used to calibrate the parameters, and the model counterparts. The parameter values resulting from the calibration are shown in Table 9.

6.1 Benchmark economy

The key features of the benchmark economy that results from the calibration exercise are summarized in Figure 8. Only around 4% of firms register for a trademark - consistent with data. Figure 8(a) shows

[48] To calibrate the model, the state space is discretized into a grid. For each of the state variables, quality and firm size (customers), 100 grids are used.

the firm size at which firms of a given efficiency register their trademark. Firm size is indicated as a percentage of the steady-state size in terms of customers. The firms with the highest efficiency (or quality) level register a trademark when they are young and small. Firm with quality levels close to the highest also register, but they do so later when they get larger. Low-quality firms never register a trademark. The quality policy depicted in Figure 8(b) is monotone in efficiency, and quality increases faster for efficiency levels near the trademark threshold.

The likelihood of a sale from an ad is plotted Figure 8(c). All ads sent by the highest quality firms generate a sale, whereas the lowest quality firms' ads turn into a sale with probability 49%. Since the lowest quality firms are the least favored by consumers, this probability is equal to the mass of consumers with no product information. Higher quality firms with trademarks reach consumers more easily. They are also preferred more by consumers and their marketing activity is rewarded by a sale more often. As a result, they grow more quickly. The outstanding performance of trademark firms is illustrated in Figure 8(d), which shows the life-cycle firm size profile for the lowest quality level that registers a trademark and the counterfactual profile if that firm did not register a trademark. This firm registers its trademark at age 3, and grows much faster thereafter compared to its counterfactual, achieving nearly 50% larger size than the counterfactual by the end of 30 periods.

Despite the uniform distribution of quality development efficiency among firms, firm size distribution is positively skewed and has a relatively long and heavy right tail populated by firms with high quality and trademarks, as shown in Figure 8(e). This skewness results from the higher advertising activity and higher likelihood of sale for higher quality firms, which allow them to accumulate a large customer base. The flat portion on the left tail is attributable to small firms with low efficiency that mainly gain customers through customer diversion. High quality products constitute a considerable share of a consumer's expenditure. Figure 8(f) plots the expenditure share of goods consumed by a consumer as a function of firm efficiency. Around 6% of the consumer expenditure is dedicated to firms with the highest efficiency (quality). The concentration of expenditures in high quality firms with trademarks is an important source of welfare, as highlighted below in model simulations.

7 Simulations

The model is now simulated to conduct three thought experiments. First, to evaluate the effects of trademarks at large, an economy with no trademarks is analyzed. This is done by setting the one-time trademark cost c_T prohibitively high – all other parameters fixed at their benchmark values. This economy is called the *no-trademark economy.* The second experiment examines the effects of an increase in the customer diversion rate, δ. This experiment seeks to understand the effects of an exogenous increase in the imitation and infringement activity that results in higher customer diversion among firms. The economy with the higher δ is referred to as the *high-diversion economy.* Finally, an economy where trademarks are universal is considered, by lowering the cost c_T enough to allow all firms to obtain trademarks after entry. This *full-trademark* economy is studied to understand the effects of a policy that protects all firms from customer diversion.

7.1 No-trademark economy

The absence of a trademark system results in important changes. Figure 9(a) plots the quality policy for the benchmark and no-trademark economies. Firms with trademarks in the benchmark economy develop on average 14% higher quality compared to the no-trademark economy. However, firms without trademarks offer lower quality products. In the benchmark economy, firms without trademarks are discouraged to invest in quality because of the intense competition they face from firms with trademarks, as a result of their more efficient marketing technology and higher incentives to advertise induced by trademark protection. Figure 9(b) plots the probability of sale. Note the kink in the probability in the benchmark economy at the efficiency level corresponding to the trademark threshold. There is a large increase in the probability of sale for firms that obtain a trademark. More consumer information about firms implies more intense competition in the benchmark economy, and the probability of sale is uniformly lower than that in the no-trademark economy for all quality levels below the highest quality.

Despite this negative externality on less efficient (lower quality) firms, in the benchmark economy, consumers purchase products that are higher quality and more consumers get to enjoy them. Around 6% of consumers purchase the highest quality products, whereas this share is around 3% in the no-trademark economy (Figure 9(c)). These differences between the two economies highlight the critical role of trademarks in quality provision. Existence of trademark protection leads to significant reallocation of output and employment across firms. Figure 9(d) indicates substantial job reallocation towards high quality firms with trademarks from the firms with no trademarks – based on the comparison of two stationary equilibria. The highest quality firms experience nearly 70% increase in their employment when trademark protection is in place.

As shown in Figure 9(e), the presence of trademarks creates significant skewness in firm size in the benchmark economy, whereas the absence of trademarks induces lower inequality in firm size. The firms with trademarks are much larger than the others in the benchmark. Overall, the benchmark economy behaves more like an economy with superstar firms, where trademarks reinforce the size advantage of high quality firms.

The evolution of size for the highest quality firm types is shown in Figure 9(f). Up to age three, this firm type is larger in the no-trademark economy. In the no-trademark economy, young and small firms gain diverted customers at a higher rate, which benefits them initially. In contrast, in the benchmark economy a young high quality firm grows rapidly after registering a trademark, and it reaches a size that is more than four times that of the corresponding firm in the no-trademark case. The life-cycle size profile is very different for the lowest quality firms, as shown in Figure 9(g). This firm type has around 9% smaller eventual size in the benchmark economy compared to the no-trademark one[49].

Industry concentration is plotted in Figure 9(h). Concentration is measured by the customer share of firms above a given efficiency (quality) level. In the benchmark economy, the highest quality firms account for nearly 6% of total market share, but only around 3% in the no-trademark economy. Note that concentration is also higher at any level of quality in the benchmark economy.

Overall, the availability of trademarks improves consumer welfare. Product information available to consumers increases. Consumers enjoy more product varieties, since they are now aware of more of them.

[49]Notice also that the lowest quality firms approach their steady-state size much sooner compared to the highest quality firms. The more gradual convergence to steady-state size for high quality firms is driven by the fact that marketing technology has diminishing returns, limiting the number of new consumers a firm can acquire in any period.

Additionally, average product quality is higher. As shown in Table 10, product information available to consumers is 1.28% higher, and product variety consumed is 0.86% higher in the benchmark. Average product quality is also 8.06% higher. Higher average product quality results from two sources. First, firms protected by trademarks invest more in quality. Second, there is a reallocation of consumption towards higher quality products enabled by more consumer awareness about higher quality firms. As a result, consumer welfare (measured by compensating variation, C.V.) is 3.44% higher. At the same time, the benchmark economy exhibits an industry concentration that is significantly higher (22.4%) when measured by the Hirschman-Herfindahl Index (HHI)[50].

7.2 High-diversion economy

Customer diversion rate may vary across industries, regions, or countries depending on the incentives for imitation and infringement. For instance, customer diversion may increase when firms are exposed to foreign competition, some of which may be in the form of imitations of domestic products. The strength of institutions and regulations designed to protect brands also matter. A relaxing of regulations on product quality in an industry can reduce entry barriers and invite infringing competitors, resulting in higher customer depreciation.

Suppose now the customer diversion rate approximately doubles to 30% from its benchmark value of 14%. Figure 10(a) plots the size at which firms register their trademarks as a function of efficiency. Threshold efficiency level for registering a trademark is lower in the high-diversion economy. Since protection provided by trademarks is more valuable when the diversion rate is higher, firms with lower levels of quality (or efficiency) also register trademarks. Consequently, the fraction of firms with a trademark nearly doubles from 4% to 8%.

The quality policies pictured in Figure 10(b) indicate that high efficiency firms have roughly the same quality levels in both cases. However, firms with slightly lower efficiency than the trademark threshold in the benchmark economy now register trademarks in the high-diversion economy and increase their quality levels. At lower efficiency levels, firms reduce quality significantly – particularly those firms with slightly lower quality than the trademark threshold in the high-diversion economy. Note also that the probability of sale goes down for firms near the benchmark trademark threshold, but rises for those with low quality, as shown in Figure 10(c). Overall, high quality trademark firms make up a larger share in consumer spending in the high-diversion economy, as shown in Figure 10(d).

Higher customer diversion has important employment reallocation effects through its impact on firms' trademark decisions, as depicted in Figure 10(e). Firms with the highest efficiency have about 5% more employment in the high-diversion economy. The biggest gainers of reallocation, however, are the firms which did not have trademarks in the benchmark, but now do in the high diversion economy. Employment in these firms increase by around 50%. Lower quality firms without trademarks lose employment significantly. The largest negative effects are experienced by the relatively high quality firms with no trademarks.

[50]Since there is a continuum of firms in each industry, conventionally computed HHI would inevitably be always zero. Instead, a converted (discretized) HHI is considered, where a constant measure of firms is set to be one firm and that measure is calibrated so that the benchmark HHI matches the average 6-digit NAICS HHI in the US economy (1280). This constant measure of firms is used across all equilibria pertaining to the different thought experiments to be consistent with the benchmark measurement of HHI.

Figure 10(f) indicates that both economies exhibit significant skewness in size distribution, but the higher diversion economy has higher skewness. The highest quality firm type has only slightly higher life-cycle size profile in the high-diversion economy, as shown in Figure 10(g). On the other hand, marginal firms near the benchmark trademark threshold now register trademarks and expand. A high diversion rate and more intense competition arising from an increase in the mass of trademark firms adversely affect low quality firms: they are significantly smaller compared to their counterparts in the benchmark economy. The steady-state size of the lowest quality firm type is lower by about 20% in the high-diversion economy (Figure 10(h)). Industry concentration increases at the efficiency levels for firms around the trademark threshold – see Figure 10(i).

Both product information and the number of varieties consumed are lower, but the average product quality is higher in the high-diversion economy. As seen in Table 10, product information declines by around 4.6%, and the number of varieties consumed goes down by 3.2%. On the other hand, an increase in the share of firms that register trademarks pushes average product quality up by around 4.8%. Nevertheless, these changes almost cancel each other out, and the overall welfare impact of higher diversion environment is slightly positive. At the same time, higher diversion rate increases industry concentration by around 14%. This large rise in concentration is driven by the higher trademark registration by firms and the resulting increase in the customer share of high quality firms with trademarks.

To further assess the significance of a trademark system in an economy with high-diversion rate, consider now the following additional thought experiment. What would be the effects of a higher diversion rate if there was no trademark protection at all? Consider now the economy where the diversion rate is $\delta = 30\%$, but there is no trademark protection. Because of high customer diversion rate, customer depreciation is now higher. However, with no trademark protection available, firms do not invest as much in quality and customer accumulation. Figures A1(f) and (g) in Appendix indicate that the evolution of firm size is significantly different in the absence of a trademark system. In particular, the highest quality firms are affected the most as their steady-state mass of customers declines by around ten fold in the high-diversion economy, compared to a 15% gain for the lowest quality firms.

Since firms with high efficiency now have a lower incentive to invest in quality, quality differences across efficiency levels diminish, as shown in Figure A1(a). Consequently, consumption shifts from higher quality products to lower quality ones – Figure A1(c). Because higher quality firms now also obtain lower returns from marketing and customer accumulation, industry concentration decreases – Figure A1(h).

Overall, trademarks mitigate the welfare-reducing effect of a higher diversion rate, providing a defense in an environment with higher diversion. Without trademarks, a higher diversion rate would have a much larger impact on all dimensions critical for consumer welfare. As shown in Table 10, the availability of product information would go down by 2%, and the consumed product variety by 1.4%. Average product quality would also be lower by 16%. All of these effects would add up to a 7.6% lower welfare. At the same time, industry concentration would go down by around 36%.

The high diversion thought experiment used a diversion rate that is nearly twice the rate in the benchmark. Figure A2 in Appendix explores the robustness of the results to customer diversion rate. Almost all aggregates change monotonically as the diversion rate increases from 10% to 40%. The trademark threshold becomes lower as the diversion rate increases. Consumer information and product variety decline, but average quality increases. The latter effect dominates and welfare also increases.

Higher diversion rate also leads to higher concentration.

7.3 Full-trademark economy

Consider now an economy where all firms can register trademarks without a cost. In other words, the cost of trademark registration c_T is now so low that all firms register trademarks in their first period. Because firms are now insulated from diversion and have a more efficient marketing technology, aggregate advertising increases. Consumer information goes up, as shown in Figure 11(c). In the benchmark economy, around 49% of consumers are uninformed, as opposed to 35% in the full-trademark economy.

Firms around the threshold quality level for trademark registration in the benchmark economy now invest more in quality, as they are protected from diversion – see Figure 11(a). However, the expenditure share of different qualities becomes less skewed (Figure 11(d)), and the average quality of products consumed is lower.

