



**On the Importance of Household versus Firm Credit Frictions in the  
Great Recession \***

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Abstract:

Although a credit tightening is commonly recognized as a key determinant of the Great Recession, to date, it is unclear whether a worsening of credit conditions faced by households or by firms was most responsible for the downturn. Some studies have suggested that the household-side credit channel is quantitatively the most important one. Many others contend that the firm-side channel played a crucial role. We propose a model in which both channels are present and explicitly formalized. Our analysis indicates that the household-side credit channel is quantitatively more relevant than the firm-side credit channel. We then evaluate the relative benefits of a fixed-sized transfer to households and to firms that improves each group's access to credit. We find that the effects of such a transfer on employment are substantially larger when the transfer targets households rather than firms. Hence, we provide theoretical and quantitative support to the view that the employment decline during the Great Recession would have been less severe if instead of focusing on easing firms' access to credit, the government had expended an equal amount of resources to alleviate households' credit constraints.

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# 1. Introduction

A large body of research on the Great Recession in the United States supports the view that a tightening of credit was a crucial factor behind the downturn. This research can be grouped into work that emphasizes the *household-side channel*, which argues that a tightening of credit to households was most responsible for the observed decline in output and employment, and work that emphasizes instead the alternative *firm-side channel*, which argues that a tightening of credit to firms mostly accounted for the fall in output and employment.

In terms of evidence on the household-side channel, Mian and Sufi (2011, 2014) have documented that in the United States, regions that experienced the largest declines in household debt and housing prices also experienced the largest contractions in consumption and employment. These authors have also shown that when housing prices declined, households decreased their spending, in part because of their reduced ability to borrow against home equity. Specifically, Mian and Sufi (2014, 2015) estimated that the household-side channel accounts for approximately 65 percent of the job losses between 2007 and 2009.

As for evidence on the firm-side channel, various work has documented that a deterioration in firms' balance sheets, often emanating from a decline in lending from local banks, was an important component of the downturn. For example, Chodorow-Reich (2014), Giroud and Mueller (2017), and Huber (2018) have found that employment fell the most in firms that faced the worst credit conditions, which is consistent with the view that a tightening of credit to firms played a significant role in the transmission of credit shocks to the economy. Jermann and Quadrini (2012) have provided additional aggregate evidence that links the downturn in output during the Great Recession to firms' worsening ability to borrow in 2008 and 2009. For an early important paper on the role of firm financial frictions in amplifying the macroeconomic effects of aggregate shocks, see Cooley, Marimon, and Quadrini (2004). See also Gertler and Gilchrist (2018, 2019) and Aikman et al. (2019) for surveys of the evidence on the firm-side channel and models that have addressed it.

To date, however, the role and relative importance of these channels are debated. In this paper, we propose a model in which the household-side and firm-side channels of credit frictions can be explicitly analyzed and clearly isolated. We also investigate the impact of transfers targeted to households and to firms on the recovery of aggregate employment after a recession induced by a credit contraction that affects households, firms, or both groups. In so doing, we help remedy the relative scarcity of explicit quantitative modelling of government transfers in business-cycle research on economies with heterogeneous agents, in which even lump-sum transfers can have a large impact on output, consumption, and employment.

Our first main finding is that for the same-sized credit shock to households and firms, the household-side credit channel is quantitatively much more important than the firm-side credit channel. Our second main finding concerns the effectiveness of some of the key policies that the U.S. government implemented to counteract the deterioration of credit conditions during the Great Recession. Based on the view that providing credit to firms would have been the most effective way to boost investment, hiring, and production, several policies, such as many of those promoted as part of the Troubled Asset Relief Program, were designed to improve firms' financial conditions by directly providing them with credit. We find that, on the contrary, directing credit to easing households' debt constraints could have led to a faster recovery of employment.

Several recent empirical studies motivate us to consider targeted government transfers in downturns. Oh and Reis (2012), for example, have argued that although the vast majority of research on the fiscal response to the Great Recession has focused on government purchases, the actual response consisted mostly of targeted transfers. In particular, these authors document that from the end of 2007 to the end of 2009, only one-quarter of the increase in U.S. government expenditures was accounted for by government purchases, whereas the remaining three-quarters were accounted for by transfers. Similarly, Cogan and Taylor (2012) have provided a detailed analysis of the fiscal expenditure from the American Economic Recovery Act and showed that only a small percentage was directed at government purchases, as the vast majority took the form of transfers. Oh and Reis (2012) have shown that a similar pattern holds across OECD countries between 2007 and 2009. As for transfers specifically targeted to firms, often referred to as "bailouts," see Bianchi (2016) for a discussion of the evidence on them, the relevant literature on their impact, and a quantitative evaluation of their effects in financial-type downturns like the Great Recession.

To examine the impact of targeted government transfers in recessions, we propose a model that builds on Kehoe, Midrigan, and Pastorino (2019), who have developed a version of the Diamond-Mortensen-Pissarides (*DMP*, henceforth) model of labor market search in the presence of risk-averse consumers and human capital accumulation. In their model, hiring workers is an investment activity that requires costs to be paid up front and entails benefits that accrue gradually. Thus, when credit tightens, this investment activity naturally falls and employment declines.

Although this force is present in any dynamic model, the employment drop caused by tighter credit is negligible in the textbook search model, which features no human capital accumulation. As Kehoe, Midrigan, and Pastorino (2019) show, when workers instead acquire human capital during employment, the duration of the flows of benefits from a match between a firm and a worker is much longer than in the absence of human capital acquisition. Intuitively, by acquiring skills that

are at least partially transferable across matches, workers' benefits from matching with a firm are long lasting, as the new productive skills accumulated over the course of a match have a persistent effect on a worker's output beyond the course of the match. As a result, the value of a match is highly sensitive to credit conditions, and so the fall in employment associated with a tightening of credit is greatly amplified relative to that predicted by the textbook search model.

Formally, the magnitude of the employment decline in a recession implied by a typical search model is closely related to the sensitivity of the present value of the returns from a firm's investment in hiring to changes in discount rates. In particular, the longer the duration of surplus flows is, the higher the sensitivity of the present value of firms' returns is to discount rate changes, and so the larger the drop in employment is in response to any given increase in discount rates. Consequently, in a model with human capital acquisition, a tightening of credit leads to a much larger decline in employment than in the textbook search model.

A limitation of Kehoe, Midrigan, and Pastorino (2019), however, is that credit frictions are modelled as having symmetric effects on workers and firms. Hence, their framework is not amenable to exploring the separate channels through which credit market frictions affect an economy during a downturn, which we focus on here. We formalize these channels by considering an economy in which consumers belong to one of two types of families: families of workers who search for jobs, receive wages from supplying labor, and engage in home production, referred to as either *workers* or *households*; and families of entrepreneurs who hire workers and own firms, referred to as either *entrepreneurs* or *firms*. Since each type of family faces a separate borrowing constraint, this framework allows us to investigate the potentially different implications of a tightening of credit to workers and to entrepreneurs.

To examine the relative importance of household-side and firm-side credit frictions in accounting for the decline in consumption and employment after a credit tightening, we first consider a selective tightening of access to credit by workers and firms, and then a common tightening. Note that, because of general equilibrium effects, tighter credit for either group leads to lower consumption for both groups in equilibrium. Hence, both groups place greater value on current consumption relative to future consumption. Correspondingly, a tightening of either group's credit constraints induces an increase in the cost of diverting current resources from consumption to investment in hiring, relative to the future gains from increasing the number of matches between firms and workers. As a result, vacancies and employment fall.

The two types of tightening we consider, however, have different effects precisely because a tightening of household credit constraints mostly impacts *workers'* value of current goods, whereas

a tightening of firm credit constraints mostly impacts *entrepreneurs'* value of current goods. To see why this difference matters, note that for a firm, the value of a match with a worker accrues in the form of the profits earned over the course of the match. Therefore, once a match dissolves, a firm derives no more value from it. For a worker, on the contrary, the value of a match with a firm consists not only of the value of the wages received over its course but also of the incremental value of the wages received from all future matches, because of the increase in the worker's human capital occurring during the match.

Formally, we decompose the surplus from a match into two pieces: the *current output gains* that accrue over the course of the match and the *future output gains* that arise as the human capital accumulated during the match increases a worker's output in all future periods. In terms of the current output gains, workers' and firms' discount rates symmetrically affect match surplus. In contrast, the future output gains are discounted solely by workers' discount rates. Since the future output gains are large and persistent and thus are sensitive to changes in discounting, match surplus is much more sensitive to fluctuations in workers' marginal valuations of future relative to present goods, and so workers' discount rates, than to firms' marginal valuations.

In a decentralized equilibrium, these distinct forces behind the impact of the two types of tightening manifest themselves in different responses of the present value of wages relative to the present value of output. After a worker-side tightening, the present value of wages falls significantly less than the present value of output, whereas after a firm-side tightening, the present value of wages falls nearly as much as the present value of output. In this sense, after a tightening of household credit constraints, the present value of wages is relatively stickier than after a tightening of firm credit constraints. This differential rigidity of wages helps account for how the fall in employment after a household-side credit tightening is larger than after a firm-side credit tightening leading to equal-sized drops in household and firm discount factors.

We then turn to evaluate the relative benefits of easing credit frictions faced by households and firms. Our model implies that if the government intervenes by providing lump-sum transfers to households in response to a credit tightening affecting either households or firms, it stimulates employment substantially more than by providing the same-sized transfers to firms. The key intuition behind this result is similar to that for our first main finding. A transfer to workers has a large direct effect on workers' valuations of the associated benefits, which, as argued, determine the key discount rate of the surplus from a match between a firm and a worker. On the other hand, a transfer to firms has only a small and indirect general equilibrium effect on such valuations. Since workers' valuations of the benefits of a transfer have a larger impact on the surplus from a match

than do firms' valuations, a transfer to firms has a much smaller effect on aggregate employment than a transfer to workers.

For clarity, we emphasize that throughout we consider intertemporal transfers from the future to the present financed by government debt and eventually repaid by higher future taxes, rather than by within-period transfers to one group financed by contemporaneous higher taxes on another group. We interpret such intertemporal transfers, which are considered part of current government expenditure but not government purchases, as capturing the essential component of government expenditures that the empirical literature discussed earlier has measured.

We remark also that our results are not due to different marginal propensities to spend by households and firms. Indeed, in our model, both sets of families have a marginal propensity to spend of one.<sup>1</sup> Rather, our results depend on the relative importance of the two groups' discount factors in determining the value of the surplus from a match between a firm and a worker.

Next, we extend the model by considering an economy composed of many states, each modelled as a small open economy with tradable and nontradable goods. In this economy, meant to represent the United States, a state-specific credit tightening has two effects. The first is an *investment effect*, in that to a firm in a given state, the cost of posting a job vacancy to hire a worker increases in a downturn by more than the corresponding benefits, which leads to a reduction in the number of vacancies posted and hence to a drop in overall employment in that state. The second is a *relative demand effect* and is due to the reduction in the demand for the nontradable goods produced in the state. Such a drop in turn gives rise to a decline in the demand for workers by the nontradable goods sector, which induces workers to reallocate to the tradable goods sector. When the cost of worker reallocation is small, a substantial reallocation occurs between the two sectors. As a result, states that experience larger credit tightenings also experience larger declines in nontradable employment but not necessarily in tradable employment, as is consistent with the data.

We then assess the relative desirability of transfers to workers and to firms in response to a credit tightening that impacts both workers' and firms' credit constraints. Specifically, we investigate our model's predictions for U.S. states during the Great Recession for two policies: one that implements transfers only to workers and one that implements equally-sized transfers only to firms. As before, transfers are modelled as partially relaxing the tightened credit constraint that limits the ability of either group to borrow.

Our second main finding is that after a credit tightening, a transfer to workers has substantially

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<sup>1</sup>Specifically, in our setup, workers spend all additional income on consumption goods, whereas entrepreneurs spend all additional income on a combination of consumption and investment (job vacancies) goods.

larger positive effects on employment than does a transfer to firms. For example, for Nevada, a state that experienced an especially large decline in consumption and employment during the Great Recession, a lump-sum credit subsidy to firms that leads to a fall in employment between 2007 and 2009 that is 20 percent smaller than the observed one would lead instead to a 50 percent smaller fall if that same amount was transferred to workers. In sum, our model provides quantitative support for the view that the Great Recession would have been less severe if instead of focusing on easing credit to firms, the government had rather focused on easing credit to households.

We end with an important cautionary note. Throughout the paper, we studiously avoid formulating any statements about the welfare consequences of the transfers we consider. The reason is that we have endowed the government with policy instruments—namely, targeted lump-sum transfers—that can partially or totally counter the tightening of credit constraints, which for simplicity we have assumed as exogenous and invariant to such interventions. To be able to evaluate the welfare implications of these interventions, we would need to formalize from deeper first principles a government’s ability to easily offset constraints on the private sector that arise presumably from underlying information frictions or contracting imperfections. Therefore, we interpret our exercise as simply highlighting the importance of accounting for alternative transmission channels of financial shocks when evaluating government interventions during recessions.

## 2. Baseline Model

We consider a one-good small open economy in which labor markets are subject to frictions to the matching between firms and workers. The economy consists of a continuum of consumers who belong to either a family of workers or a family of entrepreneurs; these are often referred to as simply *households* and *firms*, respectively. We think of entrepreneurs as combining the roles of firms and banks. To hire workers, entrepreneurs post job vacancies in markets indexed by a worker’s general human capital. When employed, workers accumulate new general and firm-specific human capital.

Consumers survive each period to the next with probability  $\phi$  and are otherwise replaced by an identical measure of newborns, so that the measure of consumers of each family is constant over time and equal to one. Both families of workers and entrepreneurs insure their respective members against idiosyncratic risk, but no such insurance arrangement exists across the two types of families. Each family is subject to time-varying debt constraints. As we show in the Appendix, this formulation of the economy with debt constraints has implications for the consumption of the two groups of families, output, employment, and wages that are identical to those of a richer formulation in which debt is collateralized by housing.



In this setup, after a differential tightening of the two families' debt constraints, workers and entrepreneurs can differ in their intertemporal rates of substitution of consumption. We exploit this difference to isolate the distinct roles of debt constraints on households and firms in amplifying and propagating credit shocks as well as to examine the effect on labor market outcomes of credit assistance policies alternatively directed to households and firms. In what follows, we first consider the differential implications of credit shocks affecting the two types of families, modelled as debt tightenings, on aggregate employment. We then study the impact of a government subsidy that selectively relaxes the debt tightening faced by households and firms.

