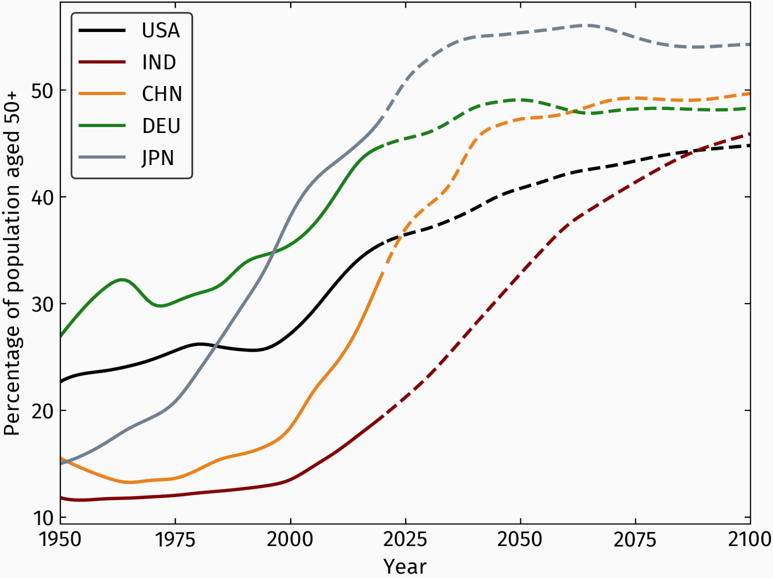
Demographics, Wealth, and Global Imbalances in the Twenty-First Century

Adrien Auclert, Hannes Malmberg, Frédéric Martenet and Matthew Rognlie

Hoover Economic Policy Working Group, March 2021

The world population is aging...

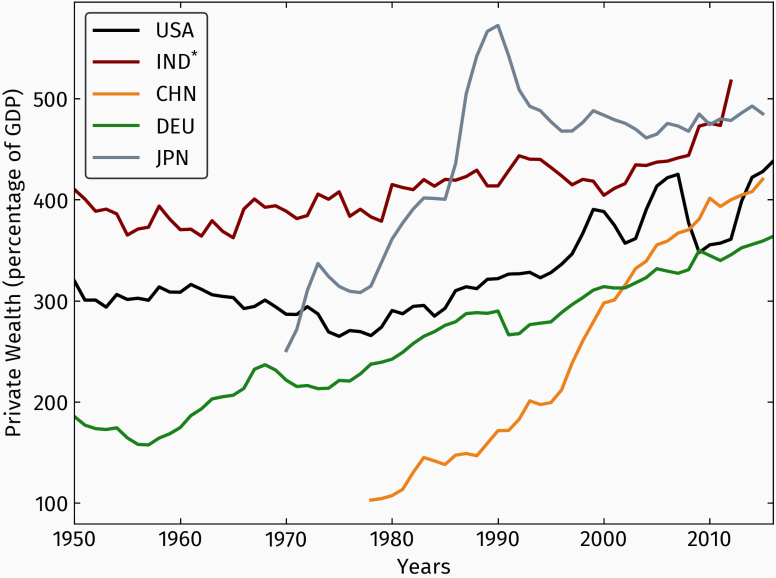
▸ 65+ ▸ World



Source: 2019 United Nations World Population Prospects

…wealth-to-GDP ratios are increasing…

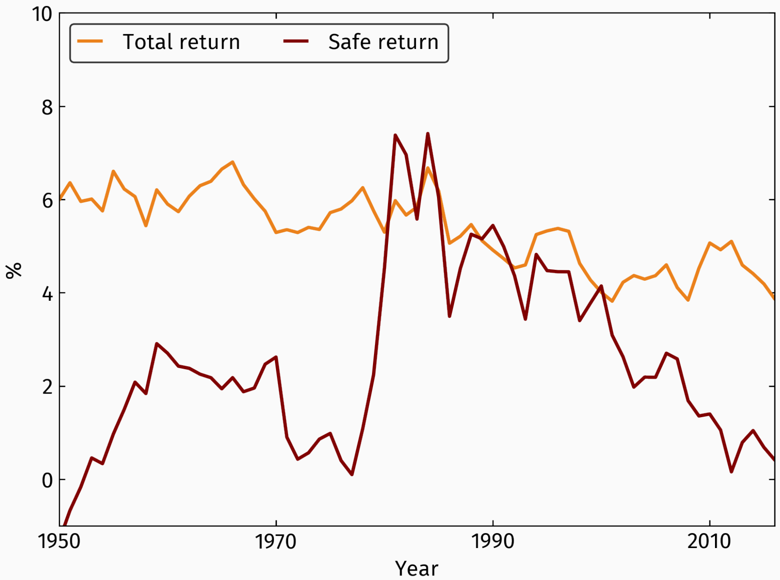
▸ National Wealth
▸ SCF vs WID



*IND: National rather than Private Wealth. Source: World Inequality Database (WID)

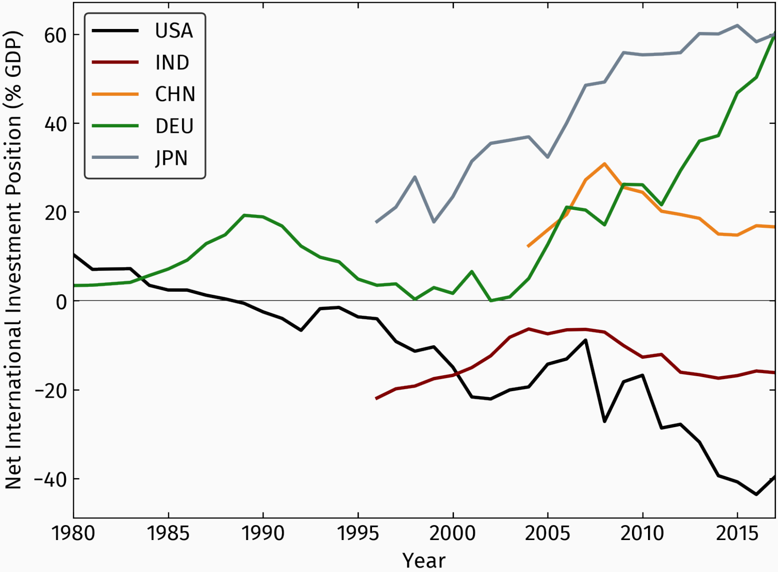
...rates of return on wealth are falling ...

▸ Definitions



Source: National Accounts, Flow of Funds, WID.

…and “global imbalances” are rising



Source: International Monetary Fund (IMF), Penn World Table (PWT) 9.1

How will demographics shape these trends in the 21st century?

- Broad agreement that demographics has contributed to historical trends in W/Y, NFA imbalances, and real returns (r)
- Much less agreement about *how much*

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- Going forward, hypothesis that these trends might revert, centered on the savings rate in an aged population:

  "While a large population cohort that is saving for retirement puts upward pressure on the total savings rate, ***a large elderly cohort may push down aggregate savings*** *by running down accumulated wealth."*

  [Lane 2020]

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  [Lane 2020]

  "**asset market meltdown**" hypothesis [Poterba 2001]

  "**great demographic reversal**" hypothesis [Goodhart-Pradhan 2020]

This paper: a sufficient statistic approach to this question

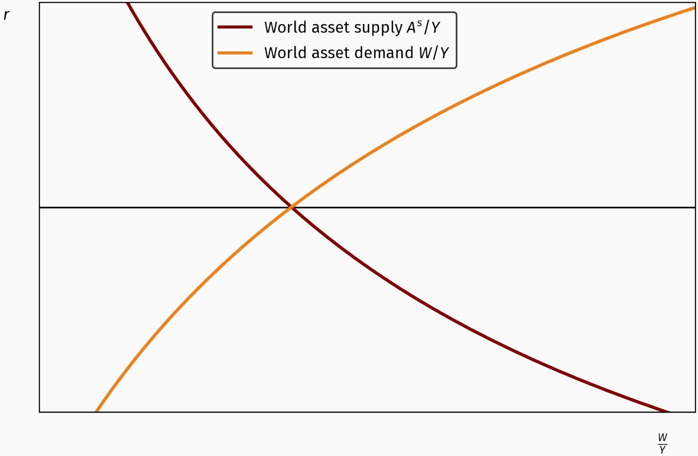
In a baseline multi-country GE OLG model, the effect of demographic change on r, W/Y and NFA depends **only** on:

1. cross-sectional age profiles of asset accumulation, labor income, and consumption
2. demographic projections
3. the elasticity of intertemporal substitution $1/\sigma$
4. the elasticity of substitution between capital and labor η

This provides a framework for measurement, which we implement

Quantitative conclusions are robust to many plausible extensions of this baseline model

Baseline model in one picture



Slope of asset supply $\bar{\epsilon}_s$: depends on η and observables

Slope of asset demand $\bar{\epsilon}_d$: depends on σ and observables (new!)

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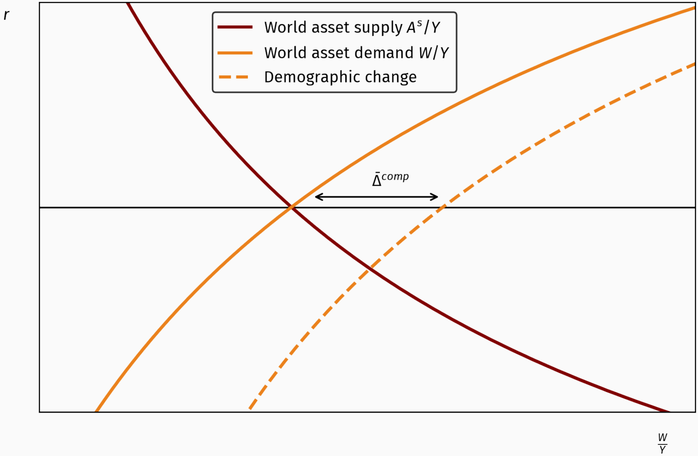
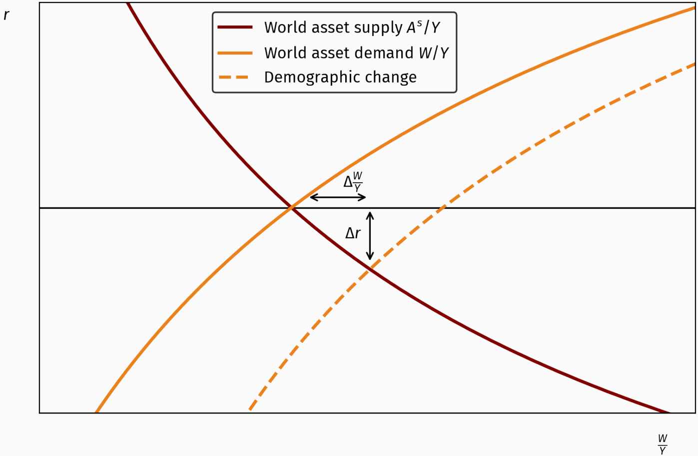


Shift in asset demand $\bar{\Delta}^{comp}$: observable from composition (new!)

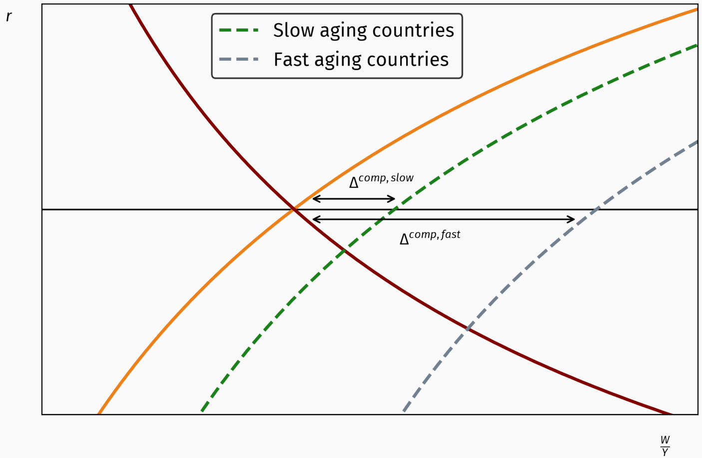
Large and positive in the data.