The increase in consumer information does not materially alter the evolution of size for the highest quality firm type, as seen in Figure 11(g). However, there is a large adverse effect on the lowest quality firm type. Figure 11(h) illustrates that over the life-cycle, the size of the lowest quality firm is nearly one third of what it would be in the benchmark economy.

Industry concentration is also lower in the full-trademark economy. The customer share of the highest quality firm goes down from around 10% in the benchmark economy to 4% in the full-trademark economy—see Figure 11(i). The ownership of trademarks by all firms creates a large reallocation across firms away from the highest quality firms to lower quality ones (Figure 11(e)). As a result, firm size distribution also becomes less skewed – Figure 11(f).

Universal trademark protection improves consumer welfare through higher consumer information and higher product variety consumed. As shown in Table 10, the number of ads per consumer goes up by 48.5% compared to the benchmark, whereas the number of varieties consumed increases by 27.6%. However, average product quality declines by 13.3%. Nevertheless, the first two effects dominate, and consumer welfare improves by about 5%. Industry concentration drops drastically (36.5%) as a result of customer reallocation from high quality firms to lower quality ones.

8 Conclusion

What is the significance of the trademark system in the U.S. economy? This paper has provided empirical evidence on the types of firms that register trademarks, and how firms with registered trademarks perform compared to those without. The evidence suggests that firms select into trademark registration based on a variety of characteristics associated with high growth and innovation. Beyond this selection, firms that register trademarks experience more growth compared to their counterparts and are more likely to achieve outcomes in the right tail of employment and revenue distributions. Trademarks also appear to protect firm value in the face of more intense competition, and are associated with higher marketing activity. These findings suggest a role for trademarks in the making of skewness in firm size and in the emergence of superstar firms.

Motivated by this evidence, a general equilibrium model of trademarks is developed. Firms invest in product quality and engage in marketing to gradually build a customer base under the threat of customer attrition due to imitation and infringement. Firms can protect their customer base from erosion by registering trademarks. Trademarks have two basic functions in the model: they protect a firm's customer base from depreciation due to imitation and infringement of its brand, and they allow firms to spread information about themselves more effectively. The analysis of the model indicates that the existing trademark system in the United States has economically significant effects. The absence of trademark protection would result in a significant loss of welfare, driven by a decline in quality provision, consumer information, and product variety consumed. Counterfactual experiments suggest that trademarks are even more critical for welfare and quality provision in environments where imitation and infringement are inherently more intense. In other words, economies with a trademark system are more resilient in the face of higher customer diversion or depreciation.

To focus on the fundamental roles of trademarks, the model has abstracted from a variety of other considerations. Some of these are left for future work, but with some modifications, the model can accommodate many of them. For instance, the model assumes that firm exit is exogenous, which means that the entry rate is also exogenously determined in the stationary equilibrium. Consequently, the model does not examine the potential adverse effects on entry arising from the dominance of trademark firms and the associated high concentration. Additionally, the model does not account for any additional utility or prestige effects from consuming products of well-known firms, which could further reinforce reallocation towards firms with more established trademarks. Trademarks do not have any direct effects on pricing and markups, but only through their impact on quality. Furthermore, product quality is perfectly revealed in advertisements, and future work can explore the effects of imperfect information about quality. Finally, the set of feasible product varieties is fixed, and the connection between new product innovations and trademarks can be introduced.

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Table 1. Some characteristics of firms with and without trademarks in 2014

Fraction of firms with trademark	4.01%
Fraction of new-born firms with trademark	0.90%
Avg. age at first trademark registration (years)	6.83
	(0.02)

	Firms with	
	TM	no TM
Avg. employment	285	12
	(10.37)	(0.06)
Avg. employment (largest 1% of firms)	19,360	427
	(939.20)	(5.68)
Employment share	49%	51%
Employment share (newborn (age zero) firms)	1.5%	98.5%

Notes: Standard errors in parentheses. Largest 1% of firms is defined separately for TM and non-TM firms.

Table 2. Early firm characteristics and the first trademark registration

	TM Indicator within 5 years of birth (0/1)	Duration to first TM (years)
	LPM	OLS
Birth Size Percentile (50-75)	0.006^{***}	-0.164^{***}
	(0.0007)	(0.0194)
Birth Size Percentile (75-95)	0.012^{***}	-0.442^{***}
	(0.0015)	(0.0230)
Birth Size Percentile (95+)	0.022	-0.963^{***}
	(0.0021)	(0.0353)
First Year Growth Percentile (50-75)	0.006^{***}	0.214^{***}
	(0.0007)	(0.0224)
First Year Growth Percentile (75-95)	0.013^{***}	0.0005
	(0.0013)	(0.0185)
First Year Growth Percentile (95+)	0.023^{***}	-0.343^{***}
	(0.0023)	(0.0325)
Born Multi-State	0.007^{**}	-0.598^{***}
	(0.0029)	(0.0358)
Born with Patent	0.168^{***}	-1.429^{***}
	(0.0063)	(0.0688)
Born Exporter	0.030^{***}	-0.238^{***}
	(0.0017)	(0.0298)
N	$8,600,000$	$231,000$
R^2	0.052	0.205

Notes: Standard errors clustered by industry in parentheses. *, **, and *** indicate significance at 10, 5, and 1%, respectively. Models include birth cohort and 6-digit NAICS industry fixed effects. N is rounded to prevent disclosure of confidential information.

Table 3. Size-age profiles for TM vs. non-TM firms

Age (years)	Avg. Size (employment)		Relative
	TM	Non-TM	Avg. Size
0	15.32	5.63	2.72
	(0.05)	(0.01)	
5	56.44	9.43	5.98
	(0.15)	(0.03)	
10	99.56	11.40	8.73
	(0.30)	(0.04)	
15	150.50	12.84	11.72
	(0.53)	(0.06)	
20	203.50	14.41	14.12
	(0.86)	(0.12)	
25	256.30	15.91	16.11
	(1.35)	(0.18)	
30	319.70	16.30	19.61
	(2.38)	(0.18)	
35	497.00	18.57	26.76
	(6.42)	(0.34)	

Notes: Standard errors in parentheses. Relative avg. size is the ratio of the average size of TM firms to that of non-TM firms.

Table 4. The effects of China trade shock on firms with and without registered TMs - second stage results from 2SLS IV estimation

	Employment (IHS)			Revenue (IHS)			Labor Productivity (IHS)		
	I	II	III	I	II	III	I	II	III
β^{TP}	0.0702***	0.0497***	0.0759***	0.1190***	0.0938***	0.2020***	0.0608***	0.0550***	0.117***
	(0.0041)	(0.0043)	(0.0061)	(0.0055)	(0.0058)	(0.0112)	(0.0050)	(0.0053)	(0.0082)
β^{TCP}	0.0008**	0.0009**	0.0010*	0.0015***	0.0014***	0.0019*	0.0009**	0.0008*	0.0016**
	(0.0003)	(0.0004)	(0.0006)	(0.0005)	(0.0005)	(0.0010)	(0.0005)	(0.0005)	(0.0008)
β^{C}	0.0013***	0.0013**	0.0029***	0.0007	0.0006	0.0030**	0.0003	0.0002	0.0016
	(0.0005)	(0.0005)	(0.0008)	(0.0007)	(0.0007)	(0.0014)	(0.0006)	(0.0006)	(0.0010)
β^{CP}	−0.0039***	−0.0038***	−0.0062***	−0.0037***	−0.0036***	−0.0067***	−0.0009**	−0.0008*	−0.0022***
	(0.0004)	(0.0004)	(0.0005)	(0.0005)	(0.0005)	(0.0009)	(0.0004)	(0.0004)	(0.0007)
N	1,010,000	1,010,000	1,070,000	745,000	745,000	776,000	745,000	745,000	776,000
R^2	0.0135	0.0131	0.0032	0.0086	0.0079	0.0015	0.0006	0.0004	0.0015

Notes: Standard errors clustered by firm in parentheses. *, **, and *** indicate significance at 10, 5, and 1%, respectively. All models include year and firm fixed effects. N is rounded to prevent disclosure of confidential information. Dependent variables are inverse-hyperbolic-sine (IHS) transformed. The estimated β's correspond to the RHS variables in equation (2): $TP = TM \times Post$, $TCP = TM \times Post \times IC$, $C = IC$, $CP = Post \times IC$. The (I) columns measure existing registered trademark ownership as of 2001 and (II) uses 1999 as the cutoff year. The (III) column includes firm deaths.

Table 5. Mean advertising expenditures and advertising intensity

All firms

	Expenditures ($K)		Intensity (%)	
	no TM	TM	no TM	TM
	12.4	147.2	0.48	0.65
	(0.3)	(9.6)	(0.001)	(0.005)
N	317,000	43,000	317,000	43,000

Firms with positive advertising

	Expenditures ($K)		Intensity (%)	
	no TM	TM	no TM	TM
	14.5	156.7	0.56	0.70
	(0.3)	(10.2)	(0.001)	(0.005)
N	271,000	40,500	271,000	40,500

Notes: Standard errors in parentheses. Advertising intensity is the ratio of advertising expenditures to revenue, (in %). N is rounded to prevent disclosure of confidential information.

Table 6. Advertising expenditures and advertising intensity over the firm life-cycle

	Pooled Cross Section			Fixed Effects		
	I	II	III	IV	V	VI
	OLS	2SLS	OLS	OLS	2SLS	OLS
ln(*Revenue*)	-0.027^{***}	-0.021^{***}		-0.063^{***}	-0.023^{**}	
	(0.002)	(0.002)		(0.007)	(0.012)	
ln(*Employment*)			-0.002^{***}			-0.010^{**}
			(0.0006)			(0.051)
TM registration	0.091^{***}	0.085^{***}	0.079^{***}			
	(0.019)	(0.019)	(0.019)			
post-TM registration	0.047^{***}	0.046^{***}	0.047^{***}	0.066^{***}	0.060^{***}	0.058^{***}
	(0.020)	(0.020)	(0.020)	(0.020)	(0.020)	(0.020)
N	395,000	395,000	395,000	395,000	395,000	395,000
R^2	0.350	0.350	0.349	0.010	0.027	0.009

Notes: Standard errors clustered by firm in parentheses. *, **, and *** indicate significance at 10, 5, and 1%, respectively. Pooled specification includes 6-digit NAICS industry, firm age, and year dummies. Fixed effects specifications include firm age dummies, and firm and year fixed effects. N is rounded to prevent disclosure of confidential information.

Table 7. Mean advertising intensity for the sample used in the analysis in Table 6.

	Mean advertising intensity (%) (excluding cases with zero advertising)	
Firms with no trademark	0.56	
	(0.001)	
	Pre-registration	Post-registration
Firms with trademark	0.70	0.79
	(0.002)	(0.006)

Notes: Standard errors corrected for firm level clustering in parentheses. Advertising intensity is the ratio of advertising costs to revenue expressed as a percentage. Registration refers to the event of first trademark registration.