## A. Human Capital

Workers are indexed by two state variables that summarize their ability to produce output. The variable  $z_t$ , referred to as *general human capital*, captures returns to labor market experience that accrue even after a match between a firm and a worker ends. The variable  $h_t$ , referred to as *firm-specific human capital*, captures returns to tenure in a match that are lost after a match ends. A worker with state variables  $(z_t, h_t)$  produces  $z_t h_t$  when employed and  $b(z_t)$  when nonemployed. When a worker is employed, general human capital evolves according to

$$(1) \quad \log(z_{t+1}) = (1 - \rho) \log(\bar{z}_e) + \rho \log(z_t) + \sigma_z \varepsilon_{t+1};$$

whereas when a worker is not employed, it evolves according to

$$(2) \quad \log(z_{t+1}) = (1 - \rho) \log(\bar{z}_u) + \rho \log(z_t) + \sigma_z \varepsilon_{t+1},$$

where  $\varepsilon_{t+1}$  is a standard Normal random variable. We assume that  $\bar{z}_u < \bar{z}_e$ . Newborn workers start as nonemployed with general human capital  $z$  with  $\log(z)$  drawn from the distribution  $N(\log(\bar{z}_u), \sigma_z^2 / (1 - \rho^2))$ . This specification of the human capital process is in the spirit of that in Ljungqvist and Sargent (1998) and is the same specification as in Kehoe, Midrigan, and Pastorino (2019). We denote the Markov processes in (1) and (2) by  $F_e(z_{t+1}|z_t)$  and  $F_u(z_{t+1}|z_t)$  in what follows. A worker's firm-specific human capital  $h_t$  equals 1 whenever a match begins; evolves on the job according to

$$(3) \quad \log(h_{t+1}) = (1 - \rho) \log(\bar{h}) + \rho \log h_t$$

with  $\bar{h} > 1$ ; and resets to 1 when a match ends.

The assumption that  $\bar{z}_u < \bar{z}_e$  implies that when a worker is employed, on average, general

human capital  $z_t$  drifts up toward the level of productivity  $\bar{z}_e$  from the level of productivity  $\bar{z}_u$  of newborn workers, which we normalize to 1. Similarly, when a worker is not employed, on average, general human capital  $z_t$  depreciates and hence drifts down toward the level of productivity  $\bar{z}_u$ . The assumption that  $\bar{h} > 1$  implies that when a worker is employed, firm-specific human capital increases over time from  $h = 1$  toward  $\bar{h}$ . The parameter  $\rho$  governs the rate at which general and firm-specific human capital converge toward their means. The higher  $\rho$  is, the slower both types of human capital accumulate during employment, and the slower general human capital depreciates during nonemployment. For simplicity, we assume that these rates are the same for the three laws of motion described. Note that allowing for idiosyncratic shocks  $\varepsilon_{t+1}$  to general human capital allows the model to reproduce the dispersion in the growth rate of wages observed in the data. For simplicity only, we assume that the process of firm-specific human capital accumulation is deterministic.

We use  $e_t(z, h)$  to denote the measure of employed workers with general human capital  $z$  and firm-specific human capital  $h$  and  $u_t(z)$  to denote the measure of nonemployed workers with general human capital  $z$ .

### 3. Insurance

We represent the insurance arrangements in the economy by assuming that each consumer belongs to either one of a large number of identical families of workers or one of a large number of identical families of entrepreneurs, each of which consists of a continuum of members. By the law of large numbers, in each period, each family of workers as a whole receives a deterministic amount of income generated by its working and nonworking members. Likewise, in each period, each family of entrepreneurs receives a deterministic claim to the profits of all firms that it owns, net of vacancy posting costs, and a deterministic endowment. This endowment allows the model to reproduce the observed labor and profit shares of income in the data, as explained below.

Risk sharing within each type of family implies that at any date  $t$ , each consumer in a worker family consumes the same amount of goods  $c_{wt}$ , regardless of the idiosyncratic shocks that such a consumer experiences. Similarly, each consumer in an entrepreneur family consumes the same amount of goods  $c_{ft}$ , regardless of the idiosyncratic shocks that the firms owned by the consumer's family experience. Worker families and entrepreneur families are subject to distinct debt constraints.

Given this setup, we can separate the problem of a worker family into two parts. The first part determines the common consumption level of each member of the family, and the second part determines the employment and nonemployment status of each member. We can also separate the problem of an entrepreneur family into two parts. The first part determines the common

consumption level of each member of the family, and the second part determines the vacancies created, the matches formed, and the matches continued by each firm owned by the family.

### A. A Family of Workers' Problem

A family of workers has access to risk-free debt but faces constraints to borrowing. Any such family chooses a sequence for consumption  $\{c_{wt}\}$  and debt  $\{d_{wt+1}\}$  in order to maximize the present discounted value of consumption  $\sum_{t=0}^{\infty} \beta^t u(c_{wt})$ , subject to the budget constraint

$$(4) \quad c_{wt} + d_{wt} \leq \int_{z,h} w_t(z, h) e_t(z, h) dz dh + \int_z b(z) u_t(z) dz + q d_{wt+1} + T_{wt},$$

where  $w_t(z, h)$  is the wage at  $t$  of an employed worker with general human capital  $z$  and firm-specific human capital  $h$ , and  $b(z)$  is the amount of home-produced goods of a nonemployed worker with general human capital  $z$ , and subject to the borrowing constraint

$$(5) \quad d_{wt+1} \leq \chi_{wt}.$$

A family takes as given the total income from the wages of its employed members, which is the first term on the right side of (4); the total income from the home production of its nonemployed members, which is the second term on the right side of (4); and government transfers,  $T_{wt}$ . Let

$$(6) \quad y_{wt} \equiv \int_{z,h} w_t(z, h) e_t(z, h) dz dh + \int_z b(z) u_t(z) dz + T_{wt}$$

denote the total income of a family of workers. A family saves and borrows at a constant world interest rate or, equivalently, at a constant world bond price  $q > \beta$ , subject to an exogenous deterministic sequence  $\{\chi_{wt}\}$  of borrowing limits. Since workers discount the future at a higher rate than that implicit in the bond price  $q$ , their borrowing limits will bind in equilibrium.

The first-order conditions for this problem imply that the intertemporal rate of substitution of consumption for a family of workers is

$$(7) \quad \beta^t u'(c_{wt}) / u'(c_{w0}) = Q_{wt},$$

the Euler equation for borrowing is

$$(8) \quad q Q_{wt} = Q_{wt+1} + Q_{wt} \theta_{wt},$$

and the complementary slackness condition is

$$(9) \quad \theta_{wt}(d_{wt+1} - \chi_{wt}) = 0$$

at each  $t$ , where  $Q_{wt}$  is the multiplier on a family's budget constraint,  $Q_{wt}\theta_{wt}$  is the multiplier on a family's borrowing constraint, and we have normalized  $Q_{w0}$  to one so that  $Q_{wt}$  is workers' shadow value of date- $t$  goods in units of date-0 goods.

A tightening of debt constraints implies that a family of workers must temporarily reduce its consumption in order to repay some of its debt. Such a tightening leads to a larger drop in consumption at  $t$  than at  $t + 1$ , which in turn induces a larger increase in the shadow price of goods at  $t$ ,  $Q_{wt}$ , than in the shadow price of goods at  $t + 1$ ,  $Q_{wt+1}$ . Hence, the value of future goods relative to present goods,

$$(10) \quad \frac{Q_{wt+1}}{Q_{wt}} = \frac{\beta u'(c_{wt+1})}{u'(c_{wt})},$$

decreases. In this sense, a debt tightening leads workers to discount future consumption more, relative to current consumption.

## B. A Family of Entrepreneurs' Problem

The problem of a family of entrepreneurs is symmetric to that of a family of workers. A family of entrepreneurs has access to risk-free debt, faces constraints to borrowing, and chooses a sequence for consumption  $\{c_{ft}\}$  and debt  $\{d_{ft+1}\}$  in order to maximize the present discounted value of consumption  $\sum_{t=0}^{\infty} \beta^t u(c_{ft})$ , subject to the budget constraint

$$(11) \quad c_{ft} + d_{ft} \leq y + \int_{z,h} [zh - w_t(z, h)] e_t(z, h) dz dh - \int_z \kappa v_t(z) dz + qd_{ft+1} + T_{ft}$$

and the borrowing constraint

$$(12) \quad d_{ft+1} \leq \chi_{ft}.$$

The period profits of a family equal the total revenues of the firms it owns,  $\int_{z,h} zhe_t(z, h) dz dh$ , minus the total wage bill of workers employed by those firms,  $\int_{z,h} w_t(z, h)e_t(z, h) dz dh$ , and the costs of posting job vacancies,  $\int_z \kappa v_t(z) dz$ . Since we will maintain free entry in the labor market, the present value of posting a vacancy is zero when evaluated at the shadow discount factor of entrepreneurs. Hence, in an Arrow-Debreu sense, the value of the right to post vacancies is zero. Through the

endowment  $y$ , then, we capture, in a reduced-form way, all of the returns to being an entrepreneur in addition to the profits from hiring workers net of vacancy costs. For simplicity only, we assume that the endowment  $y$  is constant over time. Lastly,  $T_{ft}$  are government transfers to entrepreneurs. We denote the total income of a family of entrepreneurs by

$$(13) \quad y_{ft} \equiv y + \int_{z,h} [zh - w_t(z, h)] e_t(z, h) dz dh - \int_z \kappa v_t(z) dz + T_{ft}.$$

A family saves and borrows at a constant world bond price  $q > \beta$ , subject to an exogenous deterministic sequence  $\{\chi_{ft}\}$  of borrowing limits.

The first-order conditions for this problem imply that the intertemporal rate of substitution of consumption for a family of entrepreneurs is

$$(14) \quad \beta^t u'(c_{ft}) / u'(c_{f0}) = Q_{ft},$$

the Euler equation for borrowing is

$$(15) \quad qQ_{ft} = Q_{ft+1} + Q_{ft}\theta_{ft},$$

and the complementary slackness condition is

$$(16) \quad \theta_{ft}(d_{ft+1} - \chi_{ft}) = 0$$

at each  $t$ , where  $Q_{ft}$  is the multiplier on a family's budget constraint,  $Q_{ft}\theta_{ft}$  is the multiplier on a family's borrowing constraint, and we have normalized  $Q_{f0}$  to one so that  $Q_{ft}$  is entrepreneurs' shadow value of date- $t$  goods in units of date-0 goods.

Similar to a tightening of the debt constraints of a family of workers, a tightening of the debt constraints of a family of entrepreneurs implies that the family must temporarily lower its consumption. Thus, such a tightening induces a larger drop in consumption at  $t$  than at  $t + 1$ , which in turn gives rise to a larger increase in the shadow price of goods at  $t$ ,  $Q_{ft}$ , than in the shadow price of goods at  $t + 1$ ,  $Q_{ft+1}$ . Therefore, the value of future goods relative to present goods,

$$(17) \quad \frac{Q_{ft+1}}{Q_{ft}} = \frac{\beta u'(c_{ft+1})}{u'(c_{ft})},$$

declines. In this sense, a debt tightening leads entrepreneurs to discount future consumption more,

relative to current consumption.

### C. Government

The sole role of the government is to issue debt to foreigners in order to finance transfers to workers and entrepreneurs. In our baseline, we set  $T_{wt} = T_{ft} = 0$  at each  $t$ , so that the government is inactive. In later sections, we will examine the impact of government transfers meant to partially offset a debt tightening affecting workers or entrepreneurs. When doing so, we will maintain that the government rolls forward the debt incurred beyond the horizon we consider. This way, we can examine the impact of such transfers without the need to address the fiscal considerations arising from the accumulated government debt.<sup>2</sup>

### D. Individual Worker Values

For a given wage  $w_t(z, h)$ , which we for now posit and later characterize as the outcome of Nash bargaining between firms and workers, the present value of an employed worker's income or labor earnings expressed in date- $t$  consumption units is

$$(18) \quad \begin{aligned} W_t(z, h) = & w_t(z, h) + \phi(1 - \sigma) \frac{Q_{wt+1}}{Q_{wt}} \int_{z'} \max [W_{t+1}(z', h'), U_{t+1}(z')] dF_e(z'|z) \\ & + \phi\sigma \frac{Q_{wt+1}}{Q_{wt}} \int_{z'} U_{t+1}(z') dF_e(z'|z), \end{aligned}$$

where  $U_{t+1}(z)$  denotes the present value at  $t+1$  of the income of a nonemployed worker with general human capital  $z$ . Each employed worker discounts the value of future income by the relative shadow price of future to present goods,  $Q_{wt+1}/Q_{wt}$ , of the relevant family. General human capital evolves according to the law of motion  $F_e(z'|z)$  in (1), whereas firm-specific human capital evolves according to the law of motion in (3). Note that the continuation value in (18) reflects a worker's survival probability  $\phi$ , the exogenous separation probability  $\sigma$ , and the possibility that a worker leaves an undesirable match when  $W_{t+1}(z', h') < U_{t+1}(z')$ .

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<sup>2</sup>In practice, we can think of the government as having in each period a fixed endowment of another type of (tradable) good that all consumers in the world value and that enters all consumers' utility in a separable way as government services. Then, in the future, the government pays off its foreign debt, accumulated to finance its transfers, by selling the appropriate amount of its endowments to foreigners and correspondingly reducing the amount of its endowments it provides to its citizens. Since the economies we analyze are small in the world economy, transfers will have no effect on any prices or discount factors under these assumptions.

The present value of a nonemployed worker's income is

$$(19) \quad U_t(z) = b(z) + \phi \lambda_{wt}(z) \frac{Q_{wt+1}}{Q_{wt}} \int_{z'} \max [W_{t+1}(z', 1), U_{t+1}(z')] dF_u(z'|z) \\ + \phi [1 - \lambda_{wt}(z)] \frac{Q_{wt+1}}{Q_{wt}} \int_{z'} U_{t+1}(z') dF_u(z'|z),$$

where  $b(z)$  is the amount of home-produced goods of a nonemployed worker with general human capital  $z$  in a period, and  $(z', 1)$  is the worker's human capital at  $t + 1$  if the worker begins a new match, an event that occurs with probability  $\lambda_{wt}(z)$ . A nonemployed worker also discounts the value of future income by the relative shadow price of future to present goods,  $Q_{wt+1}/Q_{wt}$ , of the relevant family. The general human capital of a nonemployed worker evolves according to the law of motion  $F_u(z'|z)$  in (2). Observe that the continuation value in (19) accounts for a worker's survival probability  $\phi$ , a worker's job-finding rate  $\lambda_{wt}(z)$ , and the possibility that a match is not formed if  $W_{t+1}(z', 1) < U_{t+1}(z')$ .

### E. Individual Entrepreneur Value

For a given wage  $w_t(z, h)$ , the present value of profits of a firm matched with a worker with general human capital  $z$  and firm-specific human capital  $h$  expressed in date- $t$  consumption units is

$$(20) \quad J_t(z, h) = zh - w_t(z, h) + \phi(1 - \sigma) \frac{Q_{ft+1}}{Q_{ft}} \int_{z'} \max [J_{t+1}(z', h'), 0] dF_e(z'|z).$$

The flow profits are given by the difference between the amount produced,  $zh$ , and the wage paid,  $w_t(z, h)$ . Observe that since a firm is owned by a family of entrepreneurs, it discounts the value of future profits by  $Q_{ft+1}/Q_{ft}$ . Note also that the continuation value on the right side of (20) captures a firm's option to end an unprofitable match when  $J_{t+1}(z', h')$  is negative.