Baseline model in one picture



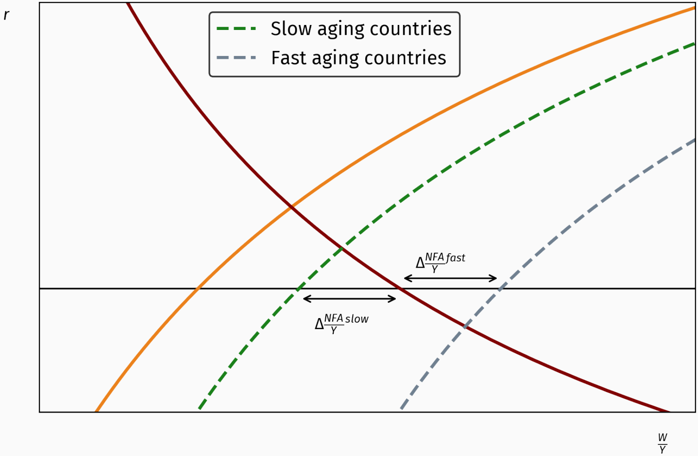
$$\Delta r \approx -\frac{\bar{\Delta}^{comp}}{\bar{\epsilon}_s + \bar{\epsilon}_d} < 0, \quad \Delta\left(\frac{\bar{W}}{Y}\right) \approx \frac{\bar{\epsilon}_s}{\bar{\epsilon}_s + \bar{\epsilon}_d}\bar{\Delta}^{comp} > 0$$

Baseline model in one picture



Country-specific shifts Δ^{comp} large and heterogeneous in data

Baseline model in one picture



$$\Delta\left(\frac{NFA}{Y}\right) \approx \Delta^{comp} - \bar{\Delta}^{comp}$$

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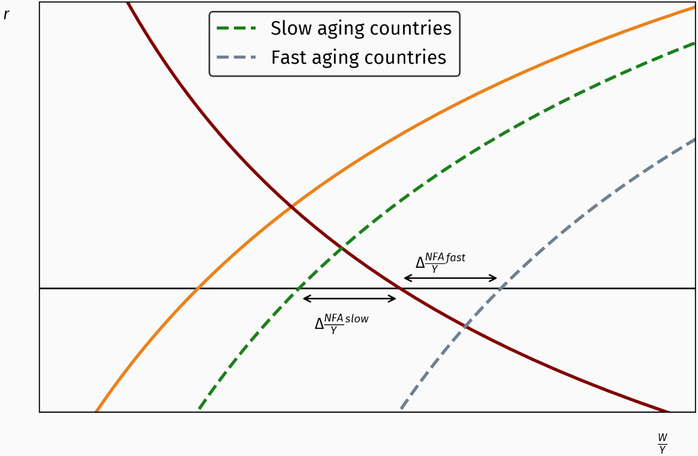

$\Rightarrow$ r **always falls,** W/Y **always rises, large global imbalances**

A bridge between reduced-form and structural approaches

- Existing literature follows two broad approaches:

1. **Reduced-form**, based on shift-share exercises
    - Projected asset demand [Poterba 2001, Mankiw-Weil 1989], projected savings rates [Summers-Carroll 1987, Auerbach-Kotlikoff 1990...]
    - Projected labor supply [Cutler et al 1990], demographic dividend literature [Bloom-Canning-Sevilla 2003...]

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2. **Structural**, based on fully specified GE OLG models
   - Demographics and **wealth** + social security [Auerback Kotlikoff 1987, İmrohoroğlu-İmrohoroğlu-Joines 1995, De Nardi-İmrohoroğlu-Sargent 2001, Abel 2003, Geanakoplos-Magill-Quinzii 2004, Kitao 2014...]
   - Demographics and **capital flows** [Henriksen 2002, Börsch-Supan-Ludwig-Winter 2006, Domeij-Flodén 2006, Krueger-Ludwig 2007, Backus-Cooley-Henriksen 2014, Bárány-Coeurdacier-Guibaud 2019...]
   - Demographics and **interest rates** [Carvalho-Ferrero-Necchio 2016, Gagnon-Johannsen-Lopez Salido 2016, Eggertsson-Mehrotra-Robbins 2019, Lisack-Sajedi-Thwaites 2017, Jones 2018, Papetti 2019, Rachel-Summers 2019...]

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- **This paper** bridges the gap between both

1. Baseline environment

Environment: demographics, production, and government

OLG model, demographic change + multiple countries facing $\{r_t\}$

Demographics (drop country subscripts)

- Exogenous, time-varying sequence of births N_{0t}
- Exogenous, constant sequence of mortality rates ϕ_j ▸ Mortality contrib.
- No migration

Production

- Aggregate production function with capital and effective labor
- Constant growth rate of labor-augmenting technology γ
- Perfect competition, free capital adjustment

Government

- Flow budget constraint

$$G_t + w_t \sum_{j=0}^{T} N_{jt} \mathbb{E} tr_j + (1 + r_t) B_t = \tau w_t \sum_{j=0}^{T} N_{jt} \mathbb{E} \ell_j + B_{t+1},$$

- Balances budget over time by adjusting G_t and B_{t+1}, not τ_t or tr_{jt}

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Environment: heterogeneous agents

Problem for **heterogeneous agents** of cohort k (age $j \equiv t - k$)

$$\max \mathbb{E}_k \left[\sum_j (\beta^j \times \psi_j \times \Phi_j) \frac{c_{jt}^{1-\sigma}}{1-\sigma} \right]$$

$$s.t \quad c_{jt} + a_{j+1,t+1} \leq w_t \left((1-\tau)\ell(z_j) + tr(z^j) \right) + \frac{(1+r_t)a_{j,t}}{\phi_j}$$

$$a_{j+1,t+1} \geq -\underline{a}$$

- $\beta^j \psi_j \Phi_j$: discounting $\times$ utility shifter $\times$ survival prob ($\prod_j \phi_j$)
- a_{jt}: annuity holdings
- $\ell(z_t)$: risky labor supply driven by arbitrary stochastic process z_t
- $\tau, tr(z^j)$: taxes and (state-contingent) government transfers

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Equilibrium

Given demographics and policy, in an integrated world equilibrium:

- Households optimize
- Firms optimize
- Global asset markets clear (with GDP weights $\omega_t^c \equiv \frac{Y_t^c}{Y_t}$)

$$\sum_c \omega_t^c \frac{W_t^c}{Y_t^c} = \sum_c \omega_t^c \left(\frac{K_t^c}{Y_t^c} + \frac{B_t^c}{Y_t^c} \right) \quad \forall t$$

Next consider two cases, each with countries facing a constant γ

1. Small country aging alone, world at steady state $\rightarrow$ r constant
2. Many countries aging together, converging to a s.s. with r_{LR}

Compositional effects as sufficient statistics

Proposition

The wealth-to-GDP ratio of a small country aging alone with constant r and γ follows

$$\frac{W_t}{Y_t} \propto \frac{\sum_j \pi_{jt} a_{j0}}{\sum_j \pi_{jt} h_{j0}}$$

where $a_{j0} \equiv \mathbb{E} a_{j,0}$ and $h_{j0} = \mathbb{E} w_0 \ell_{j,0}$ are average initial asset holdings and pretax labor income by age.

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$\Rightarrow$ G.E. demographic impact over time is $\Delta r = 0$, and

$$\Delta\left(\frac{W_t}{Y_t}\right) = \Delta\left(\frac{NFA_t}{Y_t}\right) = \Delta_t^{comp} \equiv \frac{\sum_j \pi_{jt} a_{j0}}{\sum_j \pi_{jt} h_{j0}} - \frac{\sum_j \pi_{j0} a_{j0}}{\sum_j \pi_{j0} h_{j0}}$$

measurable from demographic projections and hh. surveys

Why? Demographics do not affect (normalized) individual decisions

Long-run wealth and rate of return adjustment

Proposition

In world equilibrium, the long-run change in r and W/Y satisfy

$$r_{LR} - r_0 \simeq -\frac{1}{\bar{\epsilon}^d + \bar{\epsilon}^s} \bar{\Delta}^{comp}_{LR}$$

$$\sum_c \omega^c \left[\left(\frac{W^c}{Y^c} \right)_{LR} - \left(\frac{W^c}{Y^c} \right)_0 \right] \simeq \frac{\bar{\epsilon}^s}{\bar{\epsilon}^s + \bar{\epsilon}^d} \bar{\Delta}^{comp}_{LR}$$

- $\bar{\epsilon}^d, \bar{\epsilon}^s$: average long-run asset demand and supply sensitivities
- ω_c: initial GDP weights

Long-run change in country NFAs

$$\left(\frac{NFA^c}{Y^c} \right)_{LR} \simeq \Delta^c_{comp} - \bar{\Delta}_{comp} + [(\epsilon^{c,d} + \epsilon^{c,s}) - (\bar{\epsilon}^d + \bar{\epsilon}^s)](r_{LR} - r_0)$$

$$\simeq \Delta^c_{comp} - \bar{\Delta}_{comp} \qquad \text{(if } \epsilon_d, \epsilon_s \text{ are similar)}$$

Sensitivities of asset supply and demand

- Assuming all countries have the same capital-labor substitution elasticity η,

$$\bar{\epsilon}^s = \frac{\eta}{r_0 + \delta} \overline{\left(\frac{K_0}{Y_0}\right)}$$

$\rightarrow$ Measurable from observables and knowledge η

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Proposition

With no idiosyncratic risk, $\underline{a} = \infty$ and $r = \gamma = 0$, in each country:

$$\epsilon^d = \underbrace{\frac{C}{Y} \cdot \frac{1}{\sigma} \cdot Var\left(Age_c\right)}_{substitution\ effect} - \frac{W}{Y} \underbrace{\left(E\left[Age_a\right] - E\left[Age_c\right]\right)}_{income\ effect}$$

$\rightarrow$ Measurable from observables and knowledge of σ

2. Measurement and implications

Measuring Δ^{comp}

- Calculate shift-share Δ_t^{comp} for US and 24 other countries
- Implementation:

$$\Delta_t^{comp} \equiv \frac{W_0}{Y_0}\left(\frac{\sum \pi_{jt} a_{j0}}{\sum \pi_{jt} h_{j0}} \Big/ \frac{\sum \pi_{j0} a_{j0}}{\sum \pi_{j0} h_{j0}} - 1\right)$$

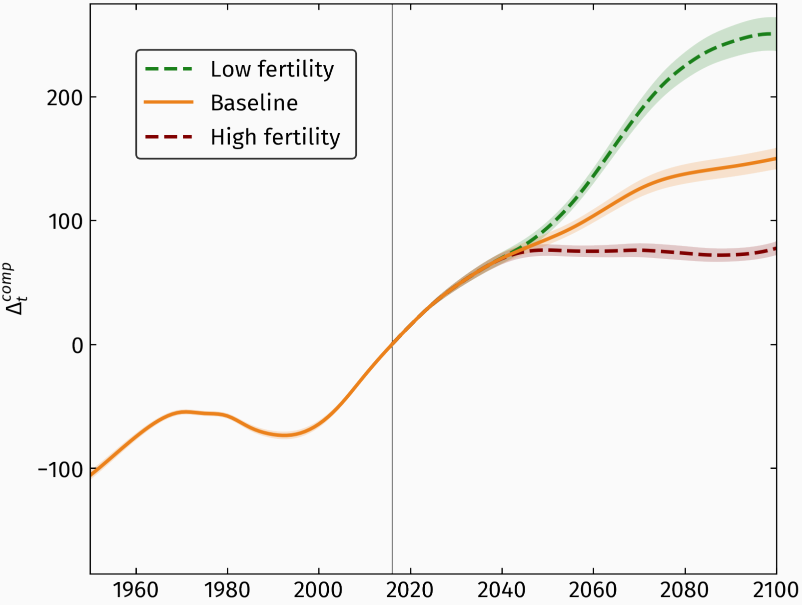
- Data:
  - π_{jt} : projections of age distributions over individuals

    2019 UN World Population Prospects
  - a_{j0}, h_{j0} : age-wealth and labor income profiles in base year

    For US: SCF, LIS/CPS, and Sabelhaus-Henriques Volz (2019)
    a_{j0} includes funded part of DB pensions
    Household $\rightarrow$ individual j by splitting wealth among adults in hh

Δ^{comp} in the United States: 1950-2100

Base year
Historical



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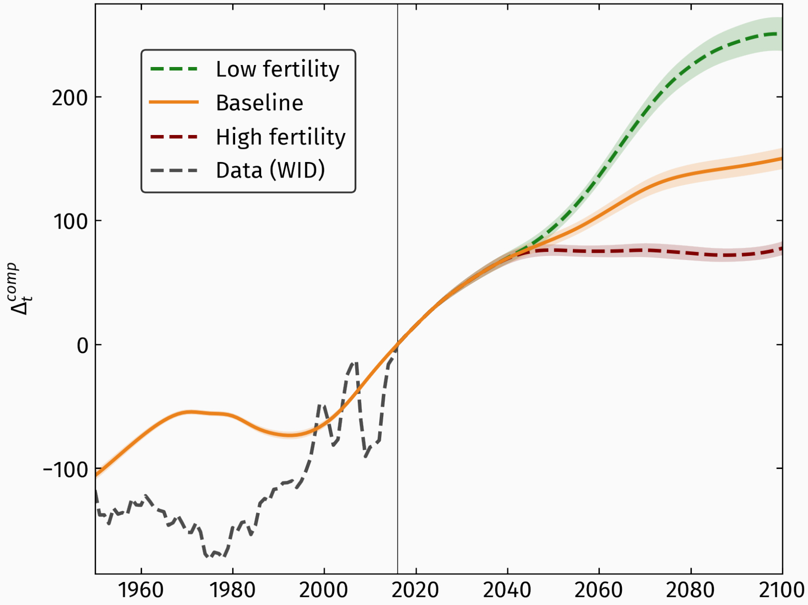
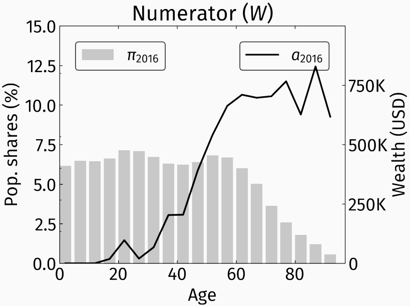


Where do these large effects come from?