Table 8. Calibration targets: model and data

Target	Model	Data	Source
Avg. annual firm exit rate	0.1	0.1	BDS
Fraction of firms with trademarks	0.039	0.039	LBD/TCFD
Ratio of avg. employment (TM firms vs. non-TM firms)	23.9	23.75	LBD/TCFD
Markup	33.3%	40%	Bundesbank - European study
Depreciation of advertising capital	0.58	0.5-0.6	Hall (2014)
Productivity dispersion across firms	0.418	0.407	Syverson (2003) - Table 1
Ratio of avg. employment (TM firms at age 35 vs. age 1)	31.8	32.4	LBD/TCFD
Ratio of avg. employment (TM vs. non-TM firms at age 1)	2.78	2.72	LBD/TCFD
Advertising cost to consumption ratio	2.16%	2%	Galbi (2001a,b)
Ratio of the slopes of Engel curves for quality vs. quantity	0.75	0.75	Bils and Klenow (2001)
Ratio of avg. employment (non-TM firms at age 35 vs. age 1)	3.4	3.3	LBD/TCFD
Ratio of avg. employment (top 1% of TM firms vs. top 1% of non-TM firms)	44.8	45.3	LBD/TCFD
Share of R&D spending to output	4.8%	3.5%	U.S. data for 2020 from OECD

Table 9. Calibrated parameter values

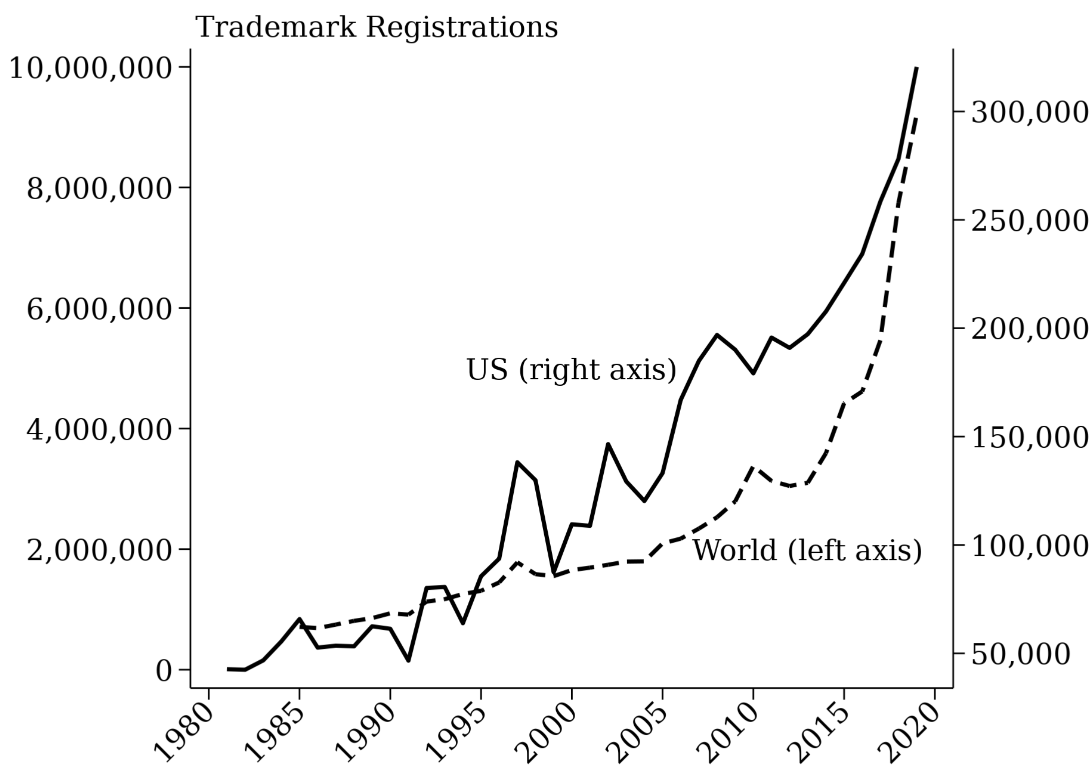
Parameter	Description	Value
Preference		
θ	Utility – exponent of quantity	0.75
ξ	Utility – exponent of quality	0.31
Technology		
A	Production function – Hicks-neutral productivity (normalized)	1
A^N	Marketing productivity – non-TM firms	2.06
B	Quality development cost – scalar (normalized)	1
δ	Customer diversion rate	0.14
λ	Customer turnover rate	0.03
η	Quality development cost – exponent of quality	4.1
ρ	Advertising technology – exponent of marketing labor	0.21
χ	Exit rate	0.10
σ	Production function – exponent of quality	0.19
ϱ	Relative marketing productivity of TM firms – A^T/A^N	1.52
Cost		
c_e	Entry cost	5.3
c_T	Trademark cost	0.72
Distribution		
$\underline{e}$	Lower bound of quality development efficiency (normalized)	1
$\overline{e}$	Upper bound of quality development efficiency	2.2

Table 10. Key aggregates for the benchmark economy and the simulated economies

	Benchmark	No Trademark		High Diversion		High Diversion (w/o Trademark)		Full Trademark	
Ad per consumer	0.736	0.726	(−1.3%)	0.702	(−4.6%)	0.688	(−2.0%)	1.093	(48.5%)
No. of varieties	0.521	0.516	(−0.9%)	0.504	(−3.2%)	0.498	(−1.4%)	0.665	(27.6%)
Average quality	3.940	3.620	(−8.0%)	4.130	(4.8%)	3.465	(−16.1%)	3.416	(−13.3%)
Welfare (C.V.)	0.751	0.736	(−3.4%)	0.753	(0.3%)	0.719	(−7.6%)	0.774	(5.0%)
HHI	1280	995	(−22.3%)	1458	(13.9%)	932	(−36.1%)	813	(−36.5%)

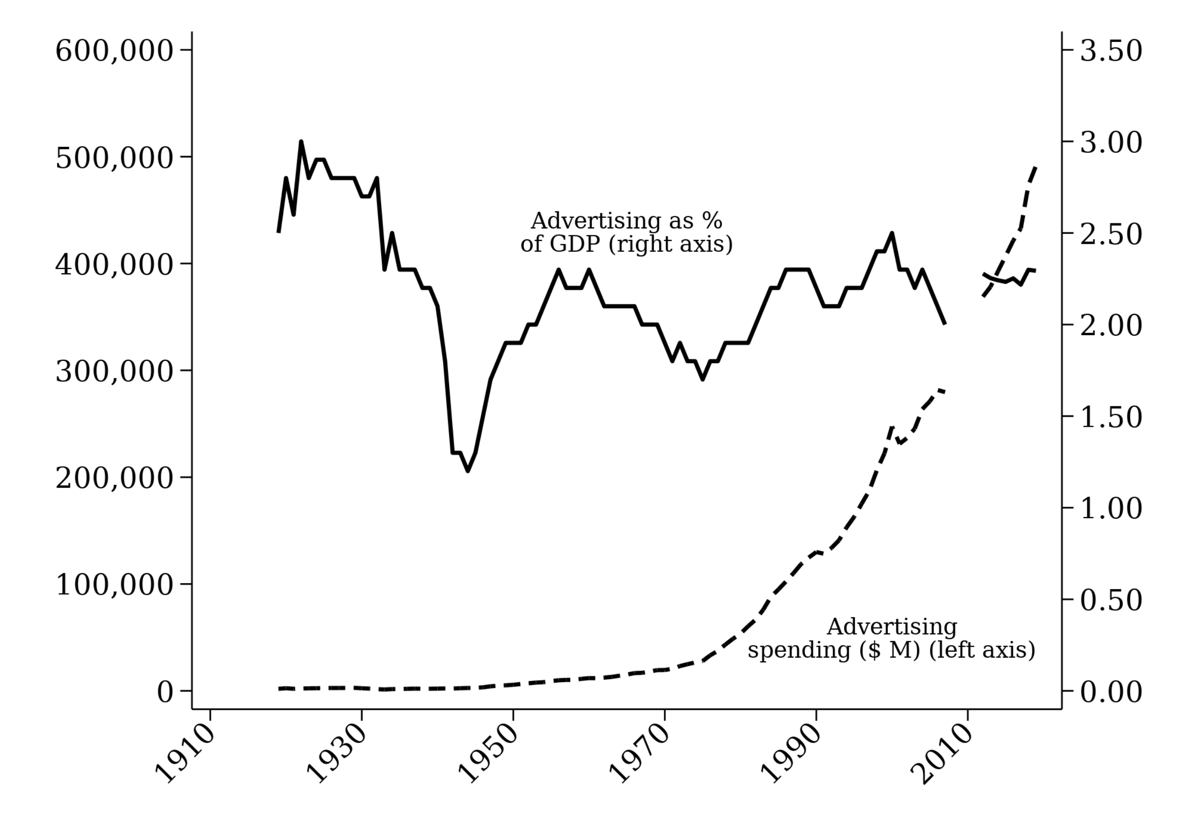
Notes: The percentages in parentheses indicate the percent change from the benchmark economy for the corresponding aggregate, except for the High Diversion (w/o Trademark) experiment where the changes are with respect to the High-Diversion economy.

Figure 1: Trademark registrations, direct and via the Madrid system, 1980-2019



Source: World Intellectual Property Organization (WIPO) Statistics Database. The Madrid System is a platform for registering and managing trademarks in several countries at once (https://www.wipo.int/madrid/en/).

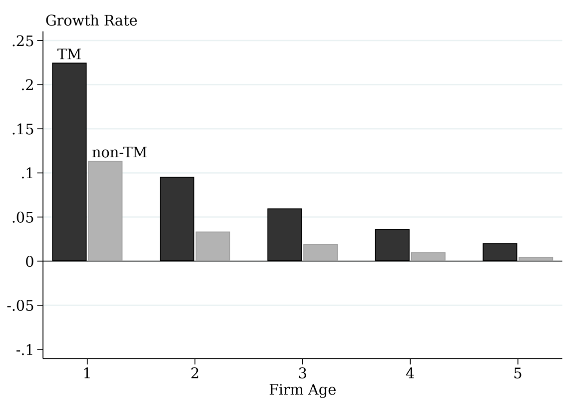
Figure 2: Advertising spending in the United States, 1919-2019



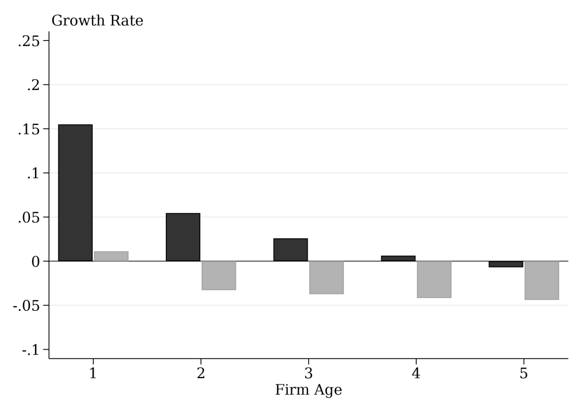
Source: The data are based on a revision of Galbi (2001a, 2001b). Advertising figures are from Coen Structured Advertising Expenditures Dataset. The GDP figures are from Johnston and Williamson (2005), Kendrick (1969), and the U.S. Bureau of Economic Analysis. The remaining GDP figures are from standard national accounts. Notes: Total advertising spending includes spending for advertising in newspapers, magazines, radio, broadcast television, cable television, direct mail, billboards and displays, Internet, and other forms. The ad spending figures are in millions of current U.S. dollars.

Figure 3: Annual DHS growth rates for firms with and without trademarks: first five years (ages 1 through 5)

(a) Conditional on survival



(b) Unconditional

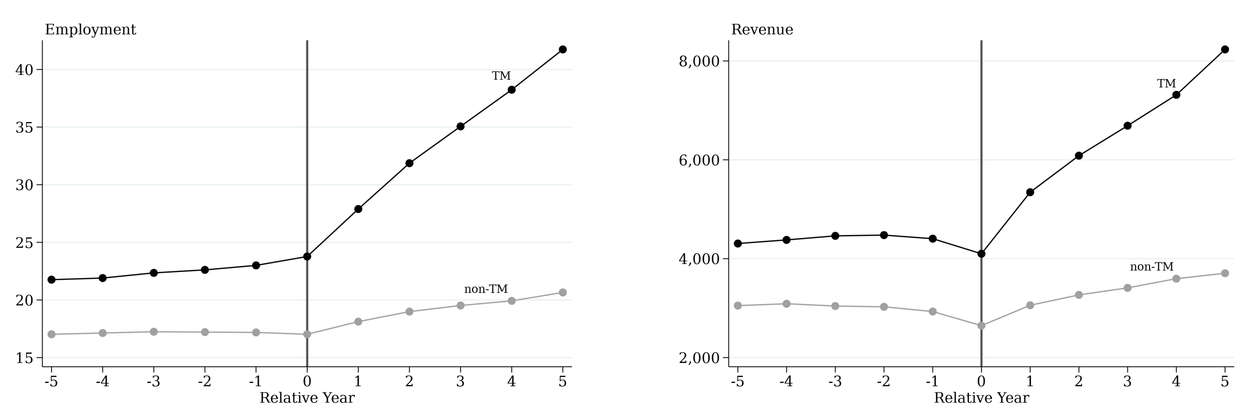


Source: LBD, USPTO Trademark Case File Database Notes: Davis-Haltiwanger-Schuh (DHS) growth rates are used. Panel (a) shows growth rates conditional on survival; Panel (b) shows growth rates inclusive of exits.