### F. Nash Bargaining, Vacancy Creation, and Resource Constraint

Entrepreneurs can direct their search for nonemployed workers with a specific level of general human capital  $z$  by posting job vacancies in the corresponding market. Matches in market  $z$  are generated by the matching function  $m_t(z) = u_t(z)v_t(z)/[u_t(z)^\eta + v_t(z)^\eta]^{1/\eta}$  specified to ensure that matching rates for firms and workers lie in the unit interval, where  $u_t(z)$  is the measure of nonemployed workers with general human capital  $z$  and  $v_t(z)$  is the measure of vacancies directed at such workers. The *tightness* of market  $z$  is given by  $\theta_t(z) = v_t(z)/u_t(z)$ . The *job-finding rate* for a worker with general human capital  $z$ —namely, the probability that a nonemployed worker with

general human capital  $z$  matches with a firm in market  $z$ —is given by

$$(21) \quad \lambda_{wt}(z) = \frac{m_t(z)}{u_t(z)} = \frac{\theta_t(z)}{[1 + \theta_t(z)\eta]^{1/\eta}}.$$

The *job-filling rate* for a firm searching for a worker with general human capital  $z$ —namely, the probability that a firm matches with a nonemployed worker with general human capital  $z$  in market  $z$ —is given by

$$(22) \quad \lambda_{ft}(z) = \frac{m_t(z)}{v_t(z)} = \frac{1}{[1 + \theta_t(z)\eta]^{1/\eta}}.$$

Notice that the job-finding rate increases with market tightness, whereas the job-filling rate decreases with it.

We now determine the wage paid by a firm to a worker with human capital  $(z, h)$  as the outcome of Nash bargaining. Formally, the Nash bargaining problem that a firm and such a worker face consists of maximizing the objective function  $[\bar{W}_t(w, z, h) - U_t(z)]^\gamma \bar{J}_t(w, z, h)^{1-\gamma}$ , where  $\gamma$  and  $1 - \gamma$  are the bargaining weights of families of workers and entrepreneurs, respectively, and  $\bar{W}_t(w, z, h)$  and  $\bar{J}_t(w, z, h)$  denote the values of employment and profits for a given  $w$ —namely, (18) and (20) with  $w$  replacing  $w_t(z, h)$ . Define the surplus from such a match as

$$(23) \quad S_t(z, h) \equiv W_t(z, h) - U_t(z) + J_t(z, h).$$

The following result is immediate.

**Lemma 1.** *Under Nash bargaining, firms and workers share match surplus according to*

$$(24) \quad W_t(z, h) - U_t(z) = \gamma S_t(z, h) \text{ and } J_t(z, h) = (1 - \gamma) S_t(z, h).$$

**Proof:** The first-order condition for the problem  $\max_w \{[\bar{W}_t(w, z, h) - U_t(z)]^\gamma \bar{J}_t(w, z, h)^{1-\gamma}\}$  is

$$\gamma[\bar{W}_t(w, z, h) - U_t(z)]^{\gamma-1} \bar{J}_t(w, z, h)^{1-\gamma} = (1 - \gamma)[\bar{W}_t(w, z, h) - U_t(z)]^\gamma \bar{J}_t(w, z, h)^{-\gamma}$$

or, equivalently, by using  $\bar{W}_t(w_t(z, h), z, h) = W_t(z, h)$  and  $\bar{J}_t(w_t(z, h), z, h) = J_t(z, h)$ ,

$$(25) \quad \gamma J_t(z, h) = (1 - \gamma)[W_t(z, h) - U_t(z)].$$

Then, (24) follows from the definition of the surplus from a match between a firm and a worker



with human capital  $(z, h)$  in (23). □

Interestingly, this result implies that although firms and workers have different discount factors, the share of surplus accruing to each of them does *not* depend on their discount factors. In particular, such a share is not higher for the more patient party.

Next, consider entrepreneurs' hiring decisions. The free-entry condition in market  $z$  ensures that vacancies are created until the cost of posting a vacancy equals the corresponding present value of profits. Specifically, the free-entry condition is given by

$$(26) \quad \kappa \geq \phi \lambda_{ft}(z) \frac{Q_{ft+1}}{Q_{ft}} \int_{z'} \max [J_{t+1}(z', 1), 0] dF_u(z'|z),$$

where the cost of posting a vacancy in any market  $z$  is  $\kappa$  units of goods. This relationship holds with equality in an active market  $z$ —that is, a market with open vacancies in that  $v_t(z) > 0$ . Since the surplus from a match increases with  $z$  under our parameterization, there exists a cutoff level of general human capital  $z_t^*$  such that firms post vacancies in all markets with  $z \geq z_t^*$  and none in any market with  $z < z_t^*$ .

Finally, the resource constraint for this small open economy is

$$c_{wt} + c_{ft} + d_{wt} + d_{ft} = y + \int_{z,h} z h e_t(z, h) dz dh + \int_z b(z) u_t(z) dz - \int_z \kappa v_t(z) dz + q(d_{wt+1} + d_{ft+1}) + T_{wt} + T_{ft}.$$

## G. Steady-State Properties

Equilibrium allocations solve the problem of workers' families, the problem of entrepreneurs' families, firms and workers' Nash bargaining problem, and clear markets. We solve for the steady-state measures of employed workers,  $e(z, h)$ , and nonemployed workers,  $u(z)$ , as a function of human capital. We solve also for the associated aggregate levels of consumption and employment. Figure 1 shows the steady-state measures of employed and nonemployed workers—the former integrated over firm-specific human capital  $h$ —and the steady-state schedules of the job-filling rate and the job-finding rate as a function of  $z$  under our parameterization. The figure displays the cutoff level of  $z$ —namely,  $z_t^*$ —such that firms post no vacancies in markets with  $z < z_t^*$  and workers have a corresponding matching probability of zero, and post a positive number of vacancies in markets with  $z \geq z_t^*$ . Note that the job-finding rate is strictly increasing with  $z$  when  $z$  is larger than  $z_t^*$ , as firms matched with workers with higher levels of  $z$  earn higher profits and thus have a greater incentive to post vacancies to attract such workers.

In anticipation of the policy experiments in later sections, we now describe the deterministic dynamics of equilibrium starting from a steady state in which both workers' and entrepreneurs' debt

constraints bind, once we subject the economy to a transitory shock to debt constraints. Specifically, we conjecture and verify that these constraints bind throughout the horizon of the experiment, in that given the inherited debt levels  $d_{w1} = \chi_{w0}$  and  $d_{f1} = \chi_{f0}$ , we posit that  $\theta_{wt} > 0$  and  $\theta_{ft} > 0$  at each  $t$  so that  $d_{wt+1} = \chi_{wt}$  and  $d_{ft+1} = \chi_{ft}$ . Given this conjecture, the multipliers on debt constraints satisfy

$$\theta_{wt} = q - \frac{\beta u'(c_{wt+1})}{u'(c_{wt})} \text{ and } \theta_{ft} = q - \frac{\beta u'(c_{ft+1})}{u'(c_{ft})},$$

where workers' and entrepreneurs' consumption satisfies the corresponding budget constraints at the binding borrowing constraints; namely,

$$(27) \quad c_{wt} = \int_{z,h} w_t(z,h)e_t(z,h)dzdh + \int_z b(z)u_t(z)dz + q\chi_{wt} - \chi_{wt-1} + T_{wt}$$

and

$$(28) \quad c_{ft} = y + \int_{z,h} zhe_t(z,h)dzdh - \int_{z,h} w_t(z,h)e_t(z,h)dzdh - \int_z \kappa v_t(z)dz + q\chi_{ft} - \chi_{ft-1} + T_{ft}$$

at each  $t$ . Note that this conjecture is verified if the relevant multipliers are indeed positive along the consumption sequences constructed based on (27) and (28) for each type of family. We guarantee that this is the case by appropriately choosing the exogenous borrowing limit sequences  $\{\chi_{wt}\}$  and  $\{\chi_{ft}\}$ , as explained below.

At an intuitive level, a tightening of workers' and entrepreneurs' debt constraints leads to a temporary decrease in workers' and entrepreneurs' consumption as each group deleverages by repaying part of its debt. This drop in period- $t$  consumption in turn increases the shadow prices  $Q_{wt}$  and  $Q_{ft}$  of period- $t$  goods in units of period-0 goods above their steady-state levels. These shadow prices then gradually decrease and revert back to these levels. The corresponding discount factors  $Q_{wt+1}/Q_{wt}$  and  $Q_{ft+1}/Q_{ft}$  thus decrease below their steady-state levels and then gradually increase back to these levels.

## H. Parameterization

Table 1 summarizes the parameterization of our model. A period in the model corresponds to one month. We follow Kehoe, Midrigan, and Pastorino (2019) in the choice of all common parameters and refer to their discussion for more details. In particular, the values of the endogenously chosen parameters are selected to match the same targets as in Kehoe, Midrigan, and Pastorino (2019). We posit an identical CRRA utility function for workers' and entrepreneurs' families,

$u(c_{wt}) = c_{wt}^{1-\alpha}/(1-\alpha)$  and  $u(c_{ft}) = c_{ft}^{1-\alpha}/(1-\alpha)$ , with relative risk aversion parameter  $\alpha$  equal to 5, and specify the home production function as  $b(z) = b_0 + b_1z$ , with parameter  $b_1$  equal to 0.25. We choose entrepreneurs' endowment  $y$  so as to reproduce the labor share of U.S. output between 2007 and 2009, as measured by the Bureau of Economic Analysis. We set the exogenous steady-state debt levels  $\chi_w$  and  $\chi_f$  to match the debt service ratios of the U.S. household and non-financial corporate sectors between 2007 and 2009, as reported by the Bank of International Settlements (BIS).<sup>3</sup>

## 4. Deleveraging Shocks

We turn to analyzing the impact of credit market disruptions on aggregate consumption and employment by examining the implications of shocks to workers' and entrepreneurs' debt limits. We then evaluate the impact of government interventions aimed at partially offsetting the effects of these shocks. To simulate the effects of a credit tightening, we assume that an unanticipated reduction in the debt limit for worker families, entrepreneur families, or both types of families occurs. We refer to these tightenings as *deleveraging shocks*, since when the relevant constraints tighten, families must reduce their debt by decreasing their consumption. In all of these experiments, we assume that the credit tightening is unexpected when it first occurs and that agents have perfect foresight afterward.

We will present and discuss our results from two distinct points of view. Specifically, we first examine the impact of these shocks on match surplus,  $S_t(z, h)$ , and then on firms' incentives to create job vacancies. To examine the impact of these shocks on match surplus, we rewrite the free-entry condition for any active market as

$$(29) \quad \kappa = \phi \lambda_{ft}(z)(1 - \gamma) \frac{Q_{ft+1}}{Q_{ft}} \int_{z'} \max[S_{t+1}(z', 1), 0] dF_u(z'|z),$$

by using Lemma 1 to express a firm's value  $J_t(z, h)$  as a fixed share of match surplus—namely,  $(1 - \gamma)S_t(z, h)$ . Using the expressions for the values of employed workers, nonemployed workers,

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<sup>3</sup>Specifically, for the labor share, we use the annual share of labor compensation to GDP at current national prices for the United States (not seasonally adjusted; the variable LABSHPUSA156NRUG from Federal Reserve Economic Data). For the debt service ratio, we rely on the BIS measure of the debt service ratio for the private non-financial sector at any quarter  $t$ ,  $DSR_t$ , computed as the ratio of the debt service cost of an installment loan to income; namely,  $DSR_t = i_t/[1 - (1 + i_t)^{-s_t}](D_t/Y_t)$ , where  $D_t$  is the total stock of debt,  $Y_t$  is quarterly income,  $i_t$  is the average interest rate on the existing stock of debt per quarter, and  $s_t$  is the average remaining maturity in quarters.

and firms—namely, (18), (19), and (20)—we can then rewrite match surplus  $S_t(z, h)$  as

$$\begin{aligned}
S_t(z, h) &= zh - b(z) \\
&+ \phi \left\{ (1 - \sigma) \left[ \gamma \frac{Q_{wt+1}}{Q_{wt}} + (1 - \gamma) \frac{Q_{ft+1}}{Q_{ft}} \right] - \gamma \lambda_{wt}(z) \frac{Q_{wt+1}}{Q_{wt}} \right\} \int_{z'} \max[S_{t+1}(z', h'), 0] dF_e(z'|z) \\
&+ \phi \frac{Q_{wt+1}}{Q_{wt}} \int_{z'} U_{t+1}(z') [dF_e(z'|z) - dF_u(z'|z)] \\
(30) \quad &+ \phi \gamma \lambda_{wt}(z) \frac{Q_{wt+1}}{Q_{wt}} \left\{ \int_{z'} \max[S_{t+1}(z', h'), 0] dF_e(z'|z) - \int_{z'} \max[S_{t+1}(z', 1), 0] dF_u(z'|z) \right\}.
\end{aligned}$$

As apparent from (30), a key reason why worker- and firm-side shocks and later government interventions directed at workers and firms have a different impact is that workers' and firms' discount factors asymmetrically affect match surplus and thus firms' and workers' incentives to create a match. Focusing on match surplus also allows us to transparently isolate the role of human capital and, correspondingly, provide an intuition for our results that does not rely on specific features of the decentralized equilibrium such as firms' profits or workers' wages.

Using the free-entry condition in (26), we complement this approach by evaluating the effect of deleveraging shocks on the decentralized equilibrium, so as to examine the impact of such shocks on firms' choices to post vacancies, which is the key margin for employment. To this end, we decompose a firm's value as

$$(31) \quad J_t(z, h) = X_t(z, h) - \mathcal{W}_t(z, h),$$

namely, as the difference between the present value of output from a match,  $X_t(z, h)$ , and the present value of wages paid over the course of a match,  $\mathcal{W}_t(z, h)$ , discounted by entrepreneurs' discount factor,  $Q_{ft+1}/Q_{ft}$ . These values are given by

$$(32) \quad X_t(z, h) = zh + \phi(1 - \sigma) \frac{Q_{ft+1}}{Q_{ft}} \int_{\{z': J_{t+1}(z', h') \geq 0\}} X_{t+1}(z', h') dF_e(z'|z)$$

$$(33) \quad \mathcal{W}_t(z, h) = w_t(z, h) + \phi(1 - \sigma) \frac{Q_{ft+1}}{Q_{ft}} \int_{\{z': J_{t+1}(z', h') \geq 0\}} \mathcal{W}_{t+1}(z', h') dF_e(z'|z).$$

Note that  $\mathcal{W}_t(z, h)$  denotes the present value of wages *only* from a current match—in contrast,  $W_t(z, h)$  denotes the present value of an employed worker's income from a current match as well as from all future periods of employment and nonemployment. In the expression for  $X_t(z, h)$  in (32) and  $\mathcal{W}_t(z, h)$  in (33), we integrate future values only over the realizations of general human capital  $z'$  for which firm value  $J_{t+1}(z', h')$  is nonnegative, since only profitable matches survive to

period  $t + 1$ . Expressions (32) and (33) make it clear how a firm's incentive to post vacancies after a deleveraging shock depends on the magnitude of the fall in the present value of output from a match relative to the fall in the present value of the wages to be paid.