▸ Alt. profiles



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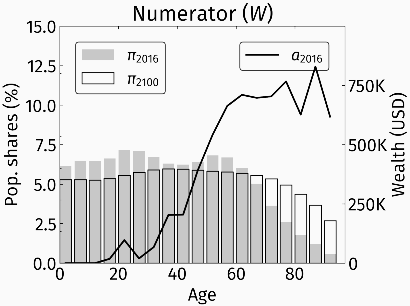

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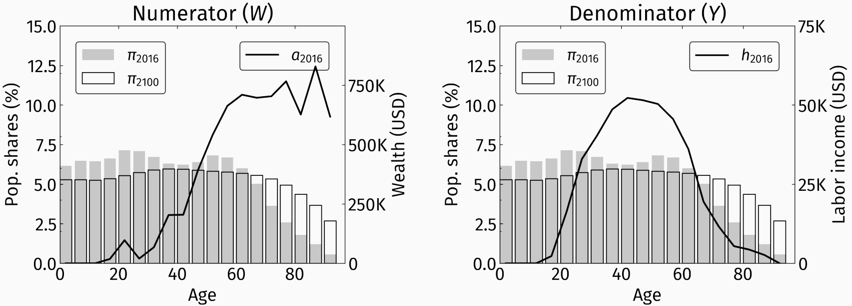

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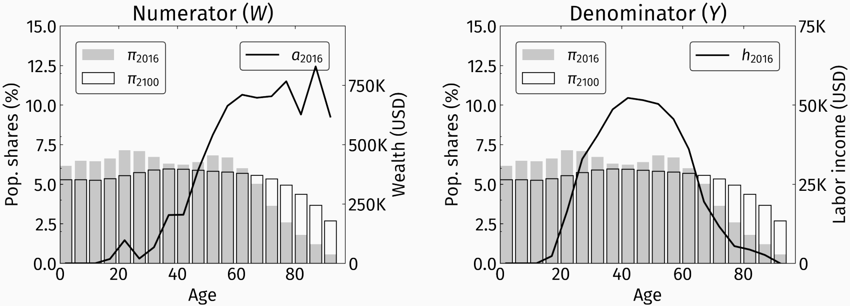
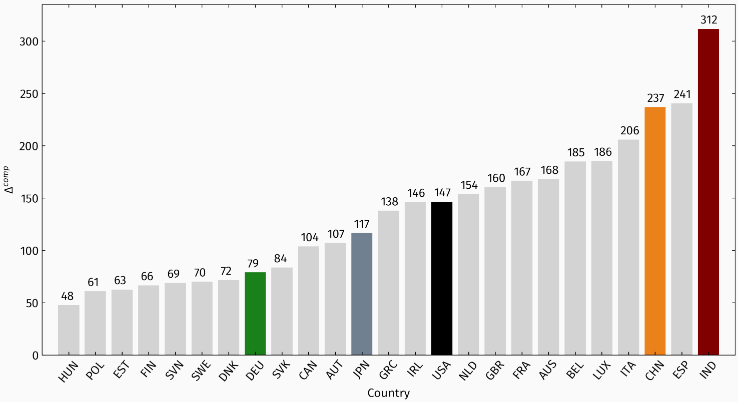


- In paper: separate contribution of numerator and denominator
  - Going forward: W contributes $\sim 2/3$, Y contributes $\sim 1/3$
  - Historically demographic dividend pushed Y up, reversed in 2010

Globally large and heterogeneous Δ^{comp} by 2100



Quantifying sensitivities

▸ USA ▸ All countries ▸ Approx.

Supply sensitivity $\bar{\epsilon}_s = \frac{\eta}{r_0+\delta}\frac{\bar{K}}{\bar{Y}}$:

- $\eta \equiv$ substitutability between capital and labor
- Range of $\eta = 0.6 - 1.25$
  [Oberfield and Raval, 2019; Karabarbounis and Neiman, 2014]

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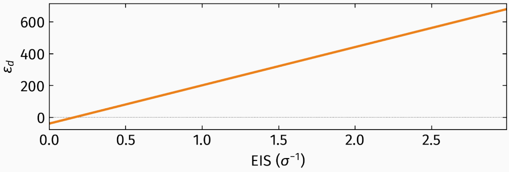


- Can also compare to literature estimates, range 5–200
  [Zoutman, 2018; Gagnon et al., 2019; Moll, Rachel and Restrepo, 2019; Eggertson et al., 2020; Jakobsen et al. 2020]

Changes in r and W/Y: 2016 to 2100

$$\Delta r \approx -\frac{\bar{\Delta}^{comp}}{\bar{\epsilon}^d + \bar{\epsilon}^s}$$

$$\Delta\left(\frac{W}{Y}\right) \approx \frac{\bar{\epsilon}^s}{\bar{\epsilon}^d + \bar{\epsilon}^s}\bar{\Delta}^{comp}$$

A. Change in world r

	EIS		
η	0.25	0.50	1.00
0.60	-3.17	-1.44	-0.69
1.00	-2.14	-1.18	-0.62
1.25	-1.78	-1.06	-0.59

B. Change in world W/Y

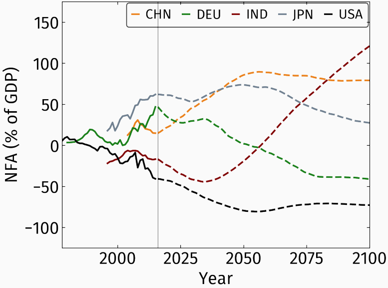
	EIS		
η	0.25	0.50	1.00
0.60	116	57	28
1.00	132	77	42
1.25	138	87	50

Demeaned compositional effect and NFAs

▸ Approx.

$$\Delta\left(\frac{NFA}{Y}\right) \approx \Delta^{comp} - \bar{\Delta}^{comp}$$

A. NFA projection



→ Data points to large global imbalances for the 21st century

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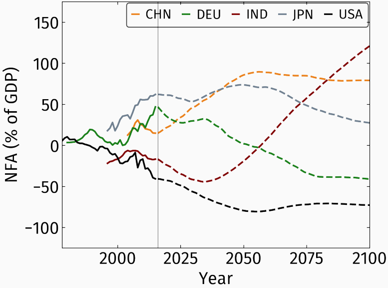
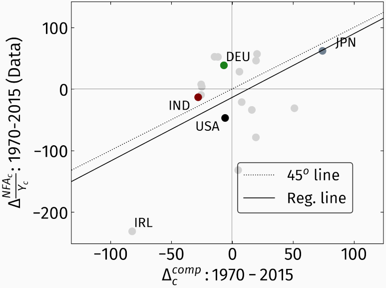


B. Historical performance



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3. Quantitative model

Updated environment

Household problem becomes

$$\max \mathbb{E}_k \sum_j \beta^j \psi_{jt} \Phi_{jk} \left[\frac{c_{jt}^{1-\sigma}}{1-\sigma} + \Upsilon Z_t^{\nu-\sigma} \left(1-\phi_{jt}\right) \frac{\left(a_{jt}\right)^{1-\nu}}{1-\nu} \right] \quad \nu \geq \sigma$$

$$\text{s.t.} \quad c_{jt} + a_{jt} \leq w_t \left((1-\tau_t)\ell_{jt}(z_j)(1-\rho_{jt}) + tr_{jt}(z_j)\right) + (1+r_t)a_{j-1,t-1} + b_{jt}^r(z_j)$$

$$a_{jt} \geq -\bar{a} Z_t$$

- From annuities to bequests:
  - assets become bequests at death, distributed as $b_{jt}^r(z_j)$
- Time-variation in mortality Φ_{jk}, utility shifters ψ_{jt} from kids in household, labor supply ℓ_{jt}, retirement age ρ_{jt}
- Fiscal rule with adjustments in taxes and transfers, income process with intergenerational persistence
- Migration

Robustness of conclusions

- Assume EIS=0.5, $\eta = 1$

	Δr	$\Delta \frac{\bar{W}}{\bar{Y}}$	$\Delta \frac{NFA^{USA}}{Y^{USA}}$	$\Delta \frac{NFA^{CHN}}{Y^{CHN}}$
Pure compositional analysis	-1.18	77	-23	59
Baseline social security	-1.13	56	-38	67
Alternative assumptions				
Only social security tax	0.47	1	32	-11
Only lower benefits	-1.68	99	-75	158

Other topics covered in paper

▸ Multiple assets

1. Multiple assets
2. Accounting for historical movements in US W/Y and r
3. Reconciling literature findings on r^* effects of demographics
4. Housing
5. Population aging and wealth inequality

4. Demographics and falling savings rates

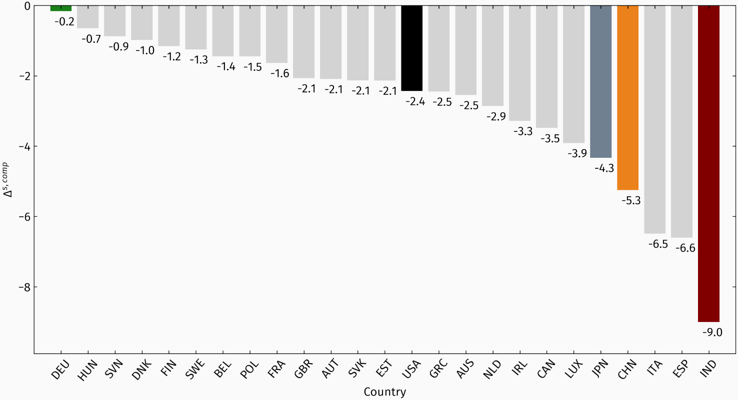
Measuring S_t^{comp}

- Possible to calculate shift-share for savings rate S/Y
  - Either directly from consumption profiles
  - Or using the budget constraint (our preferred approach)

$$S_t^{comp} \equiv \frac{1}{1+g_t} \cdot \frac{g_t \sum_j \pi_{jt} a_{jo} + \sum_j \Delta\pi_{jt} a_{jo}}{\sum \pi_{jt} h_{jo}}$$

- g_t: Growth rate of real GDP
  - Use $\sum \pi_{jt} h_{jo}$ to calculate compositional effect on labor supply

Worldwide: decreasing S_t/Y_t everywhere



Declining r despite falling savings?

- Will dissaving of the old reverse the effects of demographics? [Lane 2020, Goodhart and Pradhan 2020]
- Measured S_t^{comp} does decline
- **But**: r does not increase
- Why? Savings is misleading with declining pop. growth. In s.s.:

$$\frac{W}{Y} = \frac{1+g}{g}\frac{S}{Y}$$

With demographic change, S/Y falls but g falls by more!

Conclusion

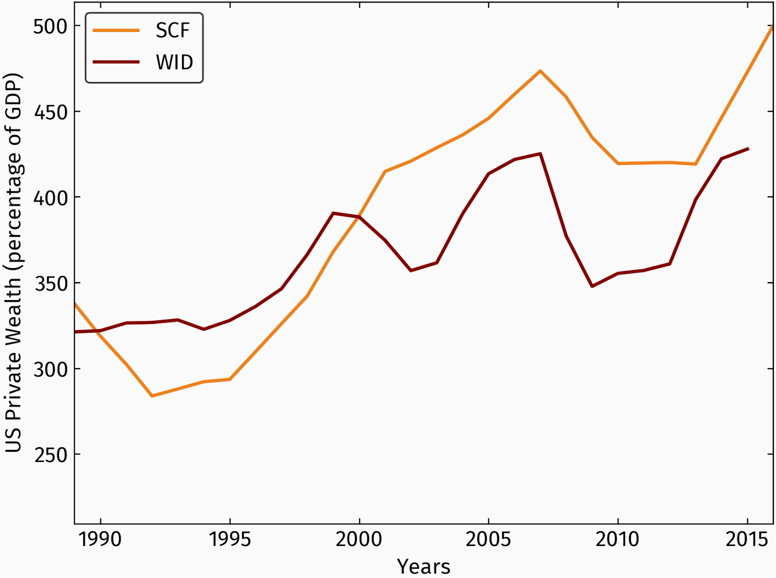
- How does population aging affect wealth-output ratios, real interest rates, and capital flows?
- Use compositional effect Δ^{comp} as starting point for forecasts
- Δ^{comp} are **large** and **heterogeneous** in the data
- For the 21st century, our approach:
    - Refutes the asset market meltdown hypothesis: r definitively falls
    - Suggests the global savings glut has just begun

Thank you!