Figure 4: Employment and revenue profiles: treated versus control firms

(a) Employment

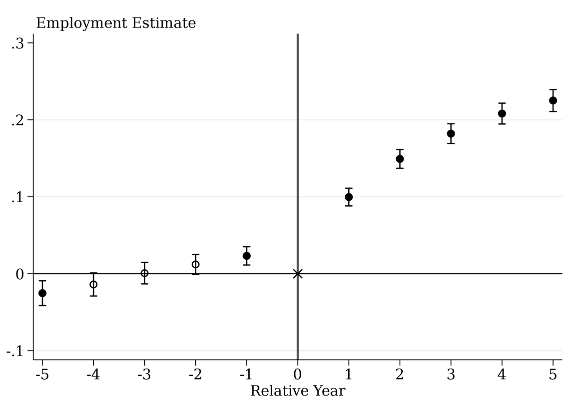
(b) Revenue



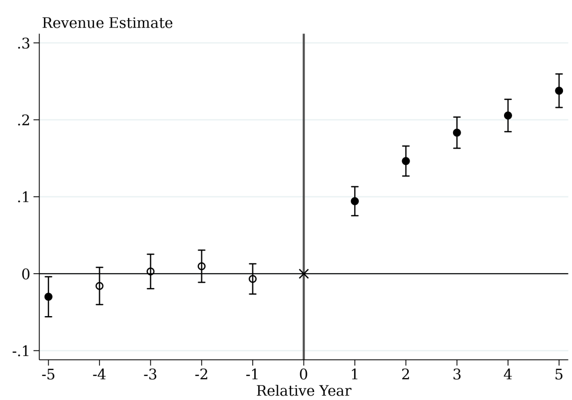
Source: LBD, USPTO Trademark Database Notes: Panel (a) and (b) show average employment and revenue, respectively. 95% confidence bands are shown in dashes. The event of trademark registration occurs at year t = 0 in x-axis.

Figure 5: Estimates of the coefficients (γ_j's) from the event study regressions for employment and revenue

(a) $ln(Employment)$



(b) $ln(Revenue)$



Source: LBD, USPTO Trademark Database Notes: Panel (a) and (b) show estimates for employment and revenue, respectively. 95% confidence bands are shown in dashes. The event of trademark registration occurs at year t = 0 in x-axis.

Figure 6: Timing of decisions by firms within a period

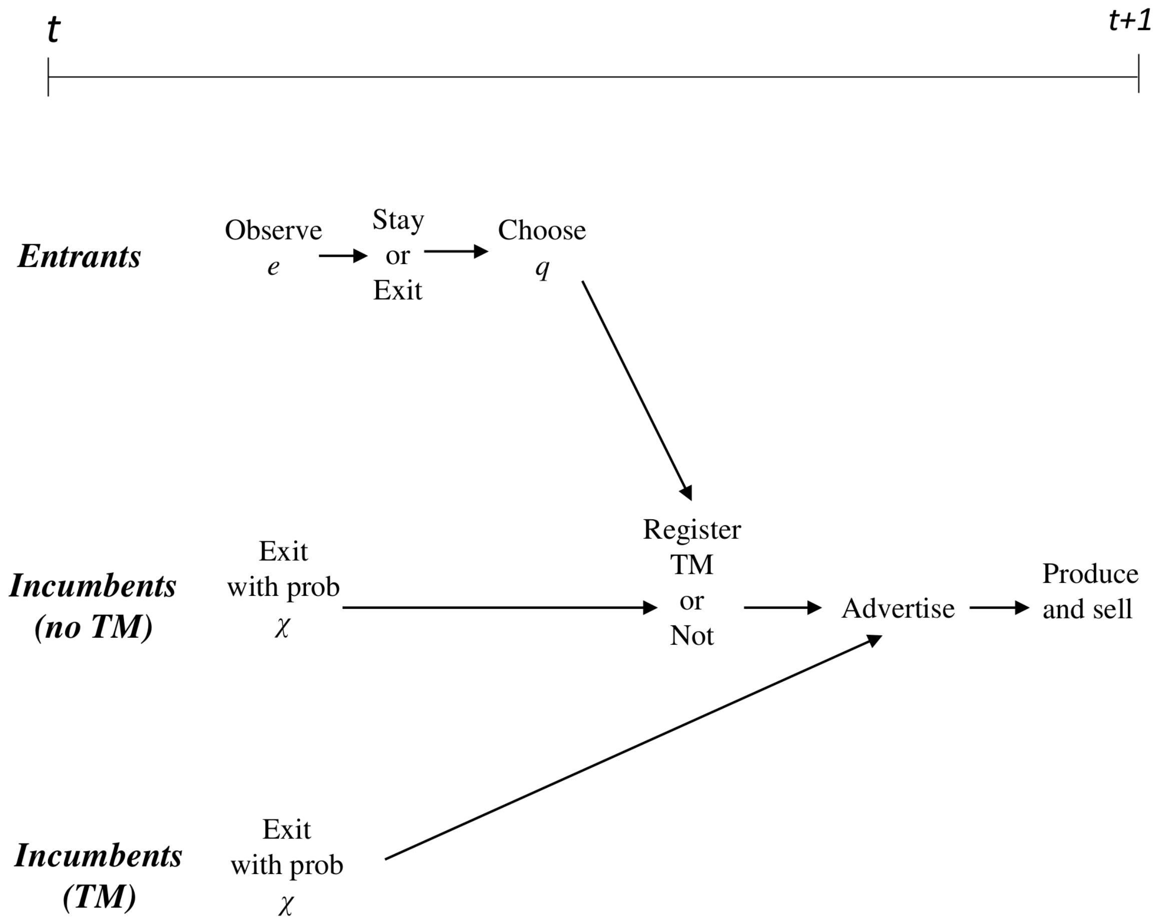


Figure 7: Trademark Policy

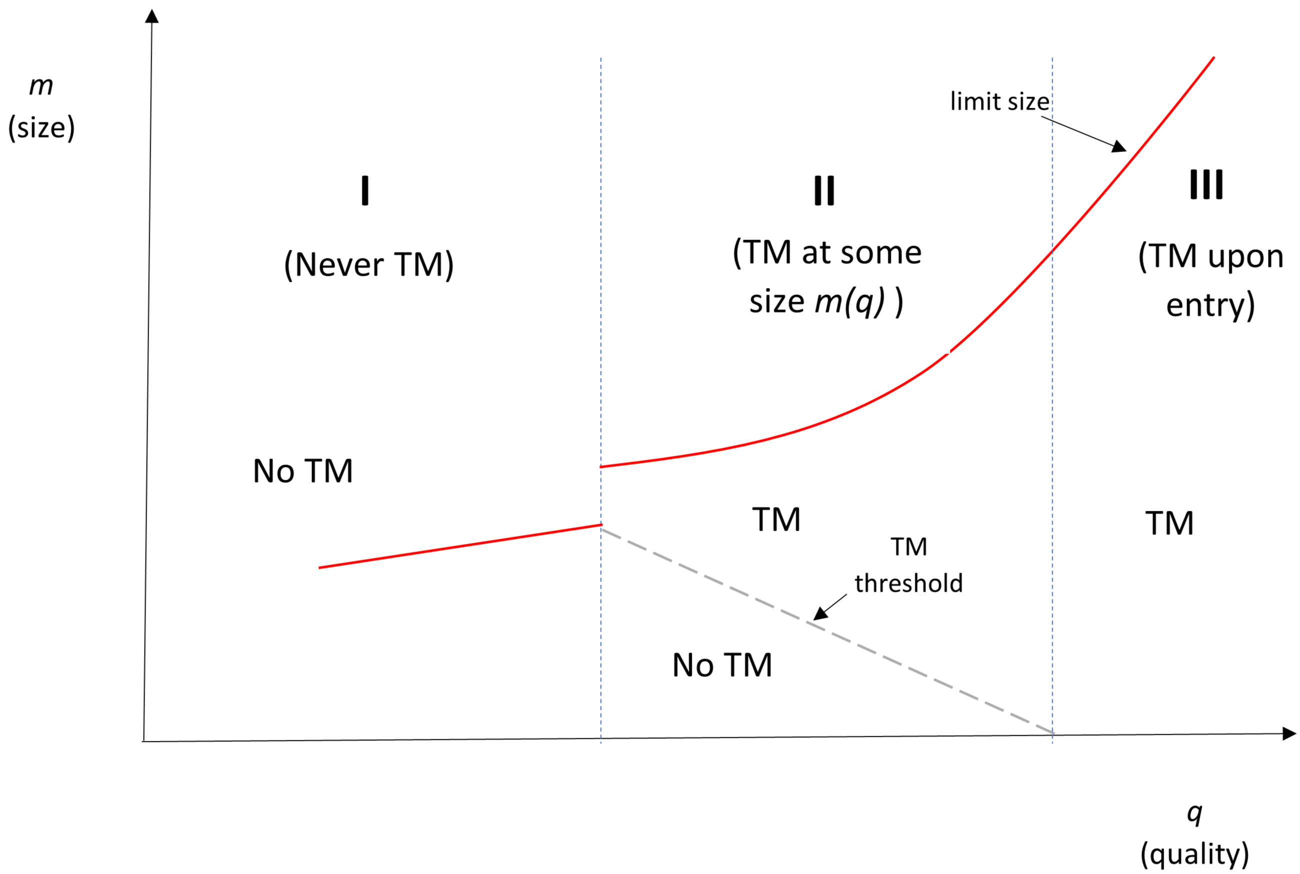
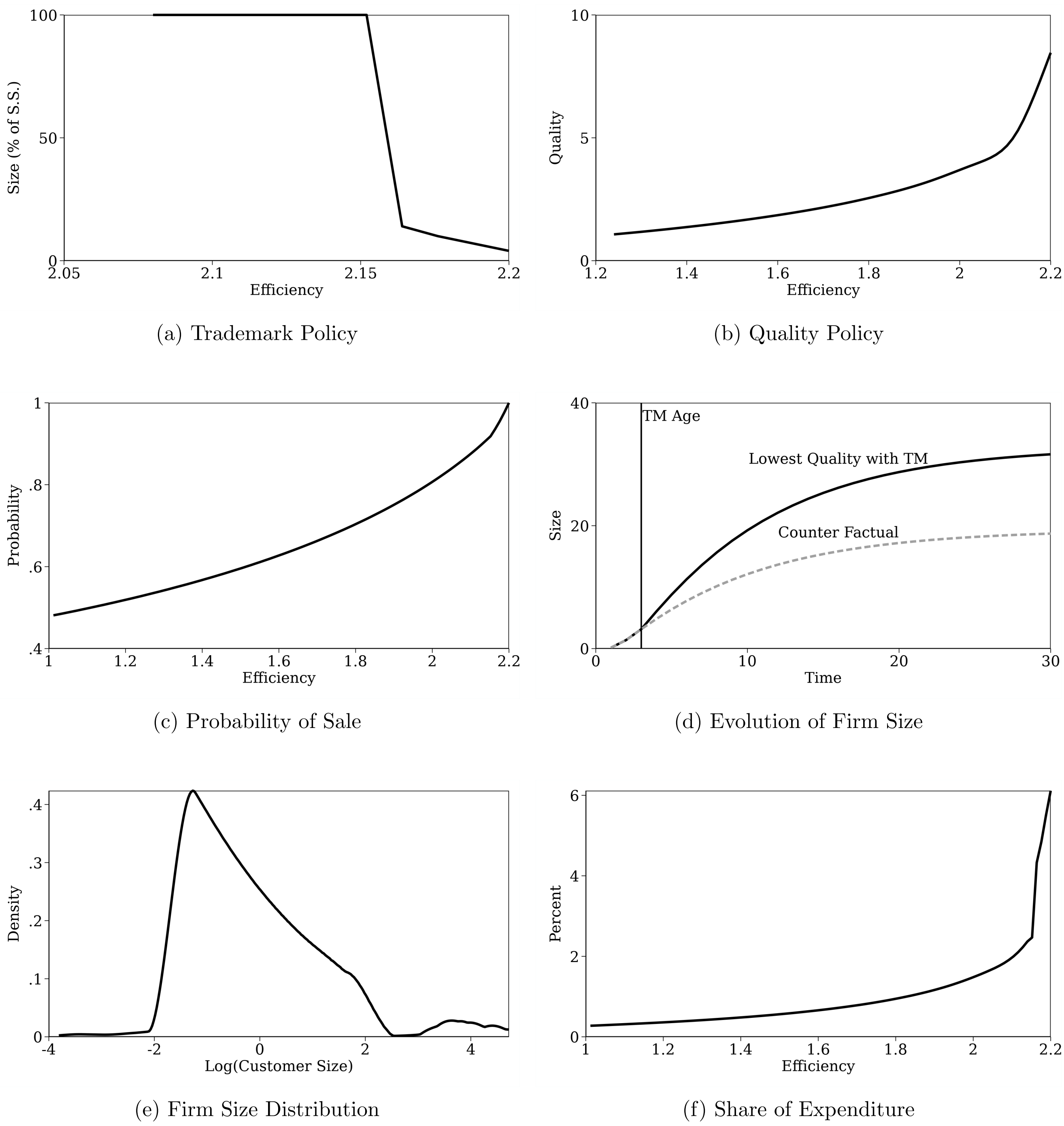