In our analysis below, we examine the effect of a deleveraging shock on both match surplus and the present values of output and wages from a match, which by (31) are related as

$$(34) \quad S_t(z, h) = \frac{X_t(z, h)}{1 - \gamma} - \frac{W_t(z, h)}{1 - \gamma},$$

since  $J_t(z, h) = (1 - \gamma)S_t(z, h)$ , which implies that match surplus can be expressed as the difference in the present values of output and wages scaled by entrepreneurs' bargaining share.

### A. Deleveraging Shocks to Workers, Firms, and Both Groups

Here, we consider deleveraging shocks that tighten the debt constraints of families of workers alone, families of entrepreneurs alone, or both types of families at once. We refer to them as *shocks to workers*, *shocks to firms*, and *shocks to both workers and firms*.

#### *Deleveraging Shocks to Workers*

Consider a deleveraging shock that tightens workers' debt constraints. After such a shock, workers reduce their consumption in order to repay part of their outstanding debt and satisfy their new debt limits. As the solid line in panel (a) of Figure 2 shows, we choose a sequence  $\{\chi_{wt}\}$  of workers' debt limits so that after the shock occurs in period 0, workers' consumption decreases in period 1 from its steady-state level of  $c_w$  to  $c_{w1} = (1 - \delta)c_w$ , with  $\delta$  equal to 5 percent, and from period 2 onward, consumption slowly adjusts back to its steady-state level at a rate of 10 percent per quarter according to

$$(35) \quad \log(c_{wt}) = (1 - \rho_c) \log(c_w) + \rho_c \log(c_{wt-1}),$$

with  $\rho_c = 0.9^{1/3}$ , as our model is monthly. We chose the value of  $\rho_c$  so as to match the speed of recovery of consumption in post-war recessions. Throughout this experiment, firms' debt limits are kept at their steady-state value of  $\chi_f$ .<sup>4</sup> The solid line in panel (c) of Figure 2 shows that this shock to consumption makes workers' one-period-ahead discount factor  $Q_{wt+1}/Q_{wt}$  drop by almost 1 percentage point (100 basis points) on impact and then slowly mean revert. The solid lines in panels (b) and (d) show how the resulting general equilibrium effects of a worker deleveraging shock

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<sup>4</sup>In practice, we posit a path for workers' consumption and solve for equilibrium including the endogenously determined path of workers' debt constraints that justifies the assumed consumption path.

impact entrepreneurs' consumption and discount factor.

As is apparent from the solid line in panel (a) of Figure 3, after a deleveraging shock to workers, match surplus sharply drops by more than 10 percent and then slowly mean reverts. In panel (b) of Figure 3, we plot the present values of output and wages from a match, both scaled by the steady-state level of output from a match,  $X$ . That is, we plot  $X_t/X$  and  $\mathcal{W}_t/X$ , which by (34) are related to match surplus scaled by its steady-state value  $S_t/S$  as

$$(36) \quad \frac{S_t}{S} = \frac{1}{1-\gamma} \frac{X}{S} \left( \frac{X_t - \mathcal{W}_t}{X} \right) \cong 10 \left( \frac{X_t - \mathcal{W}_t}{X} \right),$$

where the approximation uses that in our quantitative model  $\gamma = 1/2$  and  $X/S \cong 5$ .<sup>5</sup> From panel (b) of Figure 3, it is apparent that the present value of output falls by more than the present value of wages from a match so that both match surplus  $S_t(z, h)$  and firm value  $J_t(z, h)$  fall. Panel (d) of Figure 3 shows the resulting decline in employment.

From a firm's perspective, the mechanism behind the responses to a worker deleveraging shock is as follows. The tightening of workers' debt constraints directly reduces workers' consumption, and hence their discount factor falls, as depicted in panel (c) of Figure 2 (solid line). Two forces are at play. One force is that when workers' consumption contracts, so does their demand for goods produced by firms. By itself, such a fall in demand reduces firms' incentives to hire workers, and thus output declines. A second force is that in the bargaining between firms and workers, the sharp fall in workers' discount factor tends to make the resulting wages relatively sticky. This rigidity leads the present value of wages to fall less sharply than the present value of output from a match, which gives rise to a sizable decline in the present value of profits from hiring a worker. By the free-entry condition, this second force induces firms to reduce even further the number of vacancies they post. Overall, these two forces imply a large fall of nearly 3 percent in employment, as shown in panel (d) of Figure 3.

### *Deleveraging Shocks to Firms*

Consider next a deleveraging shock that affects families of entrepreneurs by generating the same percentage fall in entrepreneurs' consumption as in workers' consumption after a worker deleveraging shock. Namely, we simulate an initial 5 percent drop in entrepreneurs' consumption from its steady-state level, followed by a recovery to its steady-state level at rate  $\rho_c = 0.9^{1/3}$  per quarter, governed by the analogue of (35). Throughout this experiment, we maintain workers' debt

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<sup>5</sup>Note that  $S_t = \int_{z,h} S_t(z, h) e_t(z, h) dz dh$  and that  $X_t$  and  $\mathcal{W}_t$  are similarly defined.

limits at their steady-state value of  $\chi_w$ .<sup>6</sup>

The dashed line in panel (b) of Figure 2 shows the effect of this shock on entrepreneurs' consumption, and the dashed line in panel (a) of this figure shows the resulting general equilibrium effect on workers' consumption. The dashed line in panel (d) of the figure shows how on impact, this shock leads entrepreneurs' one-period-ahead discount factor  $Q_{ft+1}/Q_{ft}$  to drop by almost 1 percentage point (100 basis points) and then slowly mean revert. The dashed line in panel (c) of the figure shows the corresponding general equilibrium effect on workers' discount factor, which is much smaller.

As the dashed line in panel (a) of Figure 3 shows, after a deleveraging shock to firms, match surplus drops by roughly half as much as it does after a deleveraging shock to workers, then slowly mean reverts. Interestingly, a comparison of panels (b) and (c) of Figure 3 shows that although both the present value of output and wages from a match decrease much more sharply after a shock to firms than after a shock to workers, the *difference* between these present values declines much less after a shock to firms. In panel (a) of Figure 3, we see how this smaller drop in the difference between the present values of output and wages after a firm deleveraging shock in turn implies a smaller drop in match surplus, as is consistent with the decomposition in (36). As panel (d) of Figure 3 further shows, this smaller decline in match surplus leads to a fall in employment that is only about a third as large as the one that occurs after a worker deleveraging shock of the same magnitude.

From the perspective of the decentralized equilibrium, an intuition behind these results is that in the bargaining between firms and workers, the sharp fall in entrepreneurs' discount factor gives rise to relatively flexible wages. Hence, although a shock to entrepreneurs' credit leads to a fall in the present value of output that is nearly five times larger than the one induced by a shock to workers' credit, the relative flexibility of wages after a shock to entrepreneurs' credit implies a large fall in wages by nearly as much. Thus, by (34), match surplus and employment fall much less after a firm deleveraging shock than after a worker deleveraging shock.

### ***Deleveraging Shocks to Both Workers and Firms***

Now we consider a deleveraging shock that affects both workers and firms. In our previous experiments, we have directly simulated a decline in either workers' or entrepreneurs' consumption and then solved for the other group's induced decline in consumption for fixed debt constraints.

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<sup>6</sup>In the simulation, as in the case of a worker deleveraging shock, we posit a path for entrepreneurs' consumption and solve for equilibrium including the endogenously determined path of firms' debt constraints that justifies the assumed consumption path.

Here, we instead let worker debt constraints be those associated with the sequence of debt limits  $\{\chi_{wt}\}$  used in the worker-only deleveraging case and firm debt constraints be those associated with the sequence of debt limits  $\{\chi_{ft}\}$  used in the firm-only deleveraging case. We then solve for the resulting paths for both workers' and entrepreneurs' consumption.

Notice that because of the general equilibrium effects across the two groups, both workers' consumption and entrepreneurs' consumption fall by more than they do when just their own debt constraint is tightened. Indeed, as the solid lines in panels (a) and (b) of Figure 4 show, after a deleveraging shock to both workers and firms, workers' and entrepreneurs' total consumption drops by an amount roughly equal to the sum of the consumption drops in the worker-only and firm-only deleveraging cases. The solid lines in panels (c) and (d) show how these drops manifest themselves as declines in workers' and entrepreneurs' discount factors.

Panels (a), (b), and (c) of Figure 5 show that the drops in match surplus, the present values of output and wages, and employment when both shocks occur are all approximately equal to the sum of the corresponding drops for each shock alone. Note that the percentage drops in the present values of output and wages in panel (b) of Figure 5 are only slightly larger in absolute value than the corresponding ones in the firm-only shock case in panel (c) of Figure 3. Thus, it would not seem a straightforward exercise to construct an empirical measure of the difference in the present value of output and wages, say, during the Great Recession, and infer from it the relative magnitudes of the deleveraging shocks faced by workers and firms.

## B. Role of Human Capital

Here, we discuss the role of human capital in amplifying the effects of deleveraging shocks. To do so, we show how the impact of these shocks differs in the absence of human capital in the three scenarios just considered.

### *Deleveraging Shocks to Workers without Human Capital*

In Figure 6, we contrast the impulse responses to a deleveraging shock to workers in our baseline economy with those in an economy *without human capital*, in which we mute both general and firm-specific human capital by setting  $z_t = \bar{z}$  and  $h_t = 1$  for all  $t$ . In this economy, as is apparent from panel (a), a deleveraging shock of the same size as the shock considered earlier leads workers' consumption to drop by 5 percent on impact and then slowly mean revert, as in our baseline economy. The dashed line in panel (b) shows that in contrast to our baseline economy, without human capital, the general equilibrium effect on entrepreneurs' consumption effectively disappears. The dashed lines in panels (e) and (f) show further that without human capital, match surplus and



employment barely decline.

To understand these results, consider the expression for match surplus in the economy without human capital; namely,

$$(37) \quad S_t(\bar{z}, 1) = \bar{z} - b(\bar{z}) + \phi \left\{ (1 - \sigma) \left[ \gamma \frac{Q_{wt+1}}{Q_{wt}} + (1 - \gamma) \frac{Q_{ft+1}}{Q_{ft}} \right] - \gamma \lambda_{wt}(\bar{z}) \frac{Q_{wt+1}}{Q_{wt}} \right\} S_{t+1}(\bar{z}, 1),$$

which implies that match surplus is determined by a first-order difference equation. The term multiplying  $S_{t+1}(\bar{z}, 1)$  on the right side of (37) governs the magnitude and persistence of changes in match surplus in response to changes in workers' debt limits and so in discount factors. Up to a log-linear approximation, this term corresponds to the standard surplus root in the DMP model, in which surplus is also determined by a first-order difference equation. For standard parameterizations, this term is much smaller than one, which implies that a temporary decrease in  $Q_{wt+1}/Q_{wt}$  leads to only a modest decrease in the second term on the right-side of (37)—that is, the continuation surplus. Hence, a deleveraging shock generates only a very small and transitory decline in match surplus and thus in employment, as is apparent from the dashed lines in panels (e) and (f) of Figure 6.

Now consider our baseline economy with human capital and compare (30) with (37). The expression in (30) consists of two additional terms. The first one, in the third line of (30), is due to general human capital accumulation. As the third and fourth lines of (30) show, the expression for match surplus includes values weighted by the laws of motion of general human capital for employed and nonemployed workers,  $dF_e(z'|z)$  and  $dF_u(z'|z)$ . Since general human capital grows faster when a worker is employed, this first additional term is clearly positive. The second additional term, in the fourth line of (30), is due to both general and firm-specific human capital accumulation. Specifically, even if  $dF_e(z'|z) = dF_u(z'|z)$ , the term in brackets in this fourth line would not be zero—owing to the difference in the level of firm-specific human capital between workers in continuing matches,  $h'$ , and in new matches,  $h = 1$ —and is otherwise strictly positive.

Intuitively, since the general human capital acquired over the course of a match is transferable to other matches, the component of match surplus due to general human capital acquisition slowly decays over time. As shown in Kehoe, Midrigan, and Pastorino (2019), the slower the decay in match surplus flows is, the more sensitive match surplus is to changes in discount factors. Indeed, general, rather than firm-specific, human capital accounts mostly for the increased duration of surplus flows arising from human capital acquisition. In fact, when we mute general human capital so that  $F_e(z'|z) = F_u(z'|z)$ , the (Macaulay) duration of surplus flows decreases approximately

from 61 months in our baseline economy to 2 months.<sup>7</sup> In contrast, when we mute firm-specific human capital so that  $\bar{h} = 1$ , the (Macaulay) duration of surplus flows decreases only modestly, approximately from 61 months in our baseline economy to 59 months.

The reason for the greater response to a debt tightening that affects workers in the economy with human capital compared with that in the economy without human capital is that human capital accumulation introduces a key component to match surplus, as reflected in the terms on the third and fourth lines of the expression in (30), that is sensitive to discounting.<sup>8</sup> To see the close connection between changes in surplus and in employment, note from panel (e) of Figure 6 that in the presence of human capital, match surplus falls sharply and then mean reverts. Correspondingly, employment falls sharply, as shown in panel (f). In contrast, without human capital, match surplus barely moves and so does employment.

In panels (c) and (d) of Figure 6, by using (36), we decompose changes in surplus into changes in the present values of output and wages. As panel (c) shows, the presence of human capital implies that the present value of output falls by more than the present value of wages. Instead, panel (d) shows that absent human capital, the present values of output and wages fall by nearly the same amount. In panel (e), we scale these differences by 10, as (36) requires, to show the resulting changes in surplus. From this panel, it is apparent that with human capital, the difference in the present value of output and wages falls enough to generate a sizable decline in match surplus and thus, by the free-entry condition, a large fall in employment, as reported in panel (f). In contrast, absent human capital, the difference in the present value of output and wages barely falls, so that the resulting surplus in panel (e) is almost unchanged. Accordingly, the drop in employment in panel (f) is negligible.