Additional slides

US Wealth-to-GDP from SCF vs World Inequality Database

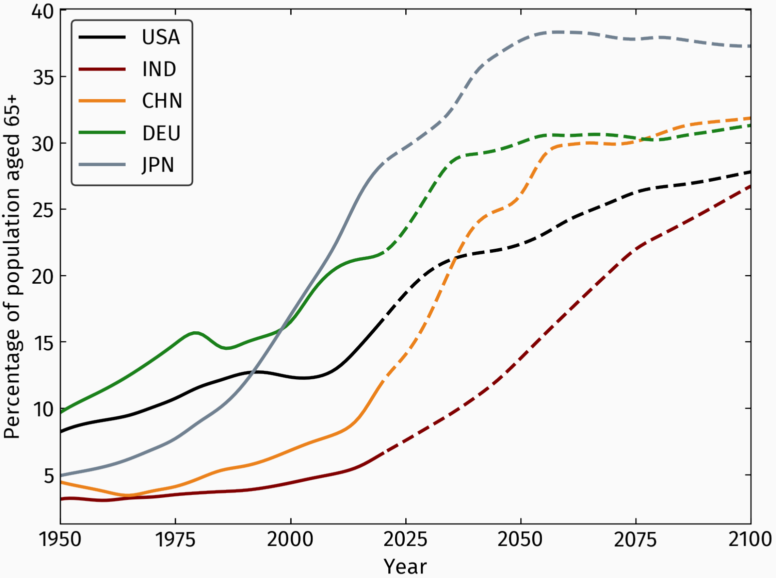
Back



Source: World Inequality Database (WID), Survey of Consumer Finances (SCF)

Share of the population aged 65+

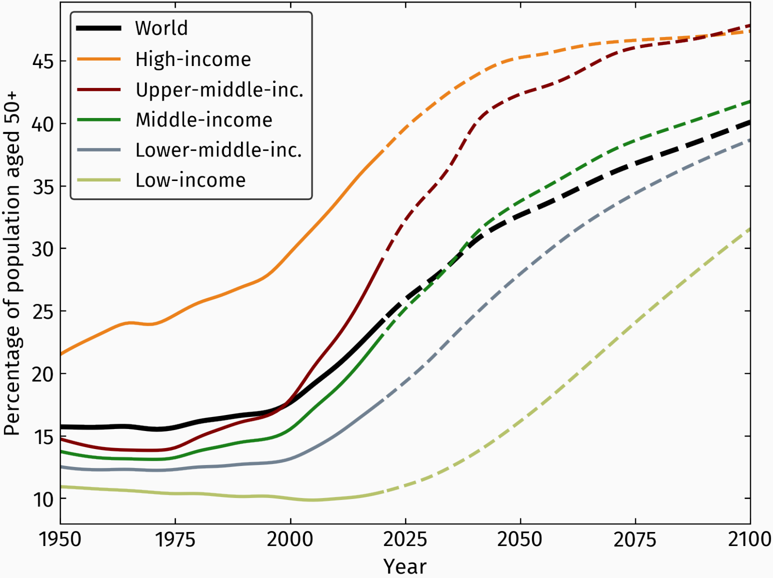
Back



Source: 2019 United Nations World Population Prospects

Countries by income group

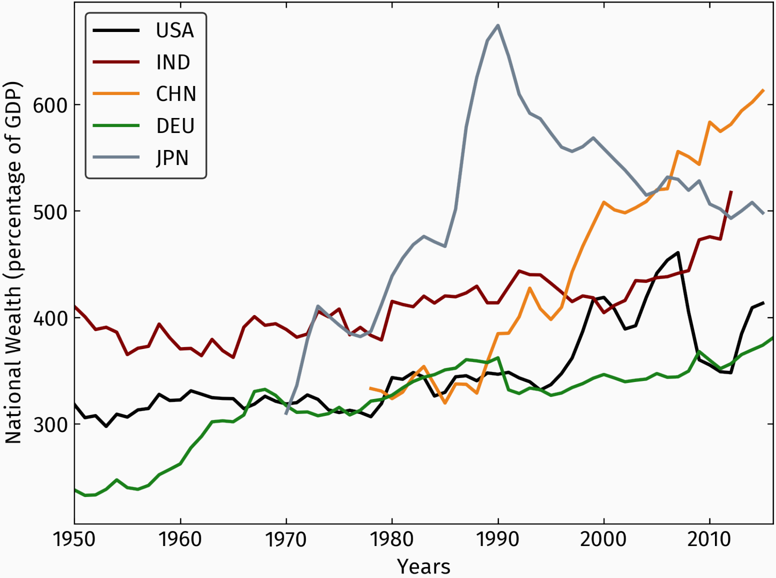
Back



Source: 2019 United Nations World Population Prospects

National Wealth over GDP

◂ Back



Source: World Inequality Database (WID)

Rates of return on wealth

◂ Back

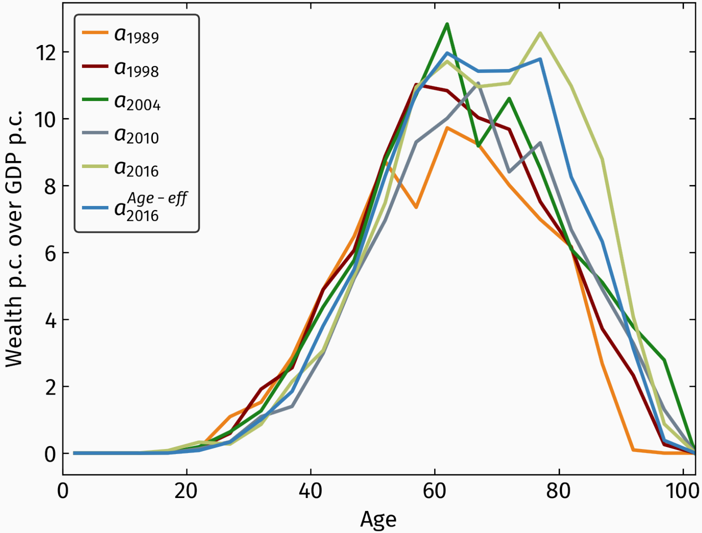
- Baseline safe return r_t^{safe} is 10 year constant maturity interest rate minus HP-filtered PCE deflator
- Baseline total return is

$$r_t = \frac{(s_K Y - \delta K)_t + r_t^{safe} B_t}{W_t - NFA_t}$$

where $(s_K Y - \delta K)_t$ is net capital income

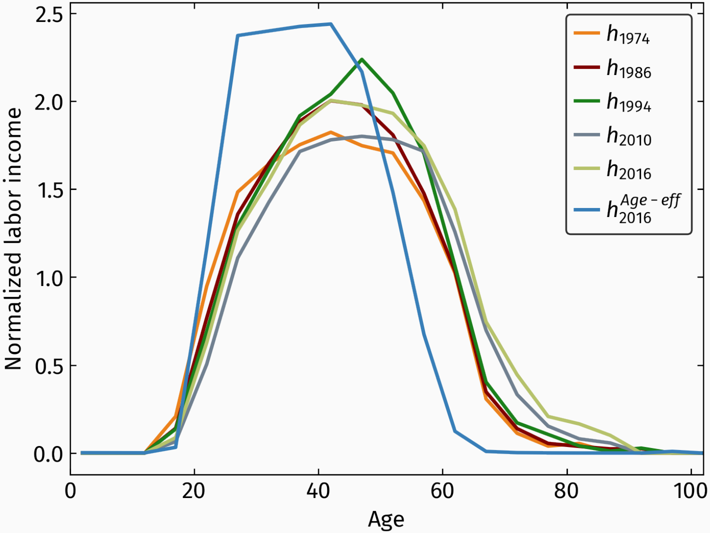
Age-wealth profiles

Back



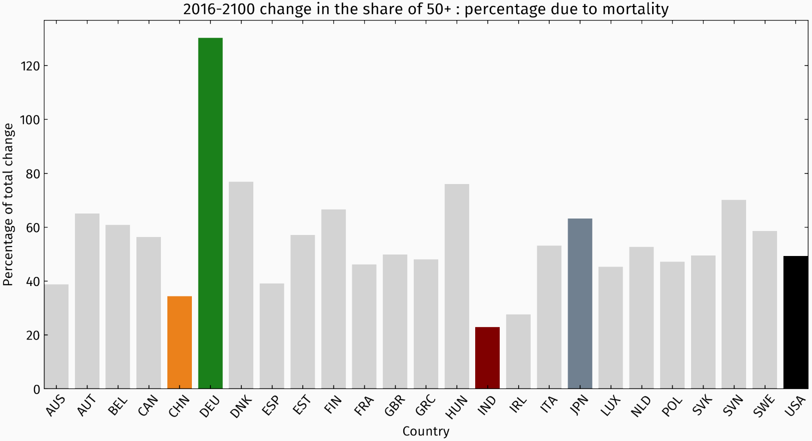
Age-labor income profiles

Back



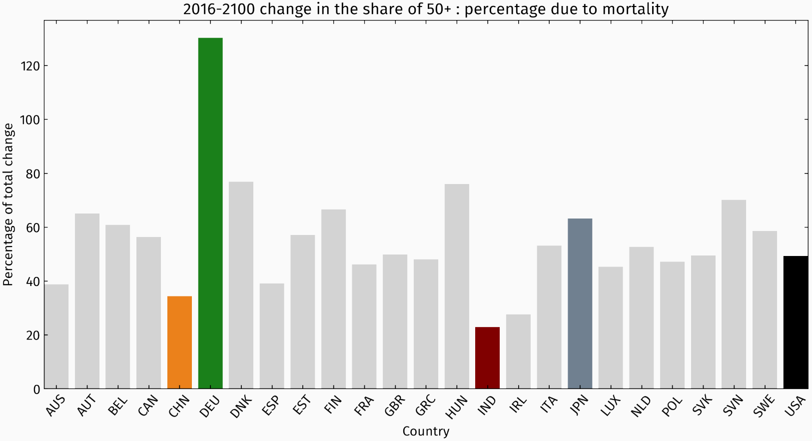
Contribution of mortality to aging since 1950

Back



Contribution of mortality to aging in 21st century

Back



Measuring income and wealth profiles

Back

- **Measuring age-labor income profiles** h_{jt}
  - Data from the Luxembourg Income Study (LIS)
  - h_{jt} is proportional to total labor income per person
  - In 2016: normalize aggregate effective labor per person

$$1 = L_{2016} = \sum_j \pi_{j,2016} h_{j,2016}$$

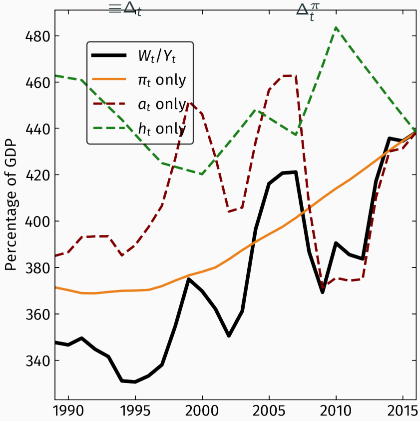
  - In t: L_t grows as aggregate labor input from the BLS $\frac{L_t^{BLS}}{L_{2016}^{BLS}}$
- **Measuring age-wealth profiles** $a_{jt} = \frac{A_{jt}}{Y_t/L_t}$
  - Data from the Survey of Consumer Finances (SCF)
  - Provide net worth by age at the household level
  - A_{jt} is aggregate household net worth over total individuals
  - Divide by Y_t/L_t^{BLS} to obtain a_{jt}

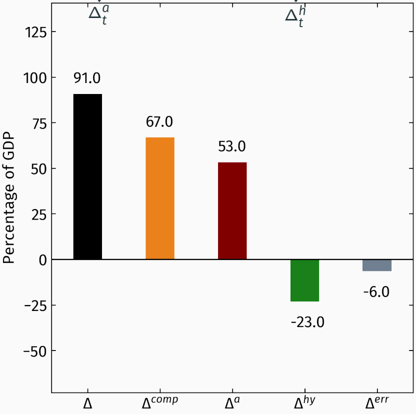
Retrospective U.S. exercise

Back

- To first order:

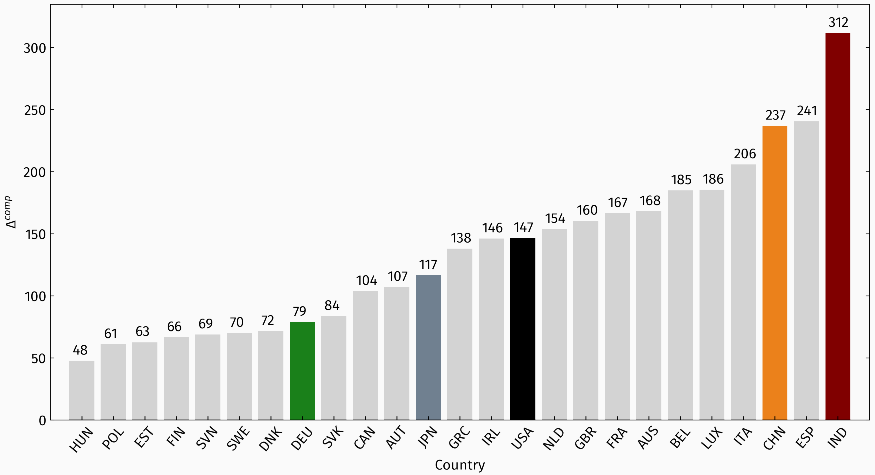
$$\underbrace{\frac{W_t}{Y_t}-\frac{W_0}{Y_0}}_{\equiv\Delta_t}=\underbrace{\frac{\sum_i \pi_{it}a_{i0}}{\sum \pi_{it}h_{i0}}-\frac{\sum_i \pi_{i0}a_{i0}}{\sum \pi_{i0}h_{i0}}}_{\Delta_t^{\pi}}+\underbrace{\sum_i \pi_{i0}\left(a_{it}-a_{i0}\right)}_{\Delta_t^{a}}-\underbrace{\sum_i \pi_{i0}\frac{W_0}{Y_0}\left(h_{it}-h_{i0}\right)}_{\Delta_t^{h}}+\Delta_t^{er}$$





Δ^{comp} around the world in 2100

Back



Robustness to baseline year for age profiles (past)

Back

Change in W/Y: 1950 to 2016

Rows: Age-wealth profile (SCF); columns: Age-labor income profile (LIS)

	1974	1979	1986	1991	1994	1997	2000	2004	2007	2010	2013	2016	DH-t	DH-c
1989	71	72	74	74	73	72	74	71	70	67	67	68	98	83
1992	71	72	75	75	73	72	74	71	70	68	67	68	99	83
1995	76	77	79	79	78	77	79	76	75	72	72	73	103	88
1998	81	82	85	85	84	82	85	82	80	77	77	78	111	94
2001	82	83	86	85	84	83	85	82	81	78	78	78	111	95
2004	88	89	92	92	91	90	92	89	88	85	84	85	118	101
2007	91	92	95	95	94	93	95	92	91	87	87	88	123	105
2010	83	84	87	87	86	85	87	85	84	81	81	81	109	95
2013	87	88	91	91	90	89	91	88	87	84	84	84	115	100
2016	97	99	102	103	101	100	103	100	98	96	95	96	129	112
DH-t	101	102	106	106	105	104	107	104	102	99	99	100	132	116
DH-c	108	110	113	114	113	112	115	112	110	108	107	108	139	123

Robustness to baseline year for age profiles (future)

Back

Change in W/Y: 2016 to 2100

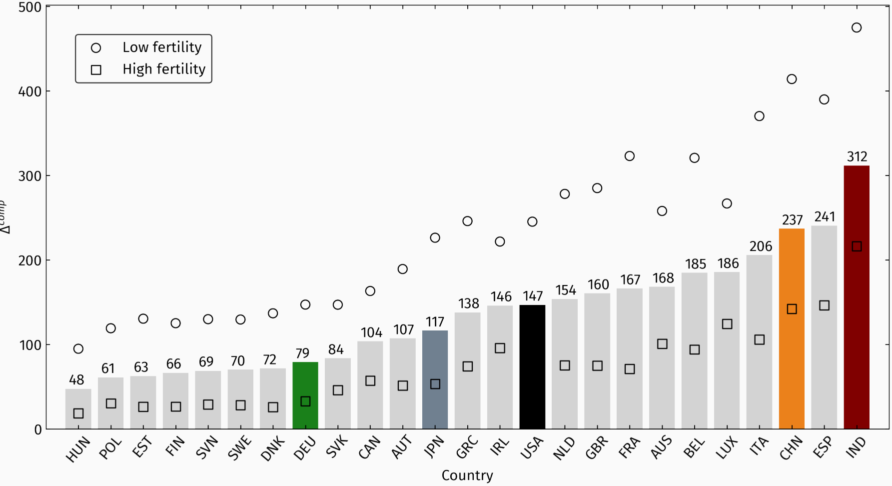
Rows: Age-wealth profile (SCF); Columns: Age-labor income profile (LIS)

Age-wealth profile (SCF)	1974	1979	1986	1991	1994	1997	2000	2004	2007	2010	2013	2016	DH-t	DH-c
1989	106	107	110	111	111	110	112	109	107	105	103	102	123	117
1992	89	89	92	92	92	92	93	90	88	86	84	83	104	98
1995	102	103	105	106	107	106	107	105	102	100	98	97	118	113
1998	98	99	101	102	102	101	103	100	98	96	93	93	115	109
2001	97	98	100	101	101	100	101	99	96	94	92	91	113	107
2004	115	116	119	120	120	119	120	118	115	113	111	110	133	127
2007	115	116	119	119	120	119	120	117	115	113	110	109	133	127
2010	113	114	117	118	119	118	120	117	115	113	111	110	131	125
2013	121	122	125	126	127	126	127	125	122	120	118	117	140	133
2016	143	145	149	151	151	150	152	149	147	144	142	141	165	159
DH-t	128	130	133	134	135	134	136	133	130	128	125	124	148	142
DH-c	146	148	152	154	155	154	156	153	150	148	145	144	169	162

Age-labor income profile (LIS)

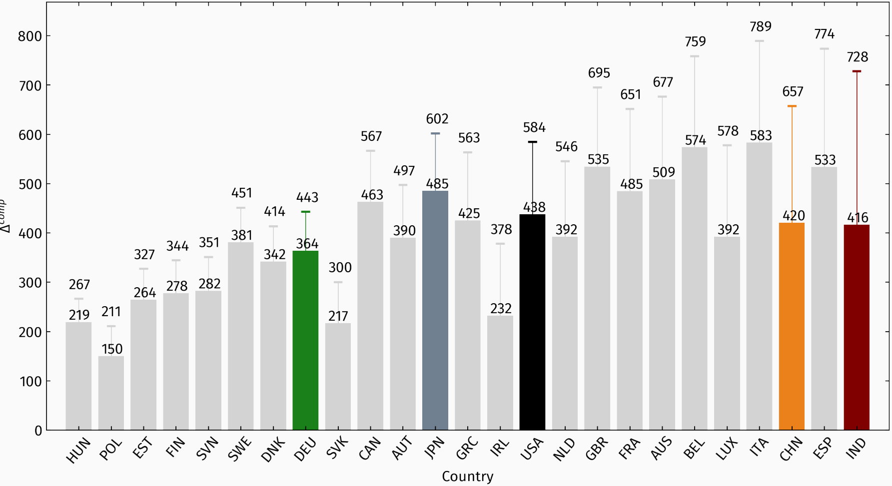
Low and high fertility scenarios

Back



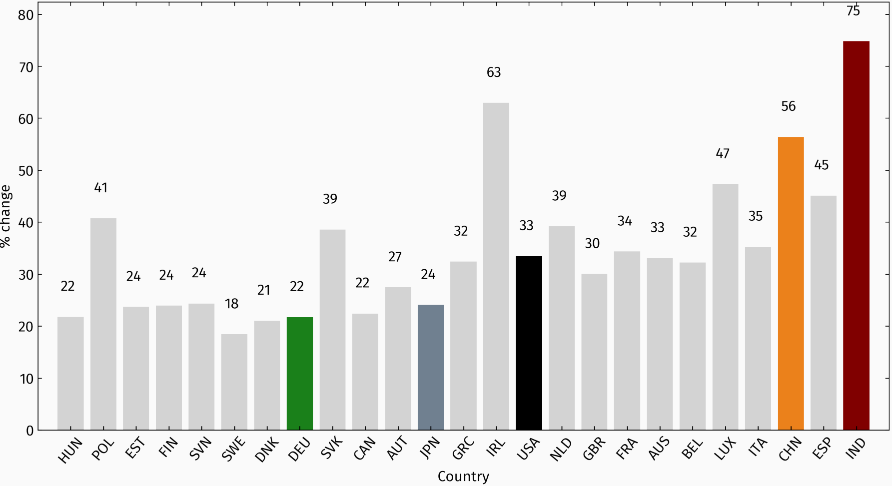
W/Y from shift-share in 2016 and in 2100

Back



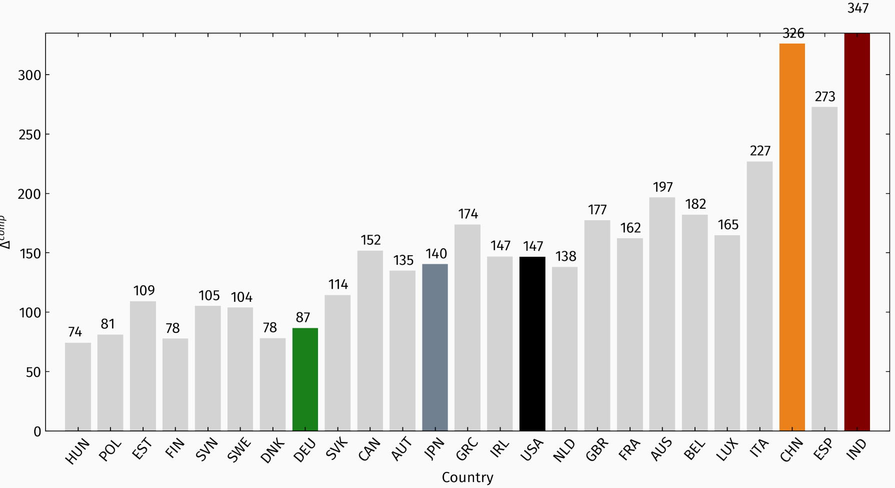
Percentage change in W/Y from shift-share

Back



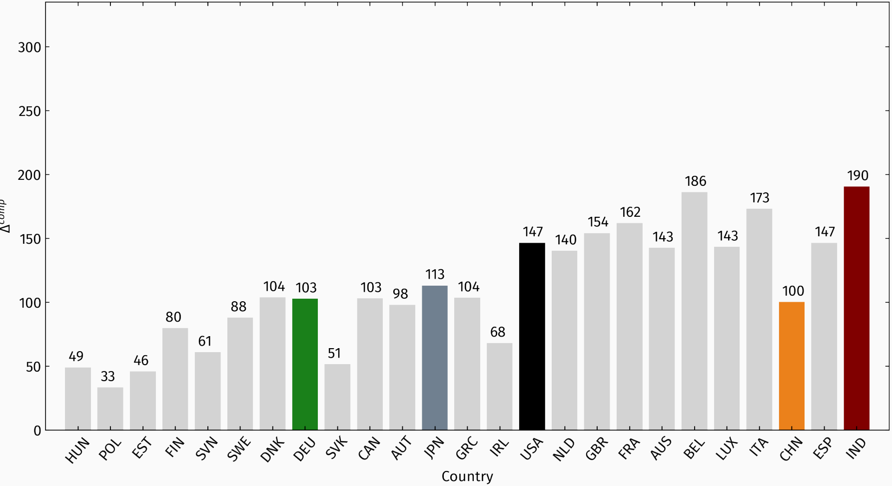
Shift-share at common age profiles (rescaled)

Back



Shift-share at common demographic change

Back



Environment: demographics

Back

- Population evolves as

$$N_{jt} = \left(N_{j-1,t-1} + M_{j-1,t-1}\right)\phi_{j-1,t-1}$$

where

  - N_{jt} denotes the numbers of individuals aged j in year t
  - $M_{j,t}$ is migration
  - $\phi_{j,t}$ are survival probabilities

- Total population is

$$N_t = \sum_j N_{jt}$$

- Population converges to a stationary distribution in the long run

◂ Back

Weight on children

- Let $c = c^P + nc^C$ be the total cons. of parent and children
- Assume flow utility function of a parent is

$$\mathcal{U}\left(c^P, c^C\right) = u\left(c^P\right) + \lambda n^{\varphi} u\left(c^C\right)$$