Figure 8: Benchmark economy



(a) Trademark Policy

(b) Quality Policy

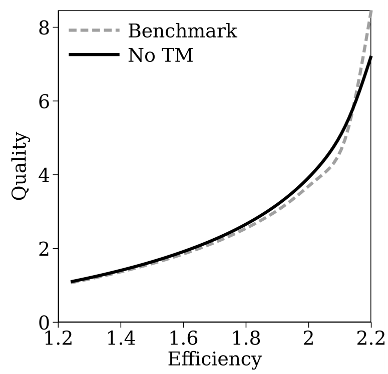
(c) Probability of Sale

(d) Evolution of Firm Size

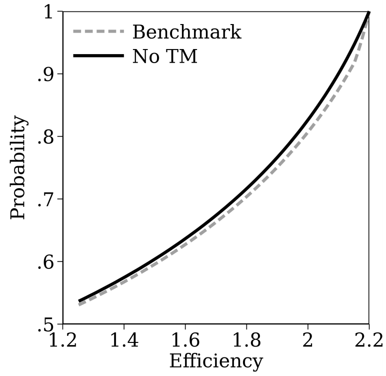
(e) Firm Size Distribution

(f) Share of Expenditure

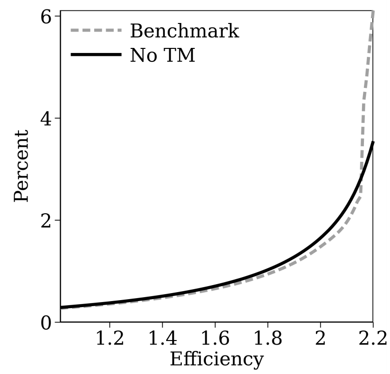
Figure 9: No-trademark economy



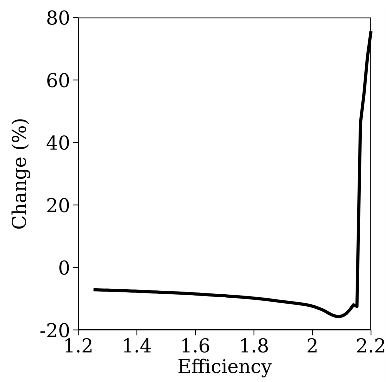
(a) Quality Policy



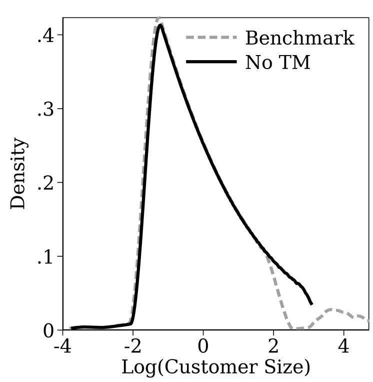
(b) Probability of Sale



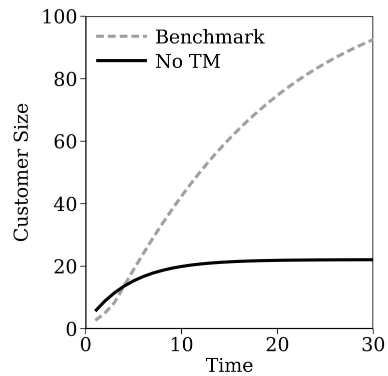
(c) Share of Expenditure



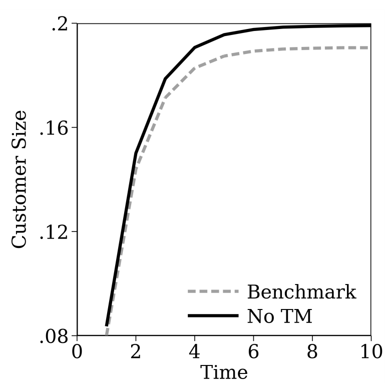
(d) Reallocation of Employment



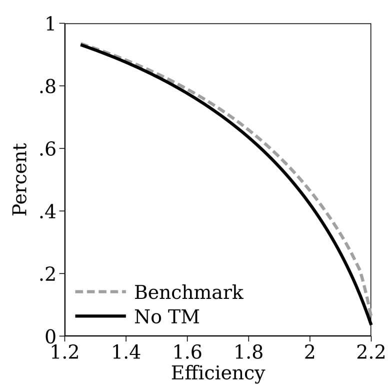
(e) Firm Size Distribution



(f) Evolution of Firm Size (Highest Quality)

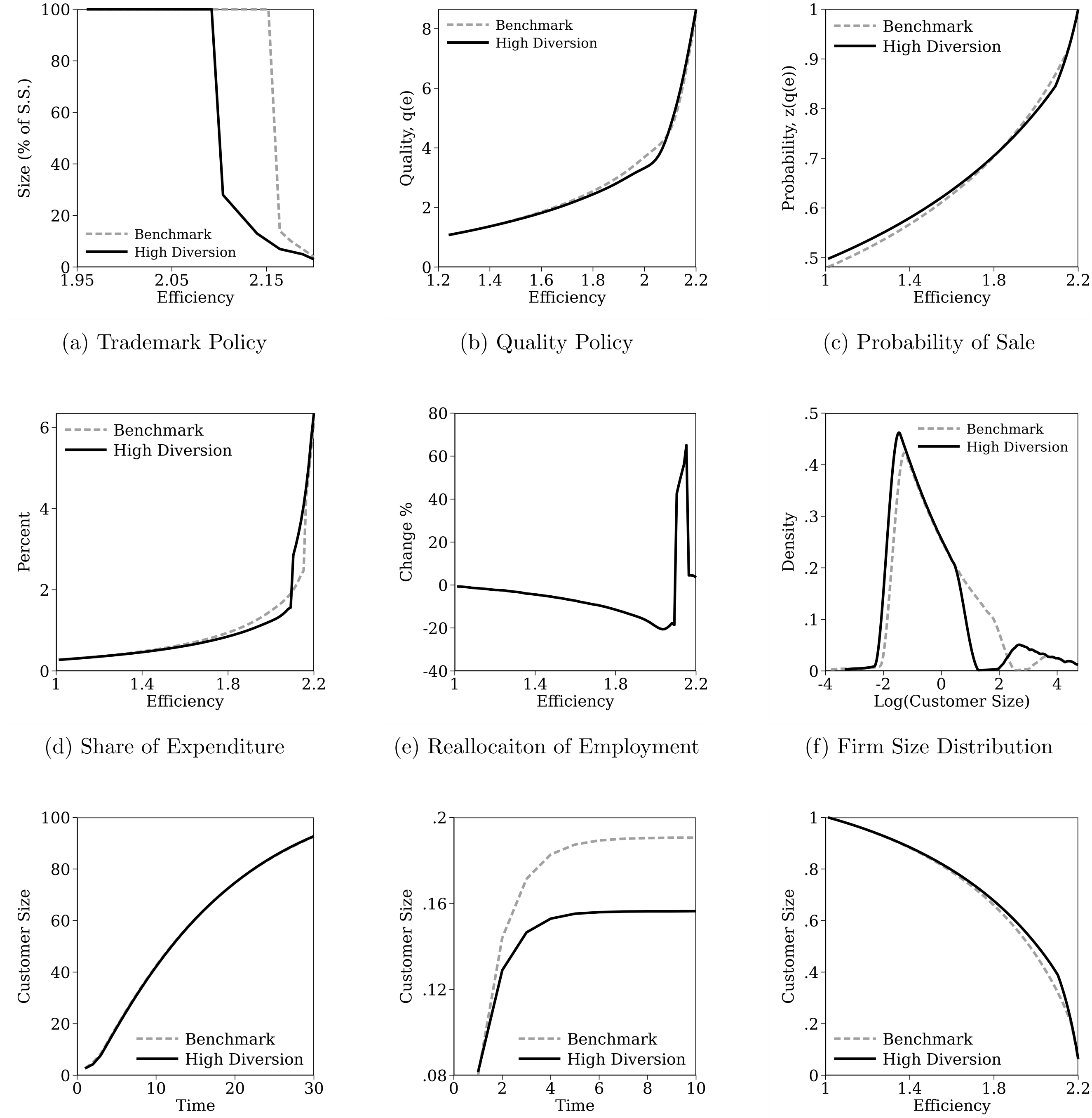


(g) Evolution of Firm Size (Lowest Quality)



(h) Concentration of Customer Size

Figure 10: High-diversion economy



(a) Trademark Policy

(b) Quality Policy

(c) Probability of Sale

(d) Share of Expenditure

(e) Reallocaiton of Employment

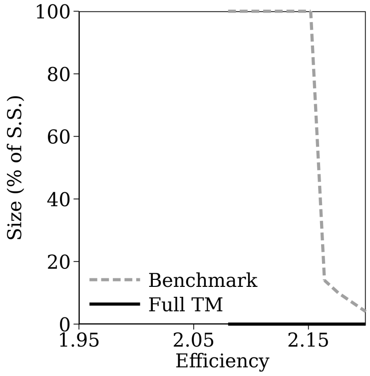
(f) Firm Size Distribution

(g) Evolution of Firm Size (Highest Quality)

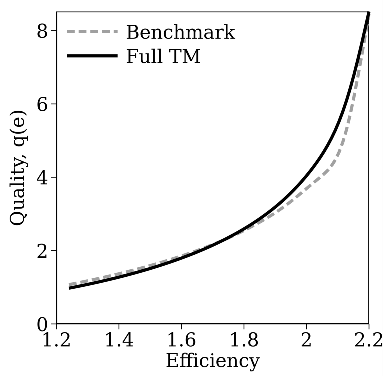
(h) Evolution of Firm Size (Lowest Quality)

(i) Concentration of Customer Size

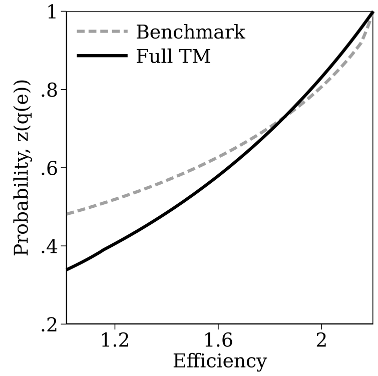
Figure 11: Full-trademark economy



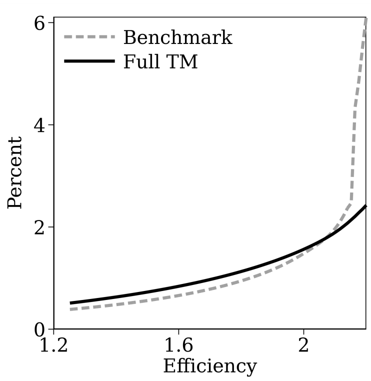
(a) Trademark Policy



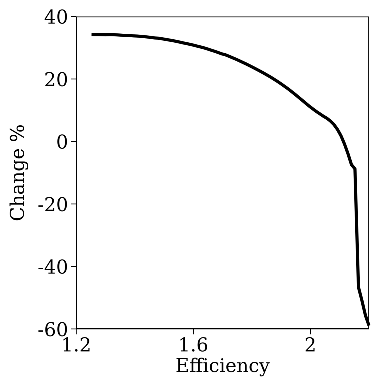
(b) Quality Policy



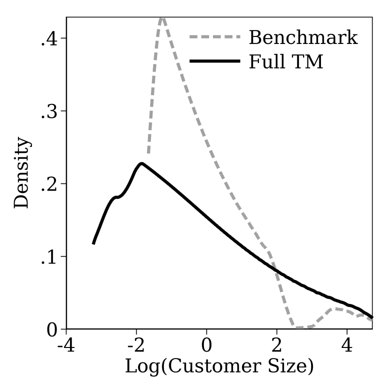
(c) Probability of Sale



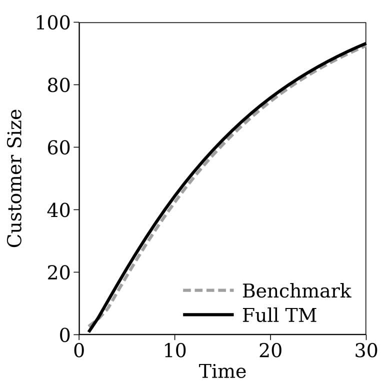
(d) Share of Expenditure



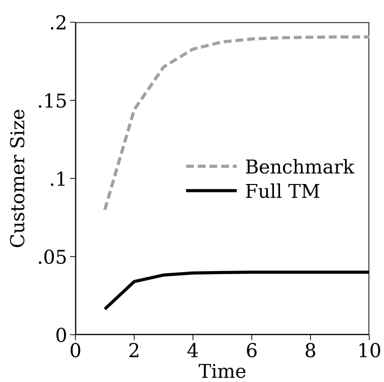
(e) Reallocaiton of Employment



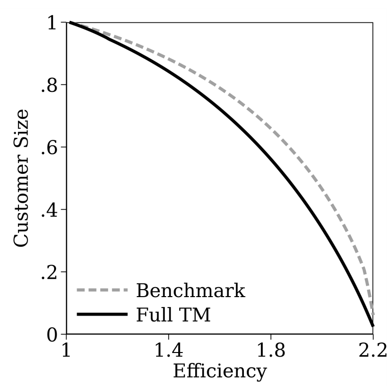
(f) Firm Size Distribution



(g) Evolution of Firm Size (Highest Quality)