### ***Deleveraging Shock to Firms without Human Capital***

In Figure 7, we compare the impulse responses to a deleveraging shock to firms in our baseline economy to those in an economy without human capital. Figure 7 shows that when human capital is muted, a deleveraging shock generates only a negligible fall in both match surplus and employment. Interestingly, as panels (c) and (d) show, both with and without human capital, a deleveraging shock to firms causes a sizable fall in the present value of both output and wages. The key difference is that with human capital, the drop in the present value of wages is slightly smaller than the drop

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<sup>7</sup>The Macaulay duration is a standard measure of the change in the present value of an intertemporal stream of payments in response to permanent changes in one-period discount rates. We compute this duration as  $\sum_{k=1}^{\infty} k\omega_k$ , where  $\omega_k = \beta^k s_k / \sum_{k=1}^{\infty} \beta^k s_k$  is the share of surplus received in the  $k$ -th period of a match.

<sup>8</sup>Without accumulation of new capital, the dispersion in general human capital at entry in the market is unimportant for these results.

in the present value of output. When scaled by 10 as (36) requires, this difference leads to about a 7 percent fall in match surplus (see panel [e]) and hence a nontrivial fall in employment (see panel [f]). In contrast, without human capital, the present values of output and wages decline by almost the same amount. Thus, the resulting change in surplus is negligible and so is the change in employment.

One way to understand these results is to contrast the impact of a deleveraging shock on match surplus in the presence of human capital in (30) and in its absence in (37). Intuitively, the crucial terms that lead to long-duration match surplus flows are those that arise from human capital accumulation. In (30), a worker’s discount factor multiplies the human capital terms in the third and fourth lines of this expression. By contrast, a firm’s discount factor appears only in the second line of this expression containing the standard terms of the match surplus equation in a DMP model, in which, as argued, surplus flows have short durations and so are relatively insensitive to changes in discounting. Because of this feature, match surplus is much more sensitive to a worker’s discount factor (and deleveraging shock) than to a firm’s discount factor (and deleveraging shock).

### *Deleveraging Shocks to Both Workers and Firms without Human Capital*

We now suppose that deleveraging shocks affecting workers and firms occur at the same time in the economy without human capital. In this case, impulse responses for the present values of output and wages display patterns similar to those arising when deleveraging shocks affect only firms. In particular, by comparing panels (c) and (d) of Figure 8, we see that both with and without human capital, the present values of output and wages fall sharply. In the presence of human capital, the fall in the two present values is sufficiently asymmetric that when scaled by 10, as (36) requires, it gives rise to a correspondingly large decrease in match surplus and hence in employment. In contrast, in the absence of human capital, these present values fall by almost exactly the same amount. As a result, the change in surplus, and thus in employment, is negligible. In particular, even when both deleveraging shocks occur, the presence of human capital makes wages slightly more rigid than in its absence.

## **5. Policy Interventions**

We turn to examine policy experiments in which the government implements lump-sum transfers to workers or entrepreneurs to partially offset the tightening of their debt constraints. These interventions are assumed to be financed by the government with foreign debt that is rolled forward, including any debt service payments, beyond the end of these experiments—and so is not repaid over their horizons. We formulate this assumption for simplicity, so as to focus solely on the

impact of simple credit assistance policies that do not entail addressing any of the associated fiscal considerations.

We first examine the same deleveraging shock to workers as before—namely, a tightening of debt constraints that gives rise to an initial 5 percent drop in consumption, which then mean reverts, at the rate of 10 percent per quarter, to its steady-state level. We suppose that the government performs a sequence of lump-sum transfers  $\{T_{wt}\}$  such that workers’ consumption in equilibrium falls by only 2.5 percent on impact and then transfers revert to zero at the rate of 10 percent per quarter.

We then consider a deleveraging shock to entrepreneurs and assume that the government transfers to entrepreneurs the same amount of resources transferred to workers in the previous experiment in that  $T_{ft} = T_{wt}$  at each  $t$ . Note that since workers earn 60 percent of the income in the economy and entrepreneurs earn only 40 percent, entrepreneurs receive larger transfers per family than workers. Indeed, in the presence of these transfers, entrepreneurs’ consumption falls by only 1.8 percent after the intervention factoring in general equilibrium effects (see panel [b] of Figure 10). Instead, when the government performs the same lump-sum transfers to workers in response to a worker deleveraging shock, workers’ consumption falls by more than 2 percent (see panel [a] of Figure 9). Hence, a transfer of the same size to each group spurs entrepreneurs’ consumption more than workers’ consumption. We nonetheless think that considering an equal sequence of transfers to both groups is the most relevant experiment if the goal is to contrast the implications of a transfer of any given size directed to households and directed to firms, since by definition, such an exercise measures the *bang for the buck* of each policy.

The solid lines in the panels of Figure 9 graph the responses to a negative deleveraging shock to workers that temporarily reduces their consumption  $c_{wt}$  by 5 percent, with fixed entrepreneurs’ exogenous debt limits  $\{\chi_{ft}\}$ —they repeat the corresponding ones from Figures 2 and 3. We then plot the responses to the same shock when workers’ credit is supported by the intervention described. As the effect on employment of the deleveraging shock to workers was sizable in the absence of the intervention, the impact of this intervention on employment is substantial. The largest decline in employment with the intervention is less than half as large as the corresponding decline without the intervention, as is apparent from panel (d) of Figure 9.

Similarly, the solid lines in the panels of Figure 10 report the responses to a negative deleveraging shock to entrepreneurs that temporarily reduces their consumption  $c_{ft}$  by 5 percent, with fixed workers’ exogenous debt limits  $\{\chi_{wt}\}$ —they repeat the corresponding ones from Figures 2 and 3. In each panel, we overlay the responses to the same shock when entrepreneurs’ credit is bolstered

through the same sequence of lump-sum transfers directed to workers underlying Figure 9. Note that the benefits of this intervention for employment are only half as large (in levels) as those of the intervention aimed at workers, despite the two interventions being of the same size.

The reason for the intervention directed at supporting worker credit's greater impact on employment is similar to that for the larger aggregate employment drop after a tightening of worker credit, compared with a tightening of firm credit. Simply, an increase in workers' consumption, through its impact on workers' discount factor, has a larger effect on match surplus than the same increase in entrepreneurs' consumption.

A similar result emerges also in a scenario in which a credit tightening affects both workers and entrepreneurs by the same measure. Figure 11 plots solid lines for a negative deleveraging shock to both workers and entrepreneurs that temporarily reduces their respective consumption by 5 percent. It thus repeats the consumption responses in panels (a) and (b) of Figure 4 (solid lines). In panel (d) of Figure 11, we further compare the effects on employment of an intervention that transfers the same lump-sum amount of resources solely to workers (dashed-dotted line) and solely to firms (dashed line). As before, in the first type of intervention, the sequence of lump-sum transfers is designed to offset half of the decline in workers' consumption. In the second type of intervention, the same amount of resources is directed alternatively to entrepreneurs. The difference between the dashed-dotted and dashed lines in the panels of Figure 11 is therefore due entirely to the group of families that receives transfers. As the figure shows, the employment recovery is faster when the intervention is directed to workers rather than firms.

So far we have considered a model in which fluctuations in workers' and entrepreneurs' ability to borrow originate directly from a change in the exogenous debt limits of the two groups. In the Appendix, we show that our simple model with debt constraints is equivalent to a model with collateralized borrowing against the value of the housing that each group owns. In particular, we show that our model is equivalent to one in which fluctuations in each group's ability to borrow are induced by fluctuations in housing prices for *fixed* parameters of the borrowing constraints of the two groups. At a more general level, our economy is equivalent to a much larger class of economies, as the labor market outcomes—including employment, nonemployment, vacancies, and wages—implied by our simple model are uniquely determined by the sequences of shadow prices of goods,  $\{Q_{wt}, Q_{ft}\}$ . Because of this feature, alternative setups for the families' problems that yield the same sequences of shadow prices of goods also imply equivalent labor market outcomes.

## 6. An Economy with Tradable and Nontradable Goods

We extend the model analyzed so far by considering an economy composed of many regions, meant to represent U.S. states, with tradable and nontradable goods produced and consumed in each region (state). We begin by reviewing the regional patterns of the Great Recession across U.S. states between 2007 and 2009 in terms of consumption, employment, employment in the nontradable goods sector, and employment in the tradable goods sector. We then augment the model of Kehoe, Midrigan, and Pastorino (2019), who showed that a version of this model with only one group of consumers accounts well for the cross-state comovement between consumption, nontradable employment, and tradable employment during the Great Recession. We use the resulting model to investigate the implications of a government intervention aimed at subsidizing households' and firms' credit in response to credit shocks of magnitude comparable to those experienced by U.S. states during the Great Recession.

### A. Regional Patterns

In the three panels of Figure 12, we reproduce the corresponding results in Kehoe, Midrigan, and Pastorino (2019) on the relationship between changes in state-level consumption and employment between 2007 and 2009, with consumption changes instrumented by changes in state-level house prices.<sup>9</sup> Intuitively, we correlate cross-state employment changes with cross-state consumption changes that are associated with changes in local credit conditions as accounted for by changes in local house prices. We find that the elasticity of changes in employment relative to changes in consumption is 0.38 across states; that is, a 10 percent decline in consumption across states is associated with a 3.8 percent decline in employment. From the figure, it is apparent that declines in consumption across states are strongly related to declines in nontradable employment across states. Declines in consumption across states, though, are essentially unrelated to declines in tradable employment across states; for the size of these tradable employment declines, see the large negative intercept in the bottom panel of the figure. In particular, a 10 percent decline in consumption across states is associated with a 5.5 percent decline in nontradable employment and a negligible (and statistically insignificant) 0.3 percent increase in tradable employment across states.

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<sup>9</sup>This figure repeats panels (a) to (c) of Figure 14 in Kehoe, Midrigan, and Pastorino (2019); we refer the interested reader to this paper for all omitted details. Like those authors, we restrict attention to the sample of U.S. (continental contiguous) states for which information about house prices is consistently available over the years of interest. State-level consumption changes are measured by projecting state-level consumption changes on the corresponding changes in state-level (Zillow) house prices. See Charles, Hurst, and Notowidigdo (2018) for a similar approach.

## B. Model with Tradable and Nontradable Goods

We consider a world economy that consists of a continuum of islands indexed by  $s \in S$ . The islands borrow and lend from one another in units of a composite tradable good. We focus on a finite subset  $S_{US}$  of these islands and interpret any  $s \in S_{US}$  as a state of the United States. Since we focus on island-specific shocks to a finite number of islands, we interpret the set of islands  $S_{US}$  as small in the world economy—that is, as a small subset of the set of all islands  $S = S_{US} \cup S_R$ , where  $S_R$  are the islands in the rest of the world. Hence, the total world output of tradable goods is assumed fixed, and borrowing and lending occur at a constant interest rate  $r$  with associated bond price  $q = 1/(1+r)$ . Each island is populated by a unit measure of worker families and a unit measure of entrepreneur families. Each island produces a differentiated variety of a tradable good that is consumed everywhere and a nontradable good that is consumed only on that island. Both of these goods are produced using intermediate goods. Workers can switch between the nontradable and tradable goods sectors but are immobile across islands. Each worker is endowed with one of two types of skills that are used with different intensities in the nontradable and tradable goods sectors. Such a differential intensity in the use of skills across sectors generates a cost of worker reallocation between them, which will prove central to each sector’s distinct employment response to credit shocks across states.

### *Preferences and Demand*

On each island  $s$ , there is a measure one of identical families of workers and a measure one of identical families of entrepreneurs that consume a composite consumption good,

$$c_{wt}(s) + c_{ft}(s) = x_t(s),$$

where  $c_{wt}(s)$  and  $c_{ft}(s)$  denote the consumption of market-produced goods of each group and  $x_t(s)$  is the total consumption of all families. This composite consumption good is a bundle of the nontradable good produced and consumed domestically,  $x_{Nt}(s)$ , and the composite tradable good,  $x_{Tt}(s)$ ,

$$x_t(s) = \left[ \tau^{\frac{1}{\mu}} x_{Nt}(s)^{\frac{\mu-1}{\mu}} + (1-\tau)^{\frac{1}{\mu}} x_{Tt}(s)^{\frac{\mu-1}{\mu}} \right]^{\frac{\mu}{\mu-1}},$$

where  $\mu$  is the elasticity of substitution between nontradable and tradable goods. The price of the composite consumption good on island  $s$  is

$$p_t(s) = \left[ \tau p_{Nt}(s)^{1-\mu} + (1-\tau) p_{Tt}^{1-\mu} \right]^{\frac{1}{1-\mu}},$$

where  $p_{Nt}(s)$  is the price of the nontradable good produced on island  $s$  and  $p_{Tt}$  is the price of the composite tradable good common across islands.

Given the demand for the composite consumption good on island  $s$ ,  $x_t(s)$ , the demand for its nontradable and tradable goods components are given by

$$(38) \quad x_{Nt}(s) = \tau \left[ \frac{p_{Nt}(s)}{p_t(s)} \right]^{-\mu} x_t(s) \text{ and } x_{Tt}(s) = (1 - \tau) \left[ \frac{p_{Tt}}{p_t(s)} \right]^{-\mu} x_t(s).$$

The tradable good consumed on island  $s$  is a composite of varieties of differentiated tradable goods produced on all islands,

$$x_{Tt}(s) = \left[ \int_S x_{Tt}(s, s')^{\frac{\mu_T - 1}{\mu_T}} ds' \right]^{\frac{\mu_T}{\mu_T - 1}},$$

where  $\mu_T$  is the elasticity of substitution between tradable good varieties, and  $x_{Tt}(s, s')$  is the amount of the variety of the tradable good produced on island  $s'$  and consumed on island  $s$ . The world price of the composite tradable good is

$$p_{Tt} = \left[ \int_S p_{Tt}(s)^{1 - \mu_T} ds \right]^{\frac{1}{1 - \mu_T}},$$

where  $p_{Tt}(s)$  is the price of the tradable good variety produced on island  $s$ . The demand on island  $s'$  for the tradable good variety produced on island  $s$  is

$$x_{Tt}(s', s) = \left[ \frac{p_{Tt}(s)}{p_{Tt}} \right]^{-\mu_T} x_{Tt}(s'),$$

where  $x_{Tt}(s')$  is the demand for the composite tradable good on island  $s'$ . Let  $y_{Tt} = y_T$  be the constant world demand for the composite tradable good. The world demand for the composite tradable good produced on island  $s$  is given by

$$(39) \quad y_{Tt}(s) = \left[ \frac{p_{Tt}(s)}{p_{Tt}} \right]^{-\mu_T} y_T.$$

We normalize the world price of the composite tradable good,  $p_{Tt}$ , to one, so that the composite tradable good is the numeraire.

### ***Technology***

We follow closely the setup in Kehoe, Midrigan, and Pastorino (2019), in which nontradable and tradable goods on each island  $s$  are produced only with locally produced intermediate goods. These



intermediate goods are used in different proportions by the nontradable and tradable goods sectors. This setup implies a curved production possibility frontier between nontradable and tradable goods sectors and hence effectively introduces endogenous costs of sectoral reallocation of workers.