- Utility maximization implies:

$$u'\left(c^P\right) = \lambda n^{\varphi-1} u'\left(c^C\right)$$

$\Rightarrow$ total value of having children

$$W(c) = u\left(c^P\right) + \lambda n^{\varphi} u\left(c^C\right) = \left(1 + \lambda^{\frac{1}{\sigma}} n^{\frac{\sigma+\varphi-1}{\sigma}}\right)^{\sigma} u(c)$$

- Hence $\psi_i = \left(1 + \lambda^{\frac{1}{\sigma}} n_i^{\frac{\sigma+\varphi-1}{\sigma}}\right)^{\sigma}$
  - Children raise the m.u.c. if $\lambda > 0$ and $\varphi > 1 - \sigma$
  - n_i comes from empirical distribution of children for parent aged i

Retirement policy

◂ Back

- Retirement is phased at age T_t^r
- At age T_t^r, agents still work a fraction $\rho_t \in [0, 1]$ of total hours
- Retirement policy is therefore

$$\rho_{jt} = 1_{j<T_t^r} + \rho_t 1_{j=T_t^r}$$

- Effective labor supply is

$$L_t \equiv \sum_{j<T_t^r} \pi_{jt} \widetilde{h_{jt}} + \rho_t \pi_{T_t^r t} \widetilde{h_{T_t^r t}}$$

- Effective share of retirees is

$$\mu_t^{ret} \equiv (1 - \rho_t) \pi_{T_t^r t} + \sum_{j \geq T_t^r} \pi_{jt}$$

Government policy

Back

- Flow budget constraint

$$B_t + T_t = (1 + r_{t-1})\, B_{t-1} + G_t$$

where B_t is debt, G_t are expenditures, T_t are net taxes

$$T_t = w_t N_t \Big((\tau_t^{ss} + \tau_t (1 - \tau_t^{ss}))\, L_t - (1 - \tau_t)\, \bar{d}_t \mu_t^{ret} \Big)$$

- Government sets retirement policy $\{\rho_{jt}\}$ and follows fiscal rules

$$\begin{aligned} \tau_t^{ss} &= \overline{\tau}^{ss} + \varphi^{ss}(B_t/Y_t - \overline{b}) \\ \tau_t &= \overline{\tau} + \varphi^{\tau}(B_t/Y_t - \overline{b}) \\ \frac{G_t}{Y_t} &= \overline{\frac{G}{Y}} - \varphi^{G}(B_t/Y_t - \overline{b}) \\ \overline{d_t} &= \overline{d} - \varphi^{d}(B_t/Y_t - \overline{b}) \end{aligned}$$

where $\overline{b}$ is the 2016 debt-to-GDP ratio

- Coefficients φ's regulate the aggressiveness of the adjustment

Extension 1: other sources of asset supply

- In simple cases, alternative assets just add to supply

- Allow for
  - Markups μ, capitalized monopoly profits
  - Government bonds with long-run rule $\frac{B}{Y} = b(r)$

- Then

$$\frac{a(r,\theta)}{y(r)} = \frac{k(r)}{y(r)} + b(r) + \left(1 - \frac{1}{\mu}\right)\frac{1}{r - (n + \gamma)}$$

- θ directly affects both W and market cap. through discounting

- Extra terms on RHS affect elasticity of asset supply ϵ^{s}
  - Similar formula still determines dr

Extension 2: Housing

Back

- Model housing by introducing Cobb-Douglas utility

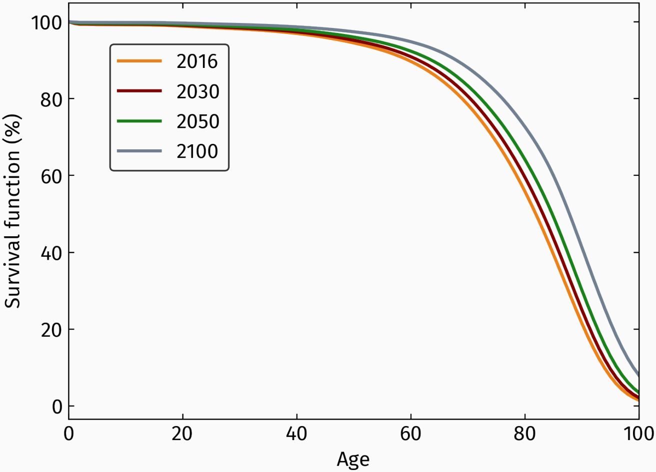
$$\frac{1}{1-\sigma}\left(c^{1-\alpha_h} h^{\alpha_h}\right)^{1-\sigma}$$

- All households rent to a REIT who owns
    - fixed supply of land L, equilibrium price P^L
    - stock of dwellings H, depreciating at δ^H, investment price $= 1$
    - $\beta = \frac{P^L L}{P^L L + H}$ is s.s. share of land
- Households invest in mutual fund that owns the REIT
- Housing supply in steady state adjusts so that

$$\frac{a(r,\theta)}{y(r)} = \frac{k(r)}{y(r)} + \frac{\alpha^h}{1-\alpha^h}\left(\frac{\beta}{r-(n+\gamma)} + \frac{1-\beta}{r+\delta^H}\right)\frac{\sum_i \pi_i(\theta)\frac{c_i(r,\theta)}{y(r)}}{\sum_i \pi_i(\theta) h_i}$$

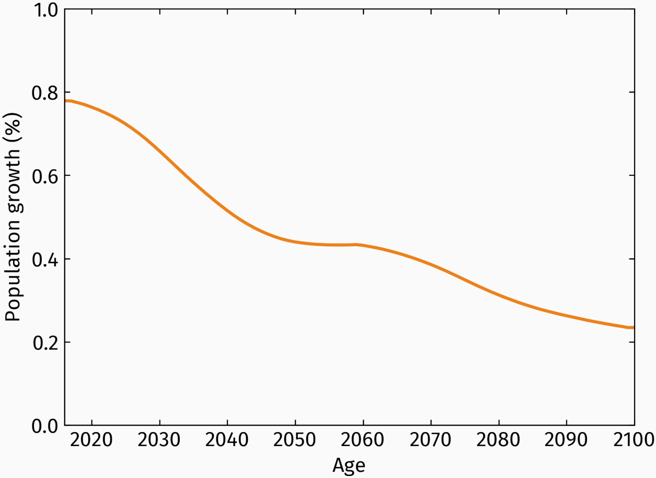
Projected survival functions

Back



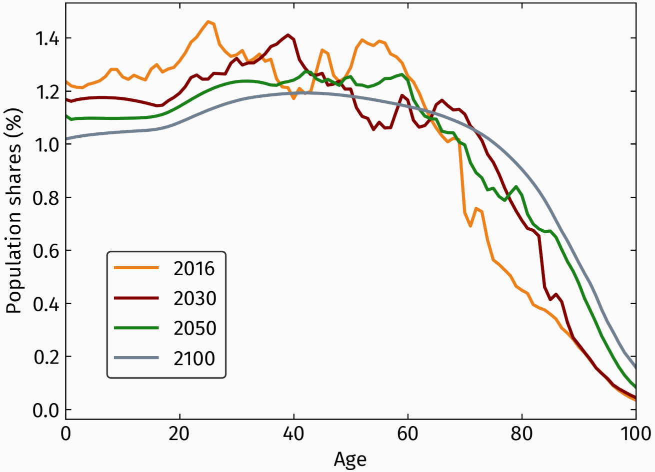
Projected population growth rate

Back



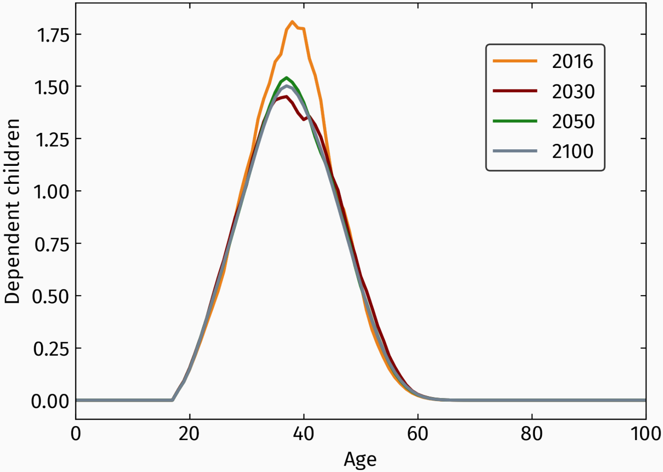
Projected population shares

Back



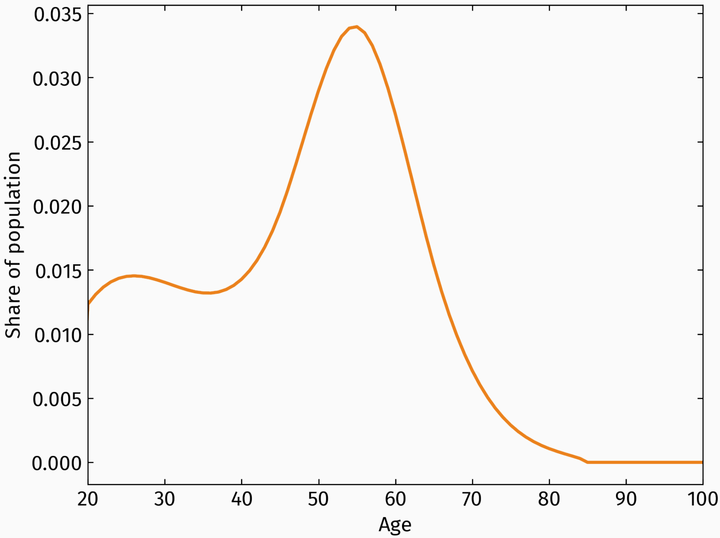
Distribution of children

Back



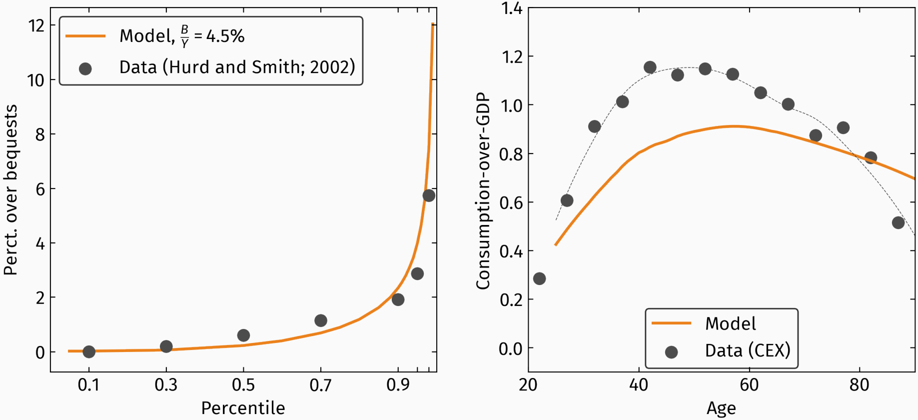
Distribution of bequests received

Back



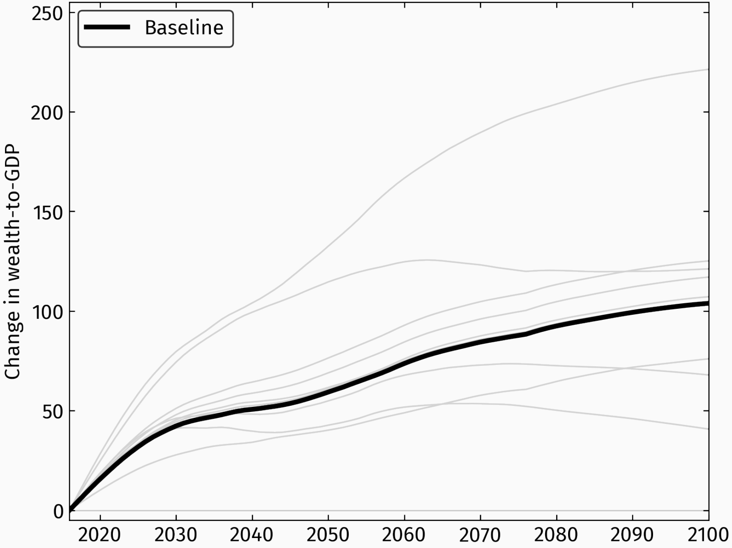
Bequests distribution and consumption profile