(h) Evolution of Firm Size (Lowest Quality)



(i) Concentration of Customer Size

A Additional empirical results

Table A1. Estimates from the event study regression (2)

	Employment (ln)		Revenue (ln)	
	No Firm FE	Firm FE	No Firm FE	Firm FE
β^{TM}	0.061***		0.251***	
	(0.004)		(0.007)	
β_{-5}	0.099***	−0.335***	0.339***	−0.127***
	(0.005)	(0.003)	(0.009)	(0.004)
β_{-4}	0.093***	−0.296***	0.331***	−0.101***
	(0.005)	(0.002)	(0.008)	(0.004)
β_{-3}	0.085***	−0.250***	0.312***	−0.072***
	(0.005)	(0.002)	(0.008)	(0.003)
β_{-2}	0.078***	−0.188***	0.280***	−0.032***
	(0.004)	(0.002)	(0.007)	(0.003)
β_{-1}	0.058***	−0.104***	0.205***	0.008***
	(0.004)	(0.001)	(0.007)	(0.002)
β_{+1}	0.064***	−0.013***	0.154***	−0.010***
	(0.004)	(0.001)	(0.007)	(0.002)
β_{+2}	0.100***	−0.020***	0.224***	−0.026***
	(0.004)	(0.002)	(0.007)	(0.002)
β_{+3}	0.123***	−0.026***	0.266***	−0.045***
	(0.004)	(0.002)	(0.007)	(0.003)
β_{+4}	0.144***	−0.028***	0.312***	−0.056***
	(0.005)	(0.002)	(0.007)	(0.003)
β_{+5}	0.162***	−0.033***	0.342***	−0.079***
	(0.005)	(0.002)	(0.008)	(0.004)
γ_{-5}	−0.025***	−0.033***	−0.030**	−0.140***
	(0.008)	(0.004)	(0.013)	(0.007)
γ_{-4}	−0.014	−0.018***	−0.016	−0.113***
	(0.008)	(0.004)	(0.012)	(0.006)
γ_{-3}	0.001	−0.002	0.003	−0.092***
	(0.007)	(0.003)	(0.011)	(0.005)
γ_{-2}	0.012	0.012***	0.010	−0.063***
	(0.007)	(0.003)	(0.01)	(0.004)
γ_{-1}	0.023**	0.032***	−0.007	−0.042***
	(0.006)	(0.002)	(0.01)	(0.003)
γ_{+1}	0.100***	0.096***	0.094***	0.061***
	(0.006)	(0.002)	(0.010)	(0.003)
γ_{+2}	0.149***	0.140***	0.146***	0.098***
	(0.006)	(0.002)	(0.010)	(0.004)
γ_{+3}	0.182***	0.171***	0.183***	0.124***
	(0.007)	(0.003)	(0.010)	(0.004)
γ_{+4}	0.208***	0.193***	0.206***	0.140***
	(0.007)	(0.003)	(0.011)	(0.005)
γ_{+5}	0.225***	0.213***	0.238***	0.164***
	(0.007)	(0.003)	(0.011)	(0.006)
α	1.805***		6.492***	
	(0.003)		(0.005)	
N	2,972,000	2,972,000	1,737,000	1,737,000
R^2	0.008	0.878	0.018	0.897

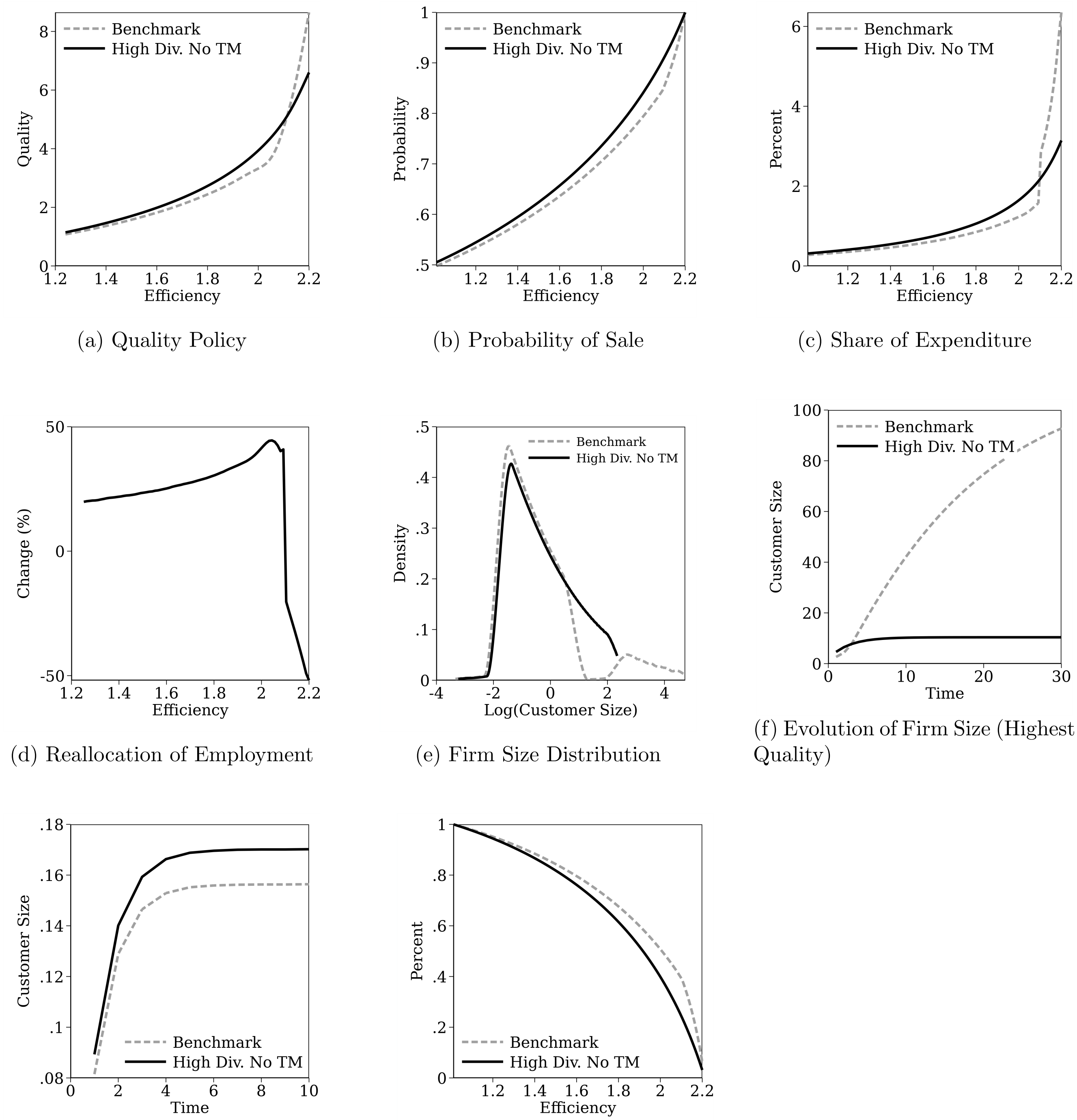
Notes: Standard errors clustered by firm in parentheses. *, **, and *** indicate significance at 10, 5, and 1%, respectively. N is rounded to prevent disclosure of confidential information.

Table A2. The effects of China trade shock on firms with and without registered TMs - first stage statistics from 2SLS IV estimation

	Employment			Revenue			Labor Productivity		
Coefficient estimate:	I	II	III	I	II	III	I	II	III
IV	0.698***	0.698***	0.699***	0.685***	0.685***	0.684***	0.685***	0.685***	0.684***
	(0.0007)	(0.0007)	(0.0007)	(0.0008)	(0.0008)	(0.0008)	(0.0008)	(0.0008)	(0.0008)
R^2	0.493	0.493	0.493	0.499	0.499	0.498	0.499	0.499	0.498
$IV_{it} \times Post_t$	0.712***	0.712***	0.713***	0.697***	0.697***	0.696***	0.697***	0.697***	0.696***
	(0.0006)	(0.0006)	(0.0006)	(0.0007)	(0.0007)	(0.0007)	(0.0007)	(0.0007)	(0.0007)
R^2	0.564	0.564	0.561	0.562	0.562	0.559	0.562	0.562	0.559
$IV_{it} \times Post_t \times TM_i$	0.698***	0.679***	0.0029***	0.688***	0.671***	0.688***	0.688***	0.671***	0.688***
	(0.0005)	(0.0005)	(0.0008)	(0.0006)	(0.0006)	(0.0006)	(0.0006)	(0.0006)	(0.0006)
R^2	0.618	0.624	0.615						
Cragg-Donald Stat.	1.4×10^5	1.4×10^5	1.5×10^5	1.1×10^5	1.1×10^5	1.1×10^5	1.1×10^5	1.1×10^5	1.1×10^5
N	$1,010,000$	$1,010,000$	$1,070,000$	$745,000$	$745,000$	$776,000$	$745,000$	$745,000$	$776,000$

Notes: Standard errors clustered by firm in parentheses. *, **, and *** indicate significance at 10, 5, and 1%, respectively. N is rounded to prevent disclosure of confidential information.

Figure A1: High-diversion economy with no trademarks



(a) Quality Policy

(b) Probability of Sale

(c) Share of Expenditure

(d) Reallocation of Employment

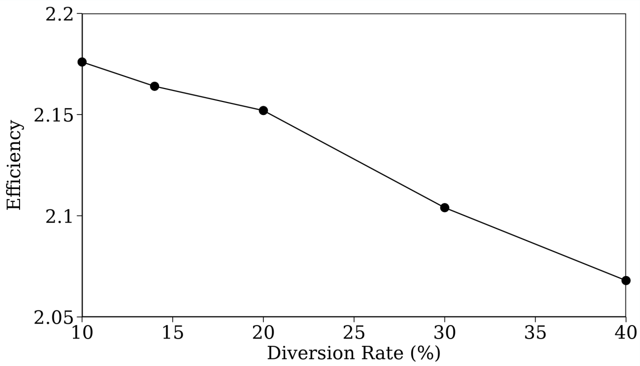
(e) Firm Size Distribution

(f) Evolution of Firm Size (Highest Quality)

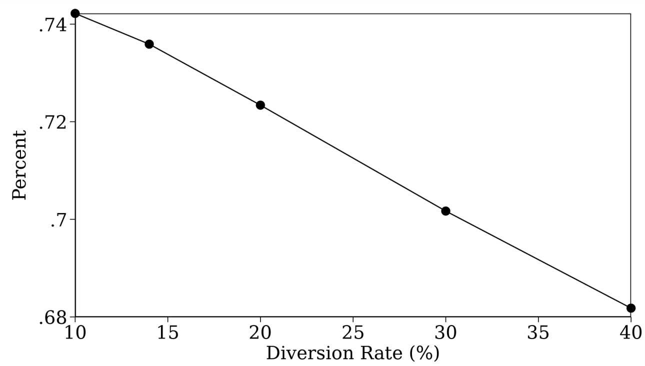
(g) Evolution of Firm Size (Lowest Quality)

(h) Concentration of Customer Size

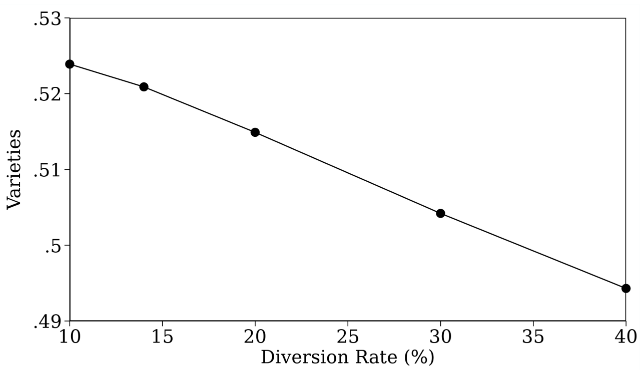
Figure A2: Key aggregates as a function of the customer diversion rate