Formally, each island  $s$  produces two types of intermediate goods, type  $\mathcal{N}$  and type  $\mathcal{T}$  goods. The technology for producing nontradable goods disproportionately uses type  $\mathcal{N}$  goods, whereas the technology for producing tradable goods disproportionately uses type  $\mathcal{T}$  goods, according to the production technologies

$$(40) \quad y_{Nt}(s) = A (y_{Nt}^{\mathcal{N}}(s))^{\nu} (y_{Nt}^{\mathcal{T}}(s))^{1-\nu} \quad \text{and} \quad y_{Tt}(s) = A (y_{Tt}^{\mathcal{N}}(s))^{1-\nu} (y_{Tt}^{\mathcal{T}}(s))^{\nu},$$

with  $\nu \geq 1/2$ . Here,  $y_{Nt}^{\mathcal{N}}(s)$  and  $y_{Tt}^{\mathcal{N}}(s)$  denote the use of intermediate goods of type  $\mathcal{N}$  as inputs in the production of nontradable and tradable goods, respectively, on island  $s$ , whereas  $y_{Nt}^{\mathcal{T}}(s)$  and  $y_{Tt}^{\mathcal{T}}(s)$  denote the use of intermediate goods of type  $\mathcal{T}$  as inputs in the production of nontradable and tradable goods, respectively, on island  $s$ . Both nontradable and tradable goods producers are competitive and take as given the price of their goods,  $p_{Nt}(s)$  and  $p_{Tt}(s)$ . The demands for intermediate goods in the nontradable goods sector are given by

$$(41) \quad y_{Nt}^{\mathcal{N}}(s) = \nu \left[ \frac{p_{Tt}^{\mathcal{T}}(s)}{p_{Tt}^{\mathcal{N}}(s)} \right]^{1-\nu} y_{Nt}(s) \quad \text{and} \quad y_{Nt}^{\mathcal{T}}(s) = (1 - \nu) \left[ \frac{p_{Tt}^{\mathcal{N}}(s)}{p_{Tt}^{\mathcal{T}}(s)} \right]^{\nu} y_{Nt}(s),$$

where  $p_{Tt}^{\mathcal{N}}(s)$  and  $p_{Tt}^{\mathcal{T}}(s)$  are the prices of the intermediate goods of type  $\mathcal{N}$  and  $\mathcal{T}$ , and we have used the convenient normalization  $A = \nu^{-\nu} (1 - \nu)^{-(1-\nu)}$ . Likewise, the demands for intermediate goods in the tradable goods sector are given by

$$y_{Tt}^{\mathcal{N}}(s) = (1 - \nu) \left[ \frac{p_{Tt}^{\mathcal{T}}(s)}{p_{Tt}^{\mathcal{N}}(s)} \right]^{\nu} y_{Tt}(s) \quad \text{and} \quad y_{Tt}^{\mathcal{T}}(s) = \nu \left[ \frac{p_{Tt}^{\mathcal{N}}(s)}{p_{Tt}^{\mathcal{T}}(s)} \right]^{1-\nu} y_{Tt}(s).$$

Adding up the demands for each type of intermediate good by the two sectors gives the total demand on island  $s$  for intermediate goods of type  $i$ ; that is,

$$(42) \quad y_t^i(s) = y_{Nt}^i(s) + y_{Tt}^i(s) \quad \text{for } i \in \{\mathcal{N}, \mathcal{T}\}.$$

Production of these intermediate goods is given by

$$(43) \quad y_t^i(s) = \int_{z,h} z h e_t^i(z, h, s) dz dh,$$

where  $e_t^i(z, h, s)$  is the measure of employed workers with human capital  $(z, h)$  producing interme-

diate goods of type  $i$  on island  $s$ . The zero-profit condition in the nontradable and tradable goods sectors implies that

$$p_{Nt}(s) = (p_t^{\mathcal{N}}(s))^{\nu} (p_t^{\mathcal{T}}(s))^{1-\nu} \quad \text{and} \quad p_{Tt}(s) = (p_t^{\mathcal{N}}(s))^{1-\nu} (p_t^{\mathcal{T}}(s))^{\nu}.$$

We assume that there exist measures  $\pi^{\mathcal{N}}$  and  $\pi^{\mathcal{T}} = 1 - \pi^{\mathcal{N}}$  of workers who supply labor to produce the two types of intermediate goods  $\mathcal{N}$  and  $\mathcal{T}$ , respectively. We refer to these workers as in *occupations*  $\mathcal{N}$  and  $\mathcal{T}$ . Workers in occupation  $i \in \{\mathcal{N}, \mathcal{T}\}$  are hired by the firms that produce intermediate goods of type  $i$ . These goods are then sold at competitive prices  $p_t^{\mathcal{N}}(s)$  and  $p_t^{\mathcal{T}}(s)$  to firms in the nontradable and tradable goods sectors. Of course, it is equivalent to interpret workers in each occupation as working in the final goods sector that purchases the intermediate goods they produce. According to this interpretation, workers are hired by employment agencies for workers in either occupation  $\mathcal{N}$  or  $\mathcal{T}$ , which rent labor services to final goods producers in sectors  $N$  and  $T$ . Thus, we can think of workers in occupation  $\mathcal{N}$  as being employed in sectors  $N$  and  $T$  and workers in occupation  $\mathcal{T}$  as also being employed in sectors  $N$  and  $T$  but in different proportions. In particular, sector  $N$  employs workers in occupation  $\mathcal{N}$  relatively more intensively, whereas sector  $T$  employs workers in occupation  $\mathcal{T}$  relatively more intensively.

This setup captures, in a simple way, the idea that switching sectors is relatively easy, whereas switching occupations is difficult. In particular, any individual worker faces no cost of switching sectors, but if a positive measure of workers reallocates from one sector to the other, then the marginal revenue product of workers in the latter sector falls and so do wages. Hence, this reduction in marginal revenue product acts like a switching cost in the aggregate.

### ***Families' Problems***

Each family of workers on island  $s$  chooses sequences for consumption  $\{c_{wt}(s)\}$  and debt  $\{d_{wt+1}(s)\}$  in order to maximize the present discounted value of consumption  $\sum_{t=0}^{\infty} \beta^t u(c_{wt}(s) + b_t(s))$ , where  $b_t(s)$  is the total amount of home-produced goods of the family's nonemployed members, subject to the budget constraint

$$p_t(s)c_{wt}(s) + d_{wt}(s) \leq y_{wt}(s) + qd_{wt+1}(s)$$

and the borrowing constraint

$$d_{wt+1}(s) \leq \chi_{wt}(s).$$

In so doing, a family of workers takes as given market prices and its income

$$y_{wt}(s) \equiv \sum_{i \in \{\mathcal{N}, \mathcal{T}\}} \int_{z,h} w_t^i(z, h, s) e_t^i(z, h, s) dz dh + T_{wt}(s),$$

where  $w_t^i(z, h, s)$  and  $e_t^i(z, h, s)$  are the wages and the measure of employed workers in occupation  $i \in \{\mathcal{N}, \mathcal{T}\}$  with general human capital  $z$  and firm-specific human capital  $h$  on island  $s$ .

Similarly, each family of entrepreneurs on island  $s$  chooses sequences for consumption  $\{c_{ft}(s)\}$  and debt  $\{d_{ft+1}(s)\}$  so as to maximize the present discounted value of consumption  $\sum_{t=0}^{\infty} \beta^t u(c_{ft}(s))$ , subject to the budget constraint

$$p_t(s)c_{ft}(s) + d_{ft}(s) \leq y_{ft}(s) + qd_{ft+1}(s)$$

and the borrowing constraint

$$d_{ft+1}(s) \leq \chi_{ft}(s).$$

A family of entrepreneurs takes as given market prices and its income

$$y_{ft}(s) \equiv y + \sum_{i \in \{\mathcal{N}, \mathcal{T}\}} \int_{z,h} [zh - w_t^i(z, h, s)] e_t^i(z, h, s) dz dh - \sum_{i \in \{\mathcal{N}, \mathcal{T}\}} \int_z \kappa v_t^i(z, s) dz + T_{ft}(s),$$

which consists of the endowment  $y$  as well as the profits from the firms that the family owns in both the nontradable and tradable goods sectors. As in the one-good economy considered earlier, through the endowment  $y$  we capture, in a reduced-form way, all the returns to being an entrepreneur in addition to the profits from hiring workers net of vacancy posting costs. From the first-order conditions for the families' problems, we can derive the shadow prices of the composite consumption good at date  $t$  in units of the composite consumption good at date 0 for the two types of families on island  $s$  as

$$Q_{wt}(s) = \frac{\beta^t u'(c_{wt}(s) + b_t(s))/p_t(s)}{u'(c_{w0}(s) + b_0(s))/p_0(s)} \text{ and } Q_{ft}(s) = \frac{\beta^t u'(c_{ft}(s))/p_t(s)}{u'(c_{f0}(s))/p_0(s)}.$$

We will consider experiments in which we tighten debt constraints on one island at a time. For simplicity, we focus on the case in which these constraints always bind, both in the steady state before the debt tightening and in the transition to the new steady state after the debt tightening. For this assumption to hold, the islands of interest need to be impatient relative to the rest of the

world in that their discount factor  $\beta$  satisfies  $\beta < q$ . To ensure that the world bond price is  $q$ , we assume that in the rest of the world, there exists a sufficient number of islands in which families have discount factor  $\beta^* = q$  and are willing to lend at price  $q$  so that the bond market clears at this price.

### **Labor Market**

Workers on each island  $s$  work for firms that produce intermediate goods and sell them to firms that produce either nontradable or tradable goods on that island. Firms that produce intermediate good  $i \in \{\mathcal{N}, \mathcal{T}\}$  post vacancies for workers in occupation  $i$  with general human capital  $z$ , who produce intermediate good  $i$  when employed. Since we assume that workers cannot switch occupations, the measure of workers in each occupation is fixed. After suppressing any explicit dependence on  $s$  for notational simplicity, the values of workers in occupation  $i$  on a given island are similar to those in our one-good model and are given by

$$(44) \quad \begin{aligned} W_t^i(z, h) &= w_t^i(z, h) + \phi(1 - \sigma) \frac{Q_{wt+1}}{Q_{wt}} \int_{z'} \max [W_{t+1}^i(z', h'), U_{t+1}^i(z')] dF_e(z'|z) \\ &+ \phi\sigma \frac{Q_{wt+1}}{Q_{wt}} \int_{z'} U_{t+1}^i(z') dF_e(z'|z) \end{aligned}$$

for employed workers with general human capital  $z$  and firm-specific human capital  $h$ , and

$$(45) \quad \begin{aligned} U_t^i(z) &= p_t b(z) + \phi \lambda_{wt}^i(z) \frac{Q_{wt+1}}{Q_{wt}} \int_{z'} \max [W_{t+1}^i(z', 1), U_{t+1}^i(z')] dF_u(z'|z) \\ &+ \phi [1 - \lambda_{wt}^i(z)] \frac{Q_{wt+1}}{Q_{wt}} \int_{z'} U_{t+1}^i(z') dF_u(z'|z) \end{aligned}$$

for nonemployed workers with general human capital  $z$ , where  $w_t^i(z, h)$  is the wage received by a worker in occupation  $i$  as a function of human capital  $(z, h)$  and  $\lambda_{wt}^i(z)$  is the job-finding rate of a worker with general human capital  $z$  in occupation  $i$ .

The value of a firm producing intermediate good  $i$  matched with a worker with human capital  $(z, h)$  in occupation  $i$  is

$$(46) \quad J_t^i(z, h) = p_t^i z h - w_t^i(z, h) + \phi(1 - \sigma) \frac{Q_{ft+1}}{Q_{ft}} \int_{z'} \max [J_{t+1}^i(z', h'), 0] dF_e(z'|z).$$

That is, a worker with human capital  $(z, h)$  in occupation  $i$  matched at date  $t$  with a firm in intermediate good sector  $i$  produces  $zh$  units of good  $i$ , which sell for  $p_t^i zh$ , and receives the wage  $w_t^i(z, h)$ . The cost of posting a vacancy in either sector is  $\kappa$  units of the composite tradable good.

The matches of firms that produce intermediate good  $i$  with workers with general human capital

$z$  are created according to the matching function  $m_t^i(z) = u_t^i(z)v_t^i(z)/[u_t^i(z)^\eta + v_t^i(z)^\eta]^{\frac{1}{\eta}}$ , where  $u_t^i(z)$  is the measure of nonemployed workers with general human capital  $z$  in occupation  $i$ , and  $v_t^i(z)$  is the measure of vacancies directed at such workers. The associated job-finding rate  $\lambda_{wt}^i(z)$  and job-filling rate  $\lambda_{ft}^i(z)$  are defined as before. The determination of wages by Nash bargaining is analogous to that in the one-good model. Finally, free entry for intermediate goods producers in the labor market for workers in occupation  $i \in \{\mathcal{N}, \mathcal{T}\}$  with general human capital  $z$  implies

$$\kappa \geq \phi \lambda_{ft}^i(z) \frac{Q_{ft+1}}{Q_{ft}} \int_{z'} \max [J_{t+1}^i(z', 1), 0] dF_u(z'|z),$$

with equality for any market  $z$  with open vacancies.

### ***Equilibrium***

Consider now the market-clearing conditions. Market clearing for the two types of intermediate goods requires that

$$(47) \quad \int_{z,h} z h e_t^i(z, h, s) dz dh = y_{Nt}^i(s) + y_{Tt}^i(s) \text{ for } i \in \{\mathcal{N}, \mathcal{T}\}$$

by (42) and (43). The left side of this equation is the total amount of intermediate goods of type  $i$  produced by employed workers with human capital  $(z, h)$  in occupation  $i$  on island  $s$ ,  $e_t^i(z, h, s)$ , whereas the right side is the total amount of these intermediate goods used by firms in the non-tradable and tradable goods sectors on that island. According to the interpretation of the economy discussed earlier, whereby workers in each occupation work in the final goods sector that purchases the intermediate goods they produce, we measure employment in the nontradable goods sector on island  $s$  as

$$(48) \quad \frac{y_{Nt}^{\mathcal{N}}(s)}{y_t^{\mathcal{N}}(s)} \int_{z,h} e_t^{\mathcal{N}}(z, h, s) dz dh + \frac{y_{Nt}^{\mathcal{T}}(s)}{y_t^{\mathcal{T}}(s)} \int_{z,h} e_t^{\mathcal{T}}(z, h, s) dz dh$$

and employment in the tradable goods sector on island  $s$  as

$$(49) \quad \frac{y_{Tt}^{\mathcal{N}}(s)}{y_t^{\mathcal{N}}(s)} \int_{z,h} e_t^{\mathcal{N}}(z, h, s) dz dh + \frac{y_{Tt}^{\mathcal{T}}(s)}{y_t^{\mathcal{T}}(s)} \int_{z,h} e_t^{\mathcal{T}}(z, h, s) dz dh,$$

where  $y_t^{\mathcal{N}}(s)$  and  $y_t^{\mathcal{T}}(s)$  are defined in (42). In our later discussion, to explain some of our results, we will rely on the idea of *relative demand effect* on employment in the two sectors. Intuitively, since  $y_{Nt}^i(s)/y_t^i(s) + y_{Tt}^i(s)/y_t^i(s) = 1$  for  $i \in \{\mathcal{N}, \mathcal{T}\}$  by (42), any shift in demand from the nontradable goods sector to the tradable goods sector on an island, holding fixed total employment on the island,

decreases employment in the nontradable goods sector and increases it in the tradable goods sector on the island.