Back



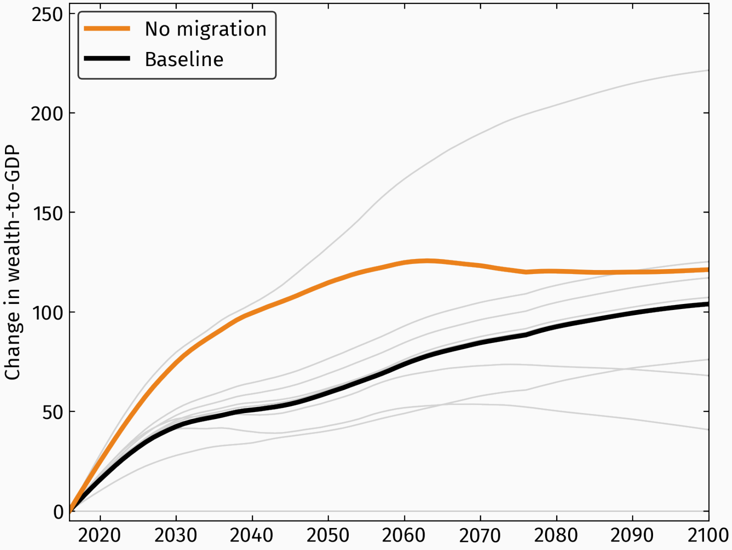
Robustness

Back

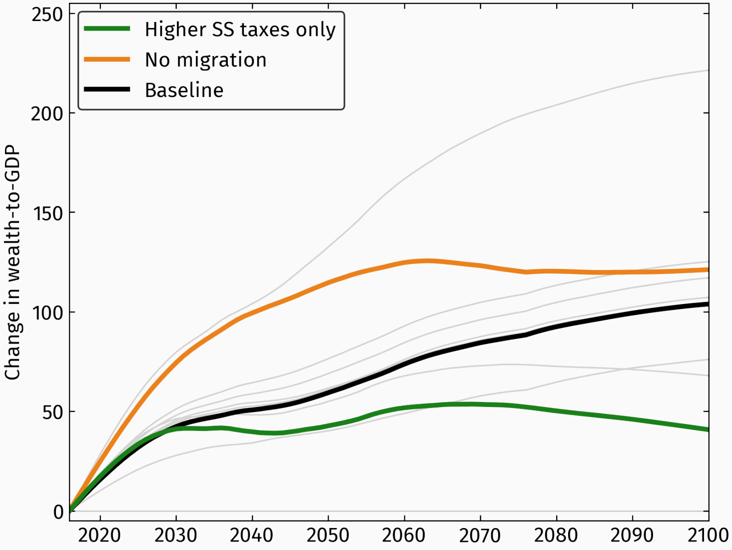


Robustness

Back

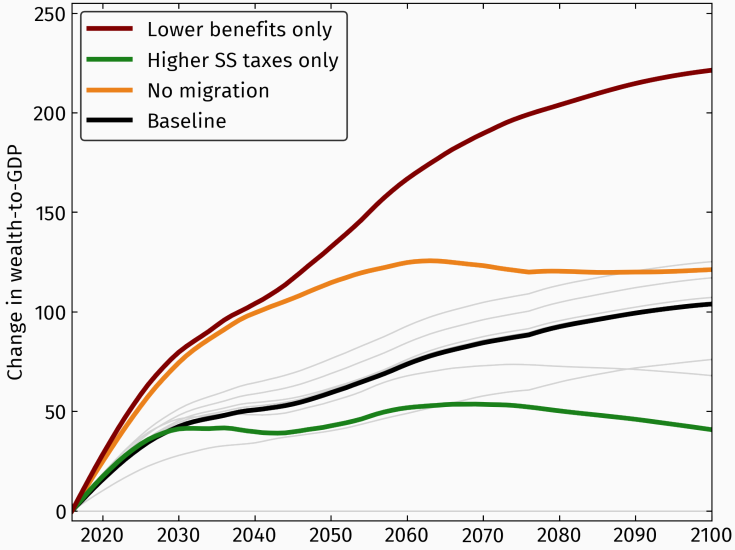


Robustness


Higher SS taxes only
No migration
Baseline
Change in wealth-to-GDP
0
50
100
150
200
250
2020
2030
2040
2050
2060
2070
2080
2090
2100

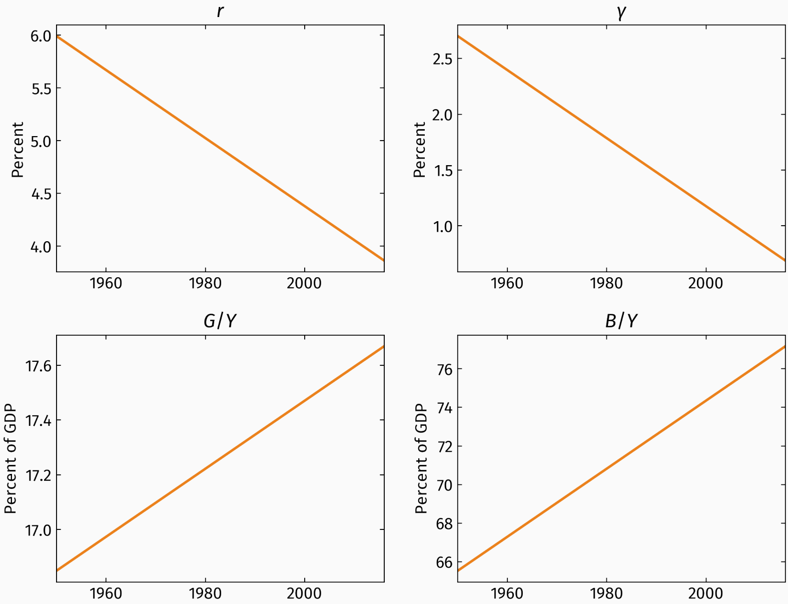
Robustness

Back



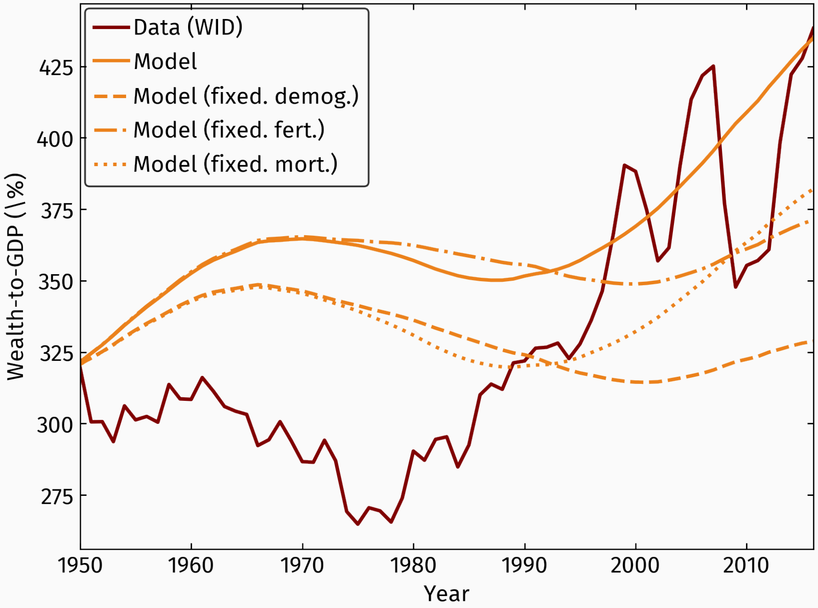
Historical exercise: inputs

Back



Historical exercise

Back

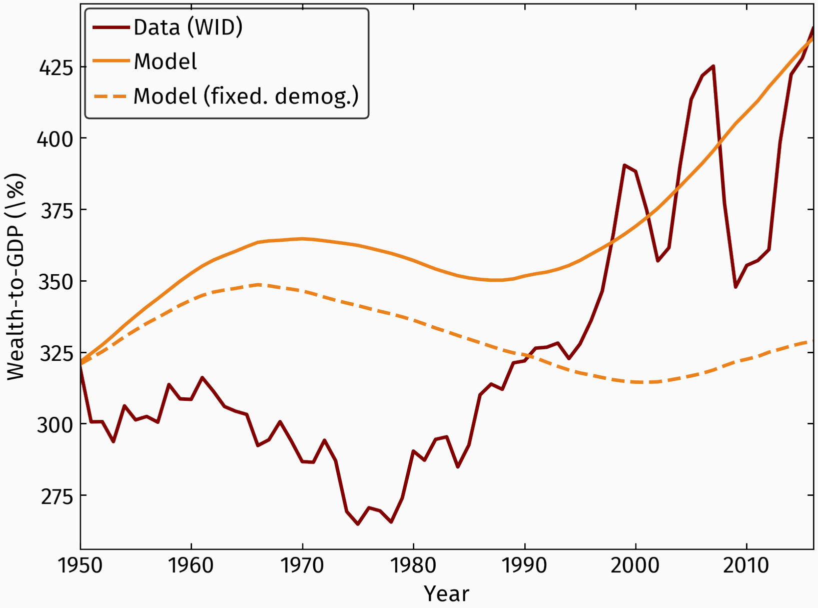


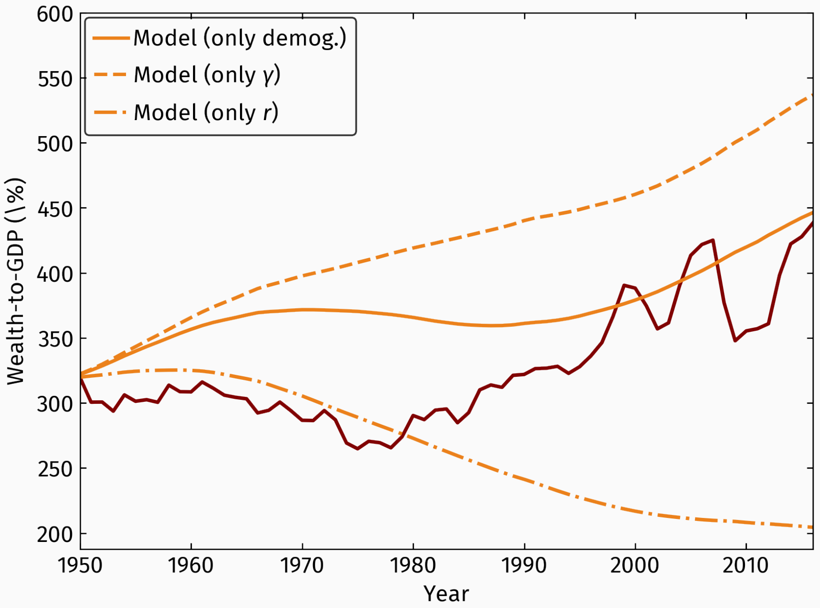
Historical trends in wealth

- We'll use our model primarily for prospective counterfactuals
- But: can the model account for trends in wealth since 1960?
- Concurrent developments to demographics over the period:
  - Falling real rates
  - Falling productivity growth
- We feed the model with observed trends in r, γ, B and G

Historical trends in wealth

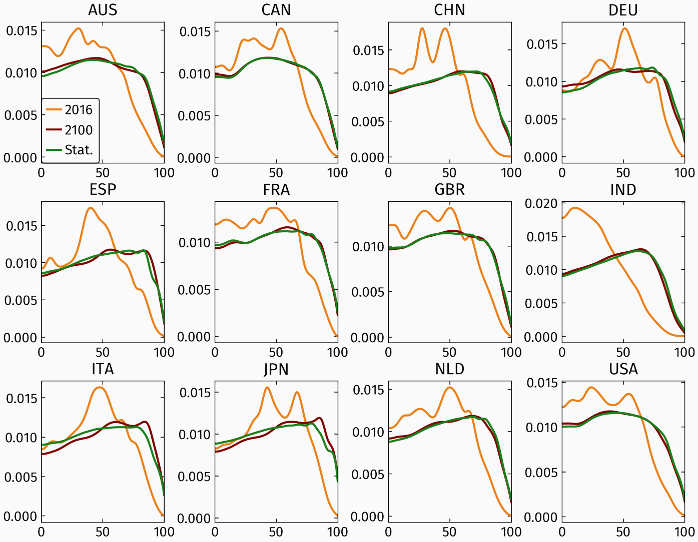
▸ Fert./Mort. ▸ Inputs ◂ Inputs




Model (only demog.)
Model (only γ)
Model (only r)
600
550
500
450
400
350
300
250
200
Wealth-to-GDP (\%)
1950
1960
1970
1980
1990
2000
2010
Year