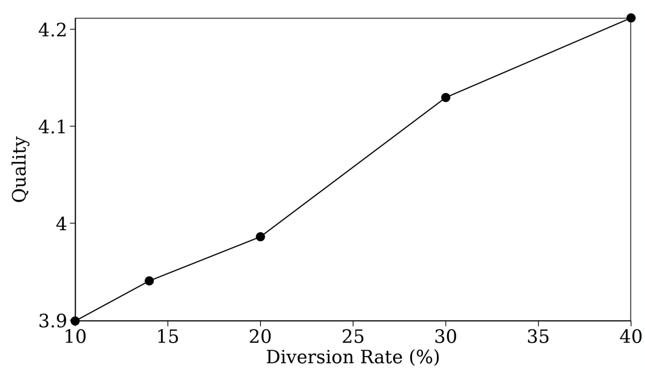
(a) Trademark Threshold



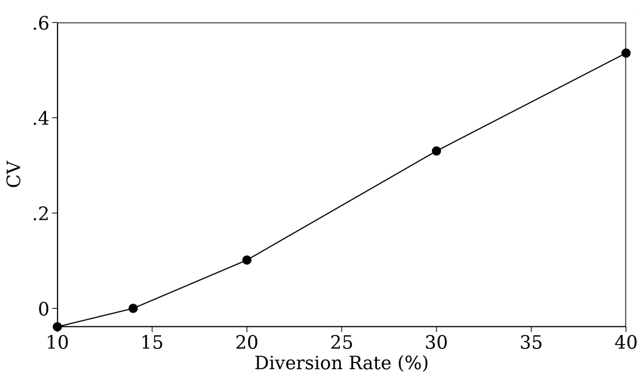
(b) Product Information



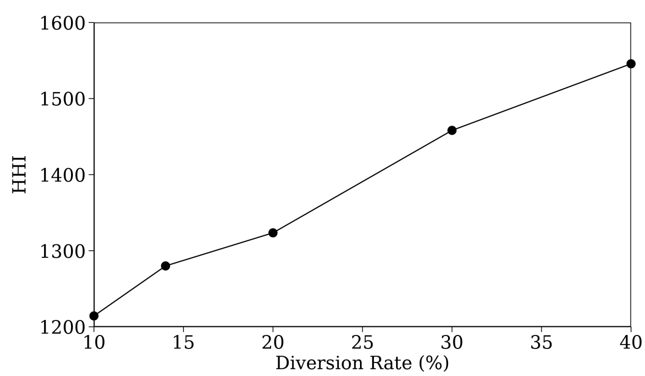
(c) Number of Product Varieties



(d) Average Product Quality



(e) Welfare (Compensating Variation)



(f) Concentration of Customer Size

B Proofs

Proof of Proposition 1. Consider the pricing decision of the firm with quality q

$$\max_p \ x(p,q)\left(p-\frac{q^\sigma}{A}\right).$$

Substituting for $x(p,q)$ using (20) yields

$$\max_p \ \left[\frac{q^\xi}{p}\right]^{\frac{1}{1-\theta}}\left[\frac{p(\underline{q})}{\underline{q}^\xi}\right]^{\frac{1}{1-\theta}} x(p(\underline{q}),\underline{q})\left[p-\frac{q^\sigma}{A}\right] \equiv \max_p \ \left[\frac{1}{p}\right]^{\frac{1}{1-\theta}}\left[p-\frac{q^\sigma}{A}\right].$$

The first order condition for p is

$$\frac{\theta}{\theta-1}p^{\frac{\theta}{\theta-1}-1}-\frac{1}{1-\theta}\frac{q^\sigma}{A}p^{\frac{1}{1-\theta}-1}=0,$$

which has the solution

$$p(q)=\frac{1}{\theta}\frac{q^\sigma}{A}.$$

■

Proof of Proposition 2. *i)* Consider two products with qualities $q_i > q_j$. Note that $\frac{x(p(q_i),q_i)}{x(p(q_j),q_j)} = \left(\frac{p(q_j)}{p(q_i)}\left[\frac{q_i}{q_j}\right]^\xi\right)^{\frac{1}{1-\theta}}$, $p(q_i) = \frac{1}{\theta}\frac{q_i^\sigma}{A}$, and $p(q_j) = \frac{1}{\theta}\frac{q_j^\sigma}{A}$. Therefore, $\widetilde{x}(q_i) = \left[\frac{q_i}{q_j}\right]^{\frac{\xi-\sigma}{1-\theta}}\widetilde{x}(q_j)$, where $\widetilde{x}(q) = x(p(q),q)$. The consumer's period surplus from these two products can then be written as

$$\widetilde{u}(q_i)=q_i^\xi\left[\frac{q_i}{q_j}\right]^{\theta\frac{\xi-\sigma}{1-\theta}}\left[\widetilde{x}(q_j)\right]^\theta-\upsilon\frac{1}{\theta}\frac{q_i^\sigma}{A}\left[\frac{q_i}{q_j}\right]^{\frac{\xi-\sigma}{1-\theta}}\widetilde{x}(q_j),$$

and

$$\widetilde{u}(q_j)=q_j^\xi\left[\widetilde{x}(q_j)\right]^\theta-\upsilon\frac{1}{\theta}\frac{q_j^\sigma}{A}\widetilde{x}(q_j).$$

where υ represents the Lagrange multiplier for the consumer's budget constraint.

Let $r_q=\frac{q_i}{q_j}>1$. The ratio of the two surplus terms is

$$\frac{\widetilde{u}(q_i)}{\widetilde{u}(q_j)}=\frac{q_i^\xi r_q^{\theta\frac{\xi-\sigma}{1-\theta}}\left[\widetilde{x}(q_j)\right]^\theta-\upsilon\frac{q_i^\sigma}{A}r_q^{\frac{\xi-\sigma}{1-\theta}}\widetilde{x}(q_j)}{q_j^\xi\left[\widetilde{x}(q_j)\right]^\theta-\upsilon\frac{1}{\theta}\frac{q_j^\sigma}{A}\widetilde{x}(q_j)}.$$

Substituting $q_i=r_q q_j$ yields

$$\frac{\widetilde{u}(q_i)}{\widetilde{u}(q_j)}=\frac{r_q^\xi q_j^\xi r_q^{\theta\frac{\xi-\sigma}{1-\theta}}\left[\widetilde{x}(q_j)\right]^\theta-\upsilon\frac{1}{\theta}\frac{r_q^\sigma q_j^\sigma}{A}r_q^{\frac{\xi-\sigma}{1-\theta}}\widetilde{x}(q_j)}{q_j^\xi\left[\widetilde{x}(q_j)\right]^\theta-\upsilon\frac{1}{\theta}\frac{q_j^\sigma}{A}\widetilde{x}(q_j)}.$$

Now divide through first by $\widetilde{x}(q_j)$ and then by q_j^ξ to obtain

$$\frac{\widetilde{u}(q_i)}{\widetilde{u}(q_j)}=\frac{r_q^{\xi+\theta\frac{\xi-\sigma}{1-\theta}}-\upsilon\frac{1}{\theta}r_q^{\sigma+\frac{\xi-\sigma}{1-\theta}}\frac{q_j^{\sigma-\varepsilon}}{A}\left[\widetilde{x}(q_j)\right]^{1-\theta}}{1-\upsilon\frac{1}{\theta}\frac{q_j^{\sigma-\xi}}{A}\left[\widetilde{x}(q_j)\right]^{1-\theta}}.$$

Since $\xi + \theta\frac{\xi-\sigma}{1-\theta} = \sigma + \frac{\xi-\sigma}{1-\theta}$, one can write

$$\frac{\widetilde{u}(q_i)}{\widetilde{u}(q_j)} = \frac{r_q^{\xi+\theta\frac{\xi-\sigma}{1-\theta}}\left(1 - \upsilon\frac{1}{\theta}\frac{q_j^{\sigma-\varepsilon}}{A}\left[\widetilde{x}(q_j)\right]^{1-\theta}\right)}{1 - \upsilon\frac{1}{\theta}\frac{q_j^{\sigma-\xi}}{A}\left[\widetilde{x}(q_j)\right]^{1-\theta}} = r_q^{\xi+\theta\frac{\xi-\sigma}{1-\theta}}.$$

Note that $r_q^{\xi+\theta\frac{\xi-\sigma}{1-\theta}} > 1$ under the assumption $\xi > \sigma$. Hence, $\widetilde{u}(q_i) > \widetilde{u}(q_j)$, implying that $\widetilde{u}(q)$ is increasing. Therefore, $\omega(q)$, $z(q)$, and $W(q)$ are also increasing in q.

ii) Consider again two quality levels q_i, q_j such that $r_q = \frac{q_i}{q_j} > 1$. The ratio of profit per customer as a function of quality is

$$\frac{\pi(q_i)}{\pi(q_j)} = \frac{\left(p(q_i) - \frac{1}{A}q_i^{\sigma}\right)\widetilde{x}(q_i)}{\left(p(q_j) - \frac{1}{A}q_j^{\sigma}\right)\widetilde{x}(q_j)} = \frac{\left(\frac{1}{\theta A}q_i^{\sigma} - \frac{1}{A}q_i^{\sigma}\right)\widetilde{x}(q_i)}{\left(\frac{1}{\theta A}q_j^{\sigma} - \frac{1}{A}q_j^{\sigma}\right)\widetilde{x}(q_j)},$$

which simplifies to

$$\frac{\pi(q_i)}{\pi(q_j)} = \frac{\left(\frac{1}{\theta A} - \frac{1}{A}\right)q_i^{\sigma}\widetilde{x}(q_i)}{\left(\frac{1}{\theta A} - \frac{1}{A}\right)q_j^{\sigma}\widetilde{x}(q_j)} = \frac{q_i^{\sigma}\widetilde{x}(q_i)}{q_j^{\sigma}\widetilde{x}(q_j)}.$$

Using $\widetilde{x}(q_i) = \left[\frac{q_i}{q_j}\right]^{\frac{\xi-\sigma}{1-\theta}}\widetilde{x}(q_j)$, one obtains

$$\frac{\pi(q_i)}{\pi(q_j)} = \left(\frac{q_i}{q_j}\right)^{\sigma}\left[\frac{q_i}{q_j}\right]^{\frac{\xi-\sigma}{1-\theta}} = r_q^{\sigma+\frac{\xi-\sigma}{1-\theta}} > 1.$$

Hence, $\pi(q)$ is increasing. Therefore, $\widetilde{\Pi}^{\Gamma}(q,m) = \max_{p,n,n_a}\Pi^{\Gamma}(p,n,n_a;q,m) = \max_{n,n_a} m_t^{\Gamma}(\Phi^{\Gamma}(n_a);q,m)\pi(q) - n - n_a$ is also increasing in q and m. Therefore, by the deterministic dynamic programming techniques in Stokey and Lucas (1989) $V^{\Gamma}(q,m)$ is increasing in q and m. ■

Proof of Proposition 3. Take two efficiency levels $e_1 > e_2$. Consider the optimal quality chocies $q(e_1)$ and $q(e_2)$. By the optimality of these choices, using (23) one can write

$$\begin{aligned} V^N(q(e_1),0) - C(q(e_1);e_1) - [V^N(q(e_2),0) - C(q(e_2);e_1)] &\geq 0, \\ V^N(q(e_2),0) - C(q(e_2);e_2) - [V^N(q(e_1),0) - C(q(e_1);e_2)] &\geq 0. \end{aligned}$$

Adding the two inequalities, one obtains

$$-C(q(e_1);e_1) - (-C(q(e_2);e_1)) - [-C(q(e_1);e_2) - (-C(q(e_2);e_2))] \geq 0$$

Note that $\left(-\frac{\partial C(q,e)}{\partial q \partial e}\right) > 0$ by the assumption that C is strictly convex and decreasing in e. Therefore, the function $(-C)$ satisfies increasing differences and the above holds with strict inequality. It follows that $q(e_1) > q(e_2)$. ■

Proof of Proposition 4. *i*) Consider the first order condition (24). The left hand side is strictly decreasing in n_a because $\rho < 1$. The left hand side is also increasing in q, because $\omega(q)$, $\pi(q)$, and $z(q)$ (hence, $l^{\Gamma(\tau)}$ and $L(q,\tau)$) are all strictly increasing in q. Therefore, a higher q implies a higher value for the left hand side. To maintain the first order condition, n_a must then increase. Therefore, $n_a^{\Gamma(\tau)}(q,\tau)$ is strictly increasing in q.

ii) The result for $\tau \geq 0$ follows from (28). For $-\infty < \tau < 0$, note that

$$\begin{aligned}
L(q,\tau) &= \sum_{t=0}^{-\tau}\left(\frac{(1-\chi)l^N(q)}{1+r}\right)^t + \left[\left(\frac{(1-\chi)l^N(q)}{1+r}\right)^{-\tau I(\tau<0)}\sum_{t=1}^{\infty}\left(\frac{(1-\chi)l^T(q)}{1+r}\right)^t\right], \\
&> \sum_{t=0}^{-\tau}\left(\frac{(1-\chi)l^N(q)}{1+r}\right)^t + \left[\left(\frac{(1-\chi)l^N(q)}{1+r}\right)^{-\tau I(\tau<0)}\sum_{t=1}^{\infty}\left(\frac{(1-\chi)l^N(q)}{1+r}\right)^t\right], \\
&= \sum_{t=0}^{-\infty}\left(\frac{(1-\chi)l^N(q)}{1+r}\right)^t \\
&= L(q,-\infty),
\end{aligned}$$

where the inequality follows from $l^T(q) > l^N(q)$. Hence, by (24) $n_a^{\Gamma(\tau)}(q,\tau) > n_a^{\Gamma(\tau)}(q,-\infty).\infty$

iii) For $\tau \geq 0$ and $\tau = -\infty$, $n_a^{\Gamma(t)}(q,\tau)$ is a constant, which follows directly from (27). For $-\infty < \tau < 0$, the difference between the left hand side of (24) evaluated at $\tau + 1$ and at τ is

$$\Delta(\tau) = \rho A^{\Gamma(\tau)}(n_a)^{\rho-1}\omega(q)\pi(q)\left[L(q,\tau+1) - L(q,\tau)\right].$$

The sign of $\Delta(\tau)$ is determined by the sign of $[L(q,\tau+1) - L(q,\tau)]$, which can be simplified to

$$\begin{aligned}
&\sum_{t=0}^{-(\tau+1)}\left(\frac{(1-\chi)l^N(q)}{1+r}\right)^t + \left(\frac{(1-\chi)l^N(q)}{1+r}\right)^{-(\tau+1)}\sum_{t=1}^{\infty}\left(\frac{(1-\chi)l^T(q)}{1+r}\right)^t \\
&-\sum_{t=0}^{-\tau}\left(\frac{(1-\chi)l^N(q)}{1+r}\right)^t - \left(\frac{(1-\chi)l^N(q)}{1+r}\right)^{-\tau}\sum_{t=1}^{\infty}\left(\frac{(1-\chi)l^T(q)}{1+r}\right)^t \\
=\; &\sum_{t=0}^{-(\tau+1)}\left(\frac{(1-\chi)l^N(q)}{1+r}\right)^t - \sum_{t=0}^{-\tau}\left(\frac{(1-\chi)l^N(q)}{1+r}\right)^t + \\
&\left(\frac{(1-\chi)l^N(q)}{1+r}\right)^{-(\tau+1)}\sum_{t=1}^{\infty}\left(\frac{(1-\chi)l^T(q)}{1+r}\right)^t - \left(\frac{(1-\chi)l^N(q)}{1+r}\right)^{-\tau}\sum_{t=1}^{\infty}\left(\frac{(1-\chi)l^T(q)}{1+r}\right)^t, \\
=\; &-\left(\frac{(1-\chi)l^N(q)}{1+r}\right)^{-\tau} + \left(\frac{(1-\chi)l^N(q)}{1+r}\right)^{-\tau}\left[\left(\frac{(1-\chi)l^N(q)}{1+r}\right)^{-1} - 1\right]\sum_{t=1}^{\infty}\left(\frac{(1-\chi)l^T(q)}{1+r}\right)^t, \\
=\; &\left(\frac{(1-\chi)l^N(q)}{1+r}\right)^{-\tau}\left\{-1 + \left[\left(\frac{(1-\chi)l^N(q)}{1+r}\right)^{-1} - 1\right]\sum_{t=1}^{\infty}\left(\frac{(1-\chi)l^T(q)}{1+r}\right)^t\right\}, \\
=\; &\left(\frac{(1-\chi)l^N(q)}{1+r}\right)^{-\tau}\left\{-1 + \left(\frac{1+r-(1-\chi)l^N(q)}{l^N(q)}\right)\left(\frac{l^T(q)}{1+r-(1-\chi)l^T(q)}\right)\right\} > 0,
\end{aligned}$$

where the last inequality follows because $l^T(q) > l^N(q)$ and the term in curly brackets is thus positive. Therefore, $\Delta(\tau) > 0$, and n_a must increase to maintain the first order condition going from τ to $\tau + 1$. Therefore, $n_a^N(q;\tau)$ is strictly increasing in τ, for $-\infty < \tau < 0$. ■

Proof of Proposition 5. Consider a firm with quality q and age A that has no trademark. Let $m = m(\text{A}, q)$ be the size of that firm at the end of the previous period. The difference between the flow profits from registering a trademark versus not registering at age A is given by

$$\begin{aligned}
\Delta(q,m) &= \widetilde{\Pi}^T(q,m) - \widetilde{\Pi}^N(q,m) \\
&= m^T(n_a^T;q,m)\pi(q) - n_a^T(q) - m^N(n_a^N;q,m)\pi(q) + n_a^N(q) \\
&= [m^T(\cdot) - m^N(\cdot)]\pi(q) - [n_a^T(q) - n_a^N(q)],
\end{aligned}$$

where the difference in the end-of-period customer sizes is

$$\begin{aligned} m^T(\cdot) - m^N(\cdot) &= m(1-\lambda)z(q) + \Phi^T(n_a^T)\omega(q) + d(q) - m(1-\lambda)(1-\delta)z(q) - \Phi^N(n_a^N)\omega(q) - d(q), \\ &= \delta m(1-\lambda)z(q) + \left(\Phi^T(n_a^T) - \Phi^N(n_a^N)\right)\omega(q) > 0. \end{aligned}$$

The last inequality follows because $\Phi^T(n_a^T) - \Phi^N(n_a^N)$. In other words, a trademark firm has higher advertising expenditures $n_a^T(q) > n_a^N(q)$, and reaches more consumers ($\Phi^T(\cdot) > \Phi^N(\cdot)$), since a trademark firm has a more efficient advertising technology.

First, it will be shown that $\Delta(q,m)$ is increasing in q and m. Consider the change in Δ as q changes. By the Envelope Theorem, $\frac{\partial \Delta}{\partial n_a^T}\frac{dn_a^T}{dq} = \frac{\partial \Delta}{\partial n_a^N}\frac{dn_a^N}{dq} = 0$. Hence,

$$\begin{aligned} \frac{d\Delta}{dq} &= \frac{d\widetilde{\Pi}^T}{dq} - \frac{d\widetilde{\Pi}^N}{dq} \\ &= \frac{d\pi(q)}{dq}\left[m^T(\cdot) - m^N(\cdot)\right] + \left[\frac{dm^T}{dq} - \frac{dm^N}{dq}\right]\pi(q), \\ &= \frac{d\pi(q)}{dq}\left[m^T(\cdot) - m^N(\cdot)\right] + \left[\left(\frac{\partial m^T}{\partial q} - \frac{\partial m^N}{\partial q}\right) + \left(\frac{\partial m^T}{\partial m} - \frac{\partial m^N}{\partial m}\right)\frac{dm}{dq}\right]\pi(q). \end{aligned}$$

Note that $m^T(\cdot) - m^N(\cdot) > 0$ and $\frac{d\pi(q)}{dq} > 0$. Next, consider the term

$$\frac{\partial m^T}{\partial q} - \frac{\partial m^N}{\partial q} = [\delta m(1-\lambda)z'(q) + \omega'(q)(\Phi^T(n_a^T) - \Phi^N(n_a^N))].$$

Here, $z'(q), \omega'(q) > 0$, since the probabilities of sale increase with quality. In addition, $\Phi^T(n_a^T) - \Phi^N(n_a^N) > 0$. Moreover,

$$\frac{dm^T}{dm} - \frac{dm^N}{dm} = \delta(1-\lambda)z(q) > 0.$$

Finally, $\frac{dm}{dq} > 0$, since the size of a non-trademark firm increases with quality (conditional on remaining a non-trademark firm at the higher quality). Therefore, $\Delta(q,m)$ is strictly increasing in q.

Next, consider the change in $\Delta(q,m)$ as age A, or equivalently, size m increases. Again, by the Envelope Theorem,

$$\begin{aligned} \frac{\partial \Delta}{\partial m} &= \frac{\partial \widetilde{\Pi}^N}{\partial m} - \frac{\partial \widetilde{\Pi}^N}{\partial m} \\ &= [\delta(1-\lambda)z(q)]\pi(q) > 0. \end{aligned}$$

Therefore, $\Delta(q,m)$ is increasing in m (or A).

Now, consider the difference

$$\begin{aligned} V_{\text{A}}^D(q) &= V_{\text{A}}^T(q) - V_{\text{A}}^{NN}(q), \\ &= \widetilde{\Pi}^T(q, m(\text{A},q)) - \widetilde{\Pi}^N(q, m(\text{A},q)) + ((1-\chi)/(1+r))[V_{\text{A}+1}^T(q) - V_{\text{A}+1}^N(q)], \\ &= \widetilde{\Pi}^T(q, m(\text{A},q)) - \widetilde{\Pi}^N(q, m(\text{A},q)) + ((1-\chi)/(1+r))[V_{\text{A}+1}^T(q) - \max\{V_{\text{A}+1}^T(q) - c_T, V_{\text{A}+1}^{NN}(q)\}], \\ &= \widetilde{\Pi}^T(q, m(\text{A},q)) - \widetilde{\Pi}^N(q, m(\text{A},q)) + ((1-\chi)/(1+r))\min\{V_{\text{A}+1}^T(q) - V_{\text{A}+1}^{NN}(q), c_T\}, \\ &= \widetilde{\Pi}^T(q, m(\text{A},q)) - \widetilde{\Pi}^N(q, m(\text{A},q)) + ((1-\chi)/(1+r))\min\{V_{\text{A}+1}^D(q), c_T\}, \\ &= \Delta(q, m(\text{A},q)) + ((1-\chi)/(1+r))\min\{V_{\text{A}+1}^D(q), c_T\} \end{aligned}$$

Let $\mathbf{T}$ be the operator such that

$$V^{D,n+1} \equiv \mathbf{T}V^{D,n} = \Delta(q, m(\text{A}, q)) + ((1-\chi)/(1+r)) \min\{V^{D,n}, c_T\}.$$

Note that $\mathbf{T}$ has a fixed point V^D by Banach fixed point theorem, as it satisfies Blackwell's sufficient conditions for a contraction mapping. $\mathbf{T}$ maps any bounded function $V^{D,n}$ into a bounded function $\mathbf{T}V^{D,n}$, and $\mathbf{T}V^{D,n}$ is continuous in its arguments because $V^{D,n}$ Δ are continuous. For any two functions $V_1^{D,n} \geq V_2^{D,n}$, it holds that $\mathbf{T}V_1^{D,n} \geq \mathbf{T}V_2^{D,n}$ because $\min\{V_1^{D,n}, c_T\} \geq \min\{V_2^{D,n}, c_T\}$. Finally, $\mathbf{T}$ satisfies the discounting hypothesis

$$\begin{aligned}
\mathbf{T}(V^{D,n} + b) &= \Delta(q, m(\text{A}, q)) + ((1-\chi)/(1+r)) \min\{V^{D,n} + b, c_T\}, \\
&\leq \Delta(q, m(\text{A}, q)) + ((1-\chi)/(1+r)) \min\{V^{D,n} + b, c_T + b\}, \\
&= \Delta(q, m(\text{A}, q)) + ((1-\chi)/(1+r))[\min\{V^{D,n}, c_T\} + b], \\
&= \mathbf{T}V^{D,n} + ((1-\chi)/(1+r))b.
\end{aligned}$$

Assume now that $V^{D,n}$ is non-decreasing in q. It will now be shown that this implies $\mathbf{T}V^{D,n}$ is increasing in q. If $V^{D,n}$ is non-decreasing in q, so is $\min\{V^{D,n}, c_T\}$. Furthemore, Δ is increasing in q. Therefore, $V^{D,n+1}$ is increasing in q. Hence, $\mathbf{T}$ maps non-decreasing functions of q into increasing functions of q. By Theorem 3.2 in Stokey and Lucas (1989), the fixed point of $\mathbf{T}$, V^D, must also be increasing in q. Similarly, assume that $V^{D,n}$ is non-decreasing in m. Then, $\min\{V^{D,n}, c_T\}$ is also non-decreasing in m. Because Δ is increasing in m, $V^{D,n+1}$ must be increasing in m, or equivalently in A. The fixed point V^D is then also increasing in A. ■