Market clearing for nontradable goods requires that the demand for nontradable goods on island  $s$  from (38) equal the amount of nontradable goods produced on island  $s$  from (40) so that

$$x_{Nt}(s) = A (y_{Nt}^N(s))^\nu (y_{Nt}^T(s))^{1-\nu} .$$

Similarly, market clearing for tradable goods requires that the world demand for tradable goods produced on island  $s$  from (39) equal the amount of tradable goods produced on island  $s$  from (40) so that

$$y_{Tt}(s) = A (y_{Tt}^N(s))^{1-\nu} (y_{Tt}^T(s))^\nu .$$

### C. Deleveraging Shocks and Policy Experiments

Here, we first describe the impact of a credit tightening affecting families of workers and entrepreneurs. Then, we discuss the effects of policy interventions that partly offset such a tightening.

#### *Deleveraging Shocks*

We adopt the parameterization in Kehoe, Midrigan, and Pastorino (2019), which matches the elasticities documented in Figure 12 and hence generates cross-state patterns for consumption and employment that are consistent with those observed in the United States during the Great Recession. In this model economy, like in the one-good model economy, a state-specific credit tightening affecting either type of family has an *investment effect*, which implies that the cost to a firm of hiring a worker by posting a job vacancy increases by more than the corresponding benefit. A novel effect that arises in this economy is that a state-specific credit tightening has also a *relative demand effect*, which reduces the demand for the nontradable goods produced in the state and hence the demand for workers by that sector. The resulting reallocation of workers from the nontradable to the tradable goods sector then reinforces the drop in nontradable employment due to the investment effect and could even lead to a relative increase in tradable employment in some states, as is consistent with the data.

#### *Policy Experiments*

We consider a class of state-level government interventions aimed at workers in the form of a sequence of transfers  $\{T_{wt}\}$ , measured in units of the composite consumption good, that would reduce the consumption decline experienced by U.S. states between 2007 and 2009 by 50 percent.

Such transfers are constructed so as to offset, by a certain percentage, the drop on impact in state-level consumption due to a state-specific credit tightening and then mean revert to zero at a rate of 10 percent per quarter. For a concrete example, consider Nevada. As depicted in Figure 13, consumption in Nevada between 2007 and 2009 fell by around 7.8 percent. For Nevada, we construct a sequence of transfers to workers  $\{T_{wt}\}$  so that state-level consumption falls by only about 3.9 percent between 2007 and 2009. To make the intervention on firms parallel, we then consider the case in which the government implements the same sequence of transfers to firms  $\{T_{ft}\}$  in that  $T_{ft} = T_{wt}$  at each  $t$ . These same transfers directed to firms, rather than to workers, lead state-level consumption in Nevada to fall a bit less—namely, by 3.4 percent rather than by 3.9 percent. In Figure 13, we graph the analogous changes in consumption for each U.S. state.<sup>10</sup> For Nevada, the blue bar (left) shows the original consumption drop of 7.8 percent, the green bar (middle) shows a drop of 3.9 percent when transfers support workers’ credit, and the orange bar (right) shows a drop of 3.4 percent when transfers support firms’ credit.

In Figure 14, we graph the corresponding changes in employment for each state. For Nevada, the blue bar shows the original drop in employment of 8.6 percent, the green bar shows that this drop would be much smaller—only 4.5 percent—if transfers supported workers’ credit, and the orange bar shows that this drop would be 6.9 percent if transfers supported firms’ credit. Perusing the figure makes it clear that for every state, a given sequence of transfers directed to workers has a larger positive effect on employment than an identical sequence of transfers directed to firms.

In Figures 15 and 16, we decompose the effect on total employment in each state into the effects on nontradable and tradable employment. Supporting workers has a larger stimulative effect on employment in both sectors. In the case of Nevada, for instance, a transfer to workers would lead to a drop in nontradable employment of only 2.7 percent, rather than the observed 8.2 percent. A transfer to firms would have a smaller impact, in that nontradable employment would fall by 5 percent rather than by 8.2 percent. To use the example of Nevada again, supporting workers would spur tradable employment by 2.6 percent, so that instead of dropping by 17.6 percent, tradable employment would drop only by 15 percent, whereas supporting firms would lead to almost no change in tradable employment. Inspecting these figures reveals that this same pattern emerges in all states: a given transfer to workers has a larger positive effect on employment in each sector than does the same-sized transfer to firms.

Either intervention leads to a much greater response in nontradable employment than in trad-

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<sup>10</sup>Note that Louisiana and Mississippi display a positive change in (instrumented) consumption between 2007 and 2009 and so receive no transfer in our experiments.

able employment because of the relative demand effect. Recall that nontradable goods produced in state  $s$  are consumed solely by residents of state  $s$ , whereas tradable goods produced in state  $s$  are consumed throughout the world. Hence, when the government transfers resources to either type of family, it causes a substantial increase in the demand for nontradable goods in a given state but essentially no increase in the demand for tradable goods produced in the state. As a result, after such an intervention, the fractions of both types of intermediate goods absorbed by the nontradable goods sector—namely,  $y_{N_t}^N(s)/y_t^N(s)$  and  $y_{N_t}^T(s)/y_t^T(s)$ —increase by (48), which in turn induces an increase in employment in the nontradable goods sector. Thus, in the nontradable goods sector, this relative demand effect reinforces the investment effect, and so employment in the nontradable goods sector rises sharply. As workers who reallocate to the nontradable goods sector come from the tradable goods sector, this flow of workers to the nontradable goods sector almost completely offsets the positive investment effect of transfers on employment in the tradable goods sector. Then, employment in the tradable goods sector overall barely changes.

To make these effects more transparent, consider the top left panel of Figure 17. Here, we graph the consumption change between 2007 and 2009 in each state (horizontal axis) against the consumption change that, according to our model, would have occurred in each state over the same period if the government had implemented transfers solely to workers, marked by an “×,” or solely to firms, marked by a “+” (vertical axis). In the top right panel of the figure, for each state, we plot the employment change observed over this same period (horizontal axis) against the employment change that our model predicts would have occurred under these two interventions (vertical axis). For each state, the two bottom panels of the figure graph the observed changes in nontradable (bottom left panel) and tradable (bottom right panel) employment between 2007 and 2009 (horizontal axis) against the changes that our model implies for these variables under the two interventions (vertical axis). As is apparent from the figure, transfers to workers lead to a larger increase in state-level employment, nontradable employment, and tradable employment than transfers to firms, as the points marked by “×” are all above those marked by “+” in the corresponding panels.

## 7. Conclusion

A major theme in Mian and Sufi (2015) and Kehoe, Midrigan, and Pastorino (2019) is that the household-side channel of the credit tightening that occurred during the Great Recession is central to understanding the consumption and employment declines experienced by the United States between 2007 and 2009. Here, we have developed a quantitative model that sheds light on the workings and relative importance of both the household-side and firm-side channels of a credit tightening and



successfully accounts for the cross-sectional patterns of consumption and employment across U.S. states during the Great Recession.

We have consistently shown that a tightening of household debt constraints leads to a larger decline in employment than an equal-sized tightening of firm debt constraints. The key feature of our model behind this result is that an additional component of the surplus from a match between a firm and a worker arises when workers accumulate human capital on the job that is even partially transferable to other matches. Since this surplus component is long-lived, it gives rise to match surplus flows that have long durations and, as such, are sensitive to changes in discounting. Because such a surplus component is discounted primarily by workers' shadow value of future to present goods rather than firms', the present value of match surplus flows is highly sensitive to any tightening of credit to households, since a credit tightening affects workers' shadow values. As a result, the household-side channel of a deleveraging shock is quantitatively more important than the firm-side channel. Likewise, we have also shown that after a credit tightening severe enough to trigger a downturn, a government intervention that transfers income to households, and thus lessens the impact of the tightening on their consumption, would have been more effective than an equal-sized intervention to firms at stimulating the recovery of employment in the cross-section of U.S. states.

One limitation of our analysis is that we consider unanticipated shocks to debt constraints. An important extension would be to make debt constraints stochastic and examine the extent to which self-insurance through precautionary savings by workers and firms can partially offset the impact of a credit tightening. Doing so would make the model more amenable to addressing the full business-cycle implications of fluctuating debt constraints and their relationship to firm dynamics. In this vein, see the canonical paper by Cooley and Quadrini (2001) on the dynamics of firms in the presence of financial frictions. Another useful extension would be to provide a foundation for the financial frictions we examine, possibly along the lines of limited commitment models as in Cooley, Marimon, and Quadrini (2004). Note that if debt constraints endogenously arise from commitment issues, then government interventions, by affecting workers' and firms' behavior, would typically lead to an endogenous response of these constraints, which is beyond the scope of our current exercise.

## **Appendix**

This appendix contains details and results omitted from the main text.

## A Collateral Constraint Economy

Here, we prove an equivalence result in terms of consumption and labor market outcomes between our economy and an economy with household mortgage debt and firm corporate debt collateralized by housing, interpreted as either private or commercial real estate. This equivalence result then allows us to interpret the responses of consumption and labor market variables in our baseline economy following a credit tightening as generated by a fall in real estate prices that leads to a tightening of collateral constraints, as many have argued occurred during the Great Recession. This result also makes it clear that a robust nexus between consumption and labor market outcomes exists across these two economies.

### A Family of Workers' Problem

In this economy, a family of workers owns houses and its borrowing is subject to collateral constraints based on the value of its houses. The problem of any such family consists of choosing sequences for consumption  $\{c_{wt}\}$ , debt  $\{d_{wt+1}\}$ , and housing stock  $\{h_{wt+1}\}$  so as to maximize the present discounted value of consumption of goods and housing services  $\sum_{t=0}^{\infty} \beta^t [u(c_{wt}) + \psi_{wt}v(h_{wt})]$ , subject to the budget constraint

$$(50) \quad c_{wt} + d_{wt} + p_{wt}h_{wt+1} \leq \int_{z,h} w_t(z, h)e_t(z, h)dzdh + \int_z b(z)u_t(z)dz + qd_{wt+1} + p_{wt}h_{wt} + T_{wt}$$

and the collateral constraint

$$(51) \quad d_{wt+1} \leq \bar{\chi}_w p_{wt} h_{wt+1}.$$

The exogenous process  $\{\psi_{wt}\}$  captures fluctuations in the desirability of housing relative to consumption goods, and  $\bar{\chi}_w$  represents the maximum mortgage loan-to-value ratio.

A family of workers that owns  $h_{wt}$  housing units has housing wealth equal to  $p_{wt}h_{wt}$ , chooses its next-period stock of housing  $h_{wt+1}$ , and faces a collateral constraint that limits the amount it can borrow to a fixed fraction  $\bar{\chi}_w$  of the value of its housing units. Such a family's budget constraint in period  $t$  reflects the fact that a family enters period  $t$  with the housing stock  $h_{wt}$  valued at  $p_{wt}$  and chooses the new housing stock to purchase  $h_{wt+1}$  at price  $p_{wt}$ . That is, a family pays  $p_{wt}(h_{wt+1} - h_{wt})$  to adjust its housing stock from size  $h_{wt}$  to size  $h_{wt+1}$  at  $t$ . The supply of residential housing is fixed at unity so in equilibrium  $h_{wt} = 1$ . Note that fluctuations in the taste for housing  $\psi_{wt}$  in this economy are a simple device to make the price of housing fluctuate. The collateral constraint parameter  $\bar{\chi}_w$ , though, is constant, so fluctuations in the maximum value of borrowing are induced

by fluctuations in the price of housing  $p_{wt}$  rather than by direct fluctuations in the debt constraint parameter  $\chi_{wt}$ , as in our baseline model.

For a family of workers, the first-order condition for consumption is

$$(52) \quad \beta^t u'(c_{wt})/u'(c_{w0}) = Q_{wt},$$

the first-order condition for debt is

$$(53) \quad qQ_{wt} = Q_{wt+1} + Q_{wt}\theta_{wt},$$

the first-order condition for housing is

$$(54) \quad Q_{wt}p_{wt} = \beta^{t+1}\psi_{wt+1}v'(h_{wt+1}) + Q_{wt+1}p_{wt+1} + Q_{wt}\theta_{wt}\bar{\chi}_w p_{wt},$$

and the complementary slackness condition is

$$(55) \quad \theta_{wt}(d_{wt+1} - \bar{\chi}_w p_{wt} h_{wt+1}) = 0$$

at each  $t$ , where  $Q_{wt}$  is the multiplier on the budget constraint, and  $Q_{wt}\theta_{wt}$  is the multiplier on the collateral constraint with the normalization  $Q_{w0} = 1$ .

### ***A Family of Entrepreneurs' Problem***

In this economy, a family of entrepreneurs owns a type of housing, referred to as *commercial real estate*, different from that owned by a family of workers, and its borrowing is subject to collateral constraints based on the value of the real estate that it owns. The problem of any such family consists of choosing sequences for consumption  $\{c_{ft}\}$ , debt  $\{d_{ft+1}\}$ , and commercial real estate  $\{h_{ft+1}\}$  in order to maximize the present discounted value of consumption of goods and housing services  $\sum_{t=0}^{\infty} \beta^t [u(c_{ft}) + \psi_{ft}v(h_{ft})]$ , subject to the budget constraint

$$(56) \quad c_{ft} + d_{ft} + p_{ft}h_{ft+1} \leq y + \int_{z,h} [zh - w_t(z, h)] e_t(z, h) dz dh - \int_z \kappa v_t(z) dz + qd_{ft+1} + p_{ft}h_{ft} + T_{ft}$$

and the collateral constraint

$$(57) \quad d_{ft+1} \leq \bar{\chi}_f p_{ft} h_{ft+1},$$

where the exogenous process  $\{\psi_{ft}\}$  captures fluctuations in the desirability of commercial housing relative to consumption goods, and  $\bar{\chi}_f$  represents the associated maximum loan-to-value ratio. A family of entrepreneurs owns commercial real estate  $h_{ft}$  with value  $p_{ft}h_{ft}$ , chooses its next-period stock of commercial real estate  $h_{ft+1}$ , and faces a collateral constraint that limits the amount it can borrow to a fraction  $\bar{\chi}_f$  of the value of its commercial real estate holding. The supply of commercial real estate is fixed at unity so in equilibrium  $h_{ft} = 1$ .

For a family of entrepreneurs, the first-order condition for consumption is

$$(58) \quad \beta^t u'(c_{ft})/u'(c_{f0}) = Q_{ft},$$

the first-order condition for debt is

$$(59) \quad qQ_{ft} = Q_{ft+1} + Q_{ft}\theta_{ft},$$

the first-order condition for housing is

$$(60) \quad Q_{ft}p_{ft} = \beta^{t+1}\psi_{ft+1}v'(h_{ft+1}) + Q_{ft+1}p_{ft+1} + Q_{ft}\theta_{ft}\bar{\chi}_fp_{ft},$$

and the complementary slackness condition is

$$(61) \quad \theta_{ft}(d_{ft+1} - \bar{\chi}_fp_{ft}h_{ft+1}) = 0$$

at each  $t$ , where  $Q_{ft}$  is the multiplier on the budget constraint,  $Q_{ft}\theta_{ft}$  is the multiplier on the collateral constraint, and we have adopted the normalization  $Q_{f0} = 1$ .

### An Equivalence Result

We now turn to establishing an equivalence result between the economy with debt constraints examined in the main text and the economy with collateral constraints described here. Consider the economy with collateral constraints. Given the exogenous sequences of taste parameters  $\{\psi_{wt}, \psi_{ft}\}$  and the world bond price  $q$ , let  $\{\bar{Q}_{wt}, \bar{Q}_{ft}\}$  denote the resulting sequences of shadow prices of the two types of family. Given these shadow prices, the labor market side of the collateral constraint economy is identical to the labor market side of the debt constraint economy. Hence, in the two economies, workers' and firms' value functions, matching and bargaining, and the free-entry conditions are identical. Intuitively, the only ingredient needed from the family problems to solve the rest of the model are the sequences of shadow prices  $\{\bar{Q}_{wt}, \bar{Q}_{ft}\}$ .

We proceed by first showing that given a debt constraint economy with sequences of borrowing limits  $\{\chi_{wt}, \chi_{ft}\}$ , we can construct sequences of taste parameters  $\{\psi_{wt}, \psi_{ft}\}$  for a collateral constraint economy such that, except for the prices and quantities of housing, equilibrium allocations and prices in the two economies coincide. We then show that given a collateral constraint economy with sequences of taste parameters  $\{\psi_{wt}, \psi_{ft}\}$ , we can construct sequences of borrowing limits  $\{\chi_{wt}, \chi_{ft}\}$  for a debt constraint economy such that, except for the prices and quantities of housing, equilibrium allocations and prices in the two economies coincide. In this construction, we set the sequences of transfers to workers and firms  $\{T_{wt}, T_{ft}\}$  to zero in the two economies for simplicity.

To start, note that in the debt constraint economy, the solutions to the worker and entrepreneur families' problems are completely characterized by the first-order and complementary slackness conditions for workers in (7), (8), and (9); worker budget and debt constraints in (4) and (5); the first-order and complementary slackness conditions for firms in (14), (15), and (16); and firm budget and debt constraints in (11) and (12). Similarly, in the collateral constraint economy, the solutions to the worker and entrepreneur families' problems are completely characterized by the first-order and complementary slackness conditions for workers in (52), (53), (54), and (55); worker budget and collateral constraints in (50) and (51); the first-order and complementary slackness conditions for firms in (58), (59), (60), and (61); and firm budget and collateral constraints in (56) and (57).

Let us provisionally assume that we can choose sequences of taste parameters so that the shadow prices of goods in the two economies coincide; that is,

$$(62) \quad \bar{Q}_{wt} = Q_{wt} \text{ and } \bar{Q}_{ft} = Q_{ft}$$

at each  $t$ . We will later show that this is the case. Under this assumption, we claim that the budget constraints in the debt constraint economy,

$$(63) \quad c_{wt} + d_{wt} \leq y_{wt} + qd_{wt+1}$$

and

$$(64) \quad c_{ft} + d_{ft} \leq y_{ft} + qd_{ft+1},$$

where  $y_{wt}$  and  $y_{ft}$  are defined in (6) and (13), are equal to the corresponding ones in the collateral constraint economy. To see why, note that in the collateral constraint economy, the terms for housing on both sides of worker and firm budget constraints cancel out, since  $h_{wt} = 1$  and  $h_{ft} = 1$

at each  $t$ . Hence, worker and firm budget constraints in the collateral constraint economy have the exact form of (63) and (64) as in the debt constraint economy with the same definitions for  $y_{wt}$  and  $y_{ft}$ . Next, observe that by our earlier argument, the labor market side of the collateral constraint economy is identical to the labor market side of the debt constraint economy under (62), and so are wages  $\{w_t(z, h)\}$  and labor market allocations  $\{e_t(z, h), u_t(z), v_t(z)\}$ . Then, family incomes  $y_{wt}$  and  $y_{ft}$  are the same in both economies at each  $t$ . Thus, families' budget constraints in the two economies coincide.

Now, under our provisional assumption, the shadow prices of goods satisfy (62) given the multipliers  $\{\theta_{wt}\}$  and  $\{\theta_{ft}\}$  from the debt constraint economy, which satisfy (8) and (15). Since the constructed multipliers  $\{\theta_{wt}\}$  and  $\{\theta_{ft}\}$  for the collateral constraint economy satisfy (53) and (59), these multipliers must be equal in the two economies. Next, note that for the debt constraints to be equivalent to the collateral constraints, we need  $\bar{\chi}_w$ ,  $p_{wt}$ ,  $\bar{\chi}_{ft}$ , and  $p_{ft}$  to satisfy

$$(65) \quad \bar{\chi}_w p_{wt} = \chi_{wt} \text{ and } \bar{\chi}_f p_{ft} = \chi_{ft},$$

which we ensure hold by setting house prices to satisfy  $p_{wt} = \chi_{wt}/\bar{\chi}_w$  and  $p_{ft} = \chi_{ft}/\bar{\chi}_f$ , respectively. Given these house prices, the intertemporal shadow prices  $\{Q_{wt}, Q_{ft}\}$ , and the multipliers  $\{\theta_{wt}, \theta_{ft}\}$ , we need the sequences of taste parameters  $\{\psi_{wt}, \psi_{ft}\}$  to satisfy the first-order conditions for worker and firm housing in the collateral constraint economy at the market-clearing values of  $h_{wt} = 1$  and  $h_{ft} = 1$ ; namely,

$$Q_{wt} p_{wt} = \beta^{t+1} \psi_{wt+1} v'(1) + Q_{wt+1} p_{wt+1} + Q_{wt} \theta_{wt} \bar{\chi}_w p_{wt},$$

and

$$Q_{ft} p_{ft} = \beta^{t+1} \psi_{ft+1} v'(1) + Q_{ft+1} p_{ft+1} + Q_{ft} \theta_{ft} \bar{\chi}_f p_{ft}.$$

The key step to guarantee that these conditions hold consists of using these expressions and (65) to set the sequences of taste parameters  $\{\psi_{wt}, \psi_{ft}\}$  to satisfy

$$\psi_{wt+1} = \frac{Q_{wt} p_{wt} - Q_{wt+1} p_{wt+1} - Q_{wt} \theta_{wt} \bar{\chi}_w p_{wt}}{\beta^{t+1} v'(1)} = \frac{1}{\beta^{t+1} v'(1)} \left( Q_{wt} \frac{\chi_{wt}}{\bar{\chi}_w} - Q_{wt+1} \frac{\chi_{wt+1}}{\bar{\chi}_w} - Q_{wt} \theta_{wt} \chi_{wt} \right)$$

and

$$\psi_{ft+1} = \frac{Q_{ft}p_{ft} - Q_{ft+1}p_{ft+1} - Q_{ft}\theta_{ft}\bar{\chi}_f p_{ft}}{\beta^{t+1}v'(1)} = \frac{1}{\beta^{t+1}v'(1)} \left( Q_{ft} \frac{\chi_{ft}}{\bar{\chi}_f} - Q_{ft+1} \frac{\chi_{ft+1}}{\bar{\chi}_f} - Q_{ft}\theta_{ft}\chi_{ft} \right).$$

We thus set the consumption sequences  $\{c_{wt}, c_{ft}\}$  and the initial debt levels  $d_{w0}$  and  $d_{f0}$  in the collateral constraint economy equal to those in the debt constraint economy. Since worker and firm budget constraints hold with equality in the debt constraint economy and, as argued, budget constraints and family incomes  $\{y_{wt}, y_{ft}\}$  in the two economies are the same, worker and firm budget constraints also hold with equality in the collateral constraint economy. But then the constructed debt sequences  $\{d_{wt}, d_{ft}\}$  in the collateral constraint economy are equal to the corresponding ones in the debt constraint economy. Clearly, if the original allocations and multipliers satisfy the complementary slackness conditions in the debt constraint economy, then these constructed allocations and multipliers necessarily satisfy these conditions in the collateral constraint economy.

Finally, note that since the shadow prices  $\{Q_{wt}, Q_{ft}\}$  in the debt constraint economy satisfy (7) and (14), then given that the consumption sequences in the two economies are equal and that (52) and (58) hold at the shadow prices  $\{\bar{Q}_{wt}, \bar{Q}_{ft}\}$ , we can conclude that our provisional assumption that  $Q_{wt} = \bar{Q}_{wt}$  and  $Q_{ft} = \bar{Q}_{ft}$  at each  $t$  holds. As our constructed allocations and prices for the collateral constraint economy satisfy all the equilibrium conditions, we have proven one direction of our equivalence result.

To prove the other direction of our equivalence result, we begin with an equilibrium for the collateral constraint economy and reverse these steps to construct an equilibrium for the debt constraint economy. As before, the key step is to choose debt constraints so that

$$(66) \quad \chi_{wt} = \bar{\chi}_w p_{wt} \text{ and } \chi_{ft} = \bar{\chi}_f p_{ft}.$$

The remaining steps consist just of matching up constraints and first-order conditions in the two economies. We summarize this discussion with a proposition.

**Proposition 1.** *The economy with debt constraints is equivalent to the economy with collateral constraints in terms of consumption, labor allocations, and wages.*

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Table 1: Parameterization

Panel A: Parameters		Panel B: Moments		
	<i>Endogenously Chosen</i>	<i>Targeted</i>	<i>Data</i>	<i>Model</i>
$y$ , exogenous endowment received by entrepreneurs	7.25	Labor share of output	0.60	0.60
$b_0$ , home production relative to mean output	0.42	Employment rate	0.63	0.63
$\eta$ , matching function elasticity	0.61	Unemployment rate	0.06	0.06
$\kappa$ , vacancy cost relative to mean output	0.15	Job-finding rate	0.45	0.45
$\sigma_z$ , std. dev. of general human capital shocks	0.06	Vacancy cost relative to mean output	0.15	0.15
$\rho$ , convergence rate of human capital	0.95 <sup>1/12</sup>	Std. dev. of annual wage growth	0.21	0.21
$\log \bar{z}_e$ , general human capital drift	2.44	Log wage difference 0-30 experience	1.21	1.19
$\log \bar{h}$ , firm-specific human capital drift	0.82			
<i>Assigned</i>				
$\beta$ , discount factor	0.96 <sup>1/12</sup>			
$q$ , bond price	0.98 <sup>1/12</sup>			
$\phi$ , survival probability	0.9979			
$\sigma$ , separation probability	0.026			
$b_1$ , replacement rate	0.25			
$\gamma$ , worker bargaining power	0.5			
$\alpha$ , coefficient of relative risk aversion	5			
Debt service ratio (workers)	0.11			
Debt service ratio (entrepreneurs)	0.43			

Figure 1: Steady-State Measures and Matching Rates

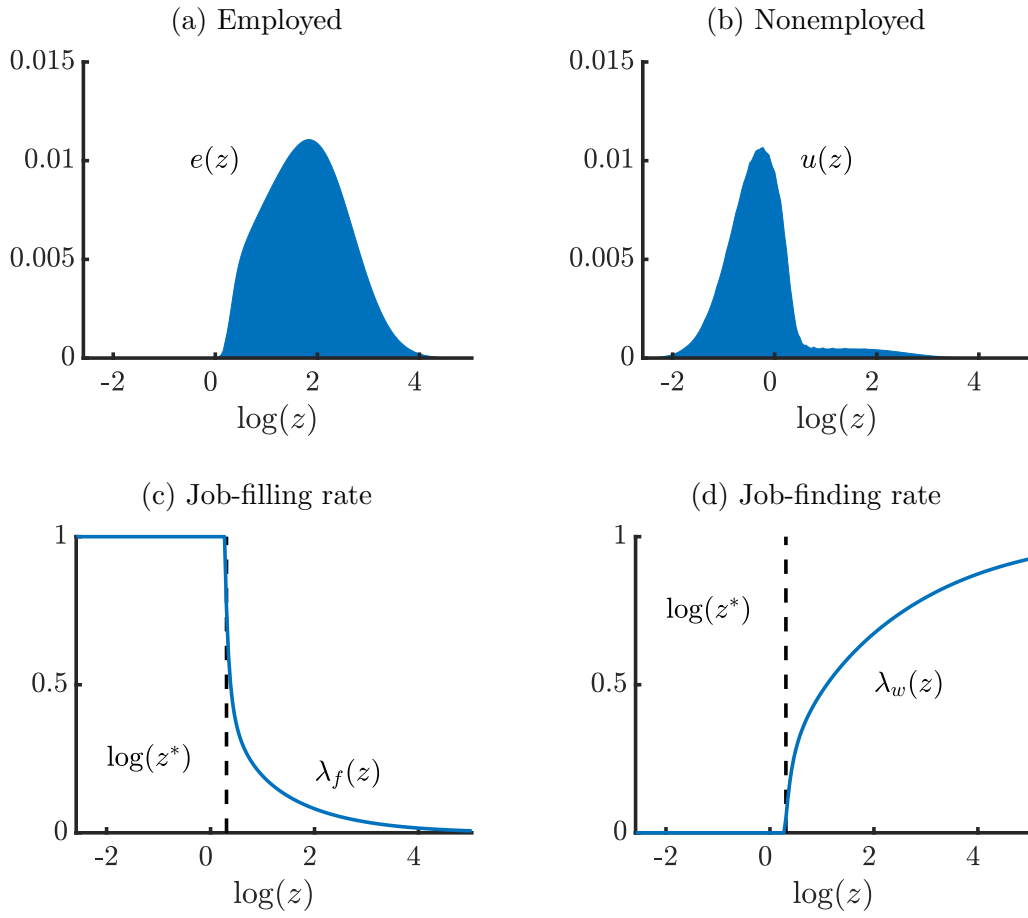


Figure 2: Responses to Deleveraging Shock to Workers or Firms