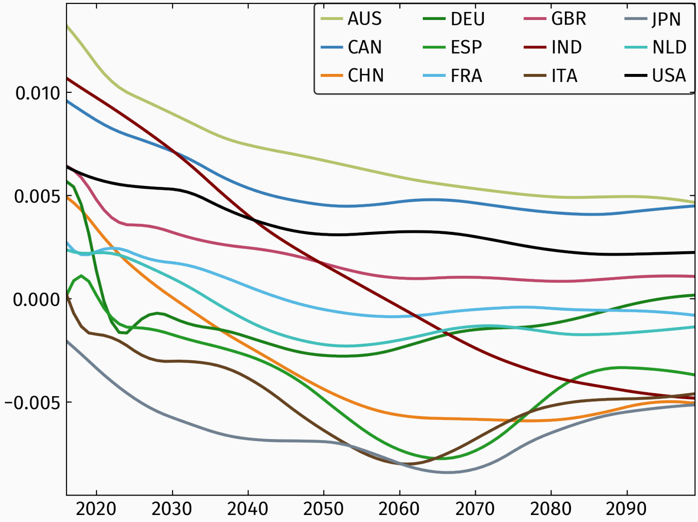
Demographics: population distributions

Back



Demographics: population growth rates

Back



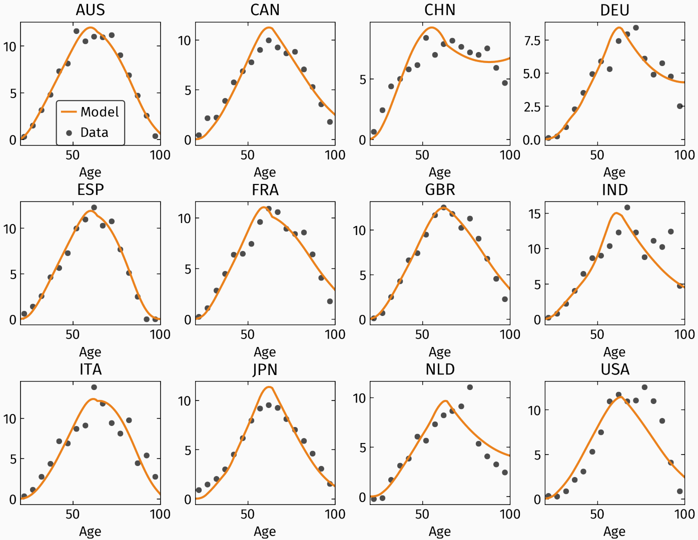
World economy calibration

Back

Country	Parameters β	Υ	$\frac{W}{Y}$ Model	Data	Δ^{comp} Model	Data
AUS	0.99	0.78	5.09	5.09	1.32	1.32
CAN	0.96	2.34	4.63	4.63	1.14	1.14
CHN	0.95	4.63	4.20	4.20	2.81	2.81
DEU	0.95	3.41	3.64	3.64	0.89	0.89
ESP	1.00	0.00	5.33	5.33	1.64	1.55
FRA	0.98	1.68	4.85	4.85	1.31	1.31
GBR	0.97	2.15	5.35	5.35	1.49	1.49
IND	0.95	3.28	3.44	3.44	3.07	3.07
ITA	1.00	0.61	5.83	5.83	1.77	1.77
JPN	0.96	1.68	4.85	4.85	0.82	0.82
NLD	0.95	3.93	3.92	3.92	1.23	1.23
USA	0.97	1.82	4.38	4.38	1.13	1.13

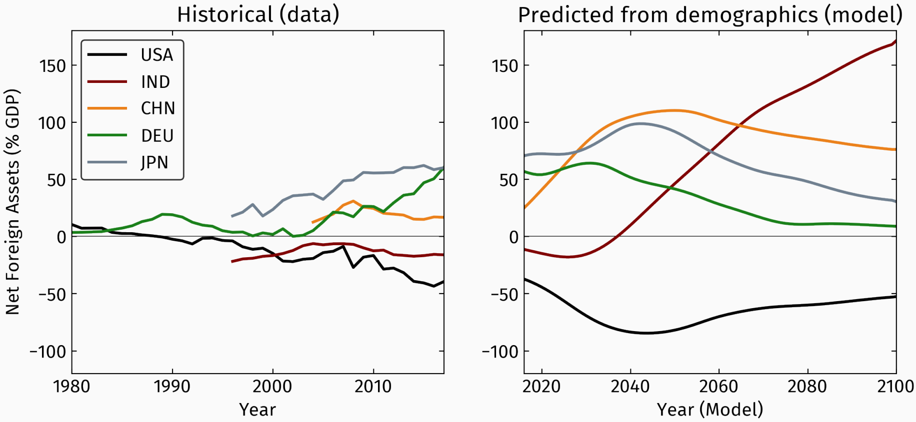
World economy calibration

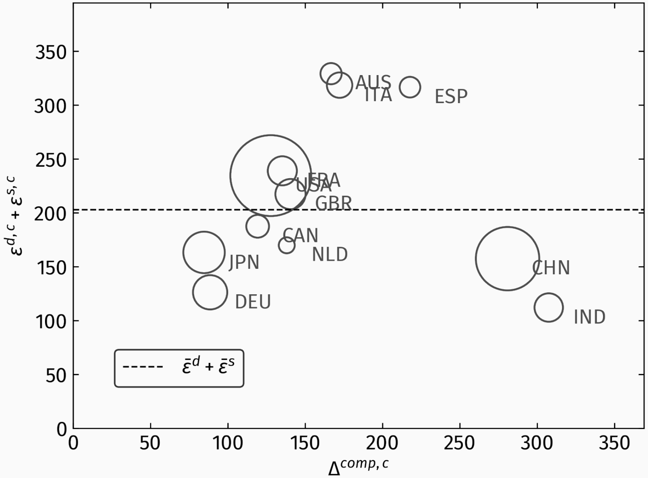
Back



Predicted NFA/Y from demographics

Back





350
300
250
200
150
100
50
0
0
50
100
150
200
250
300
350
$\varepsilon^{d,c} + \varepsilon^{s,c}$
$\Delta^{comp,c}$
AUS
ITA
ESP
FRA
USA
GBR
CAN
NLD
JPN
DEU
CHN
IND
$\bar{\varepsilon}^d + \bar{\varepsilon}^s$

Jakobsen et al. (2020) validation

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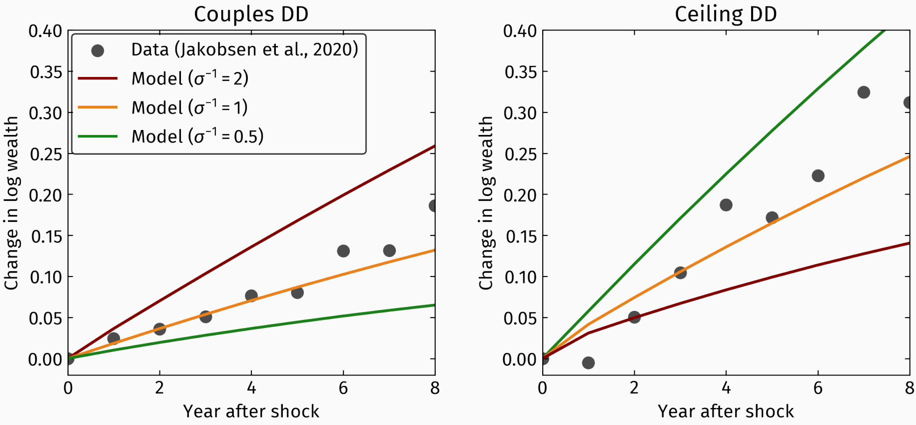

Note: Response of wealth to a reduction in the wealth tax. We replicate the model experiments of Jakobsen et al. (2020). The first (Couples DD) analyzes a reduction of the wealth tax from 2.2% to 1.2% on the top 1%. The second (Ceiling DD) analyzes the a reduction of 1.56 percentage points on the top 0.3%.

Sensitivities by country

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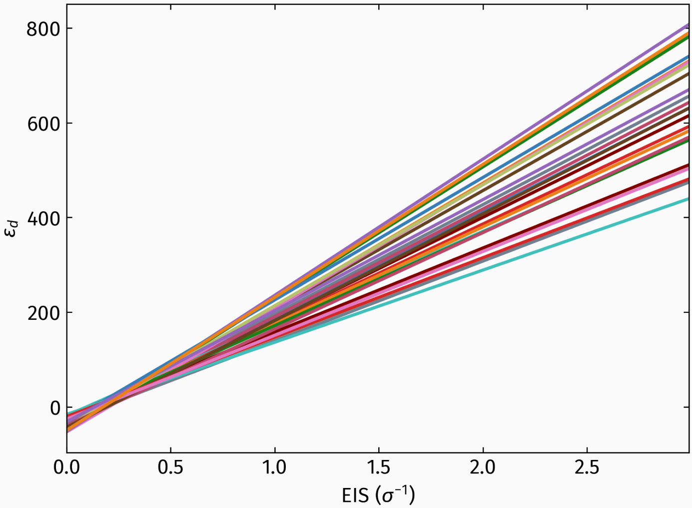

Construction of ϵ^d in the USA

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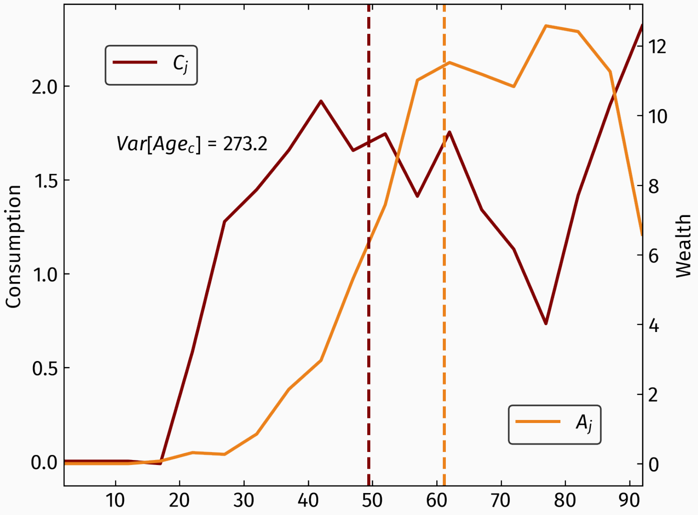

Construction of ϵ^d in the USA

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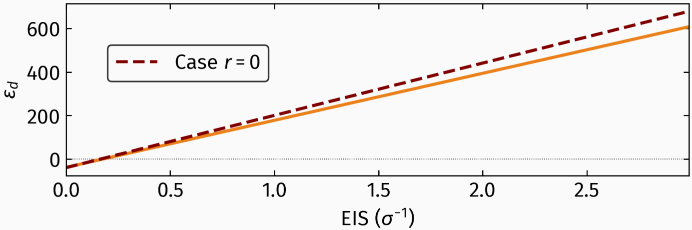

Change in NFA

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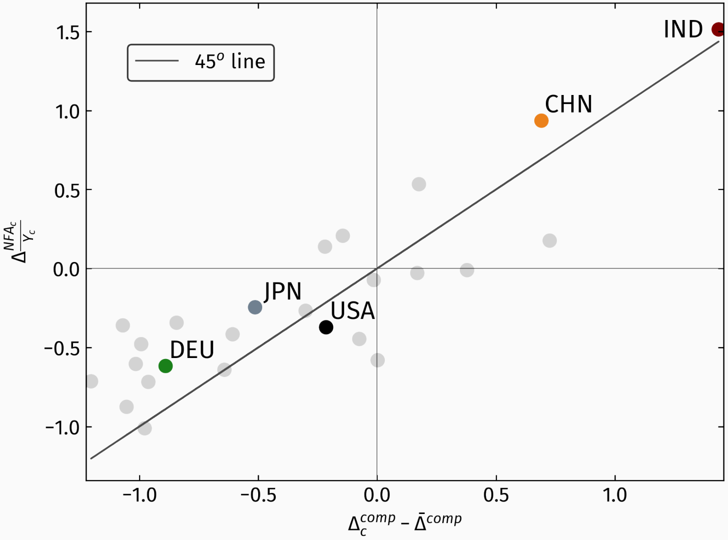

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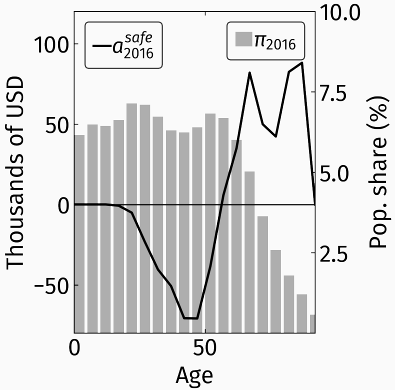
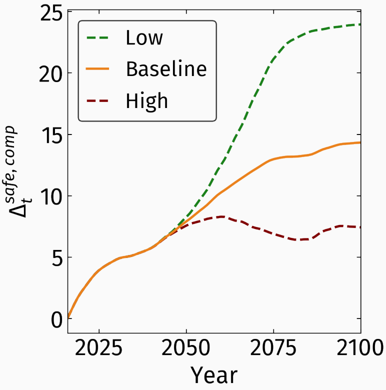
A. Net safety demand



B. Compositional effect



Multiple assets (2/2)

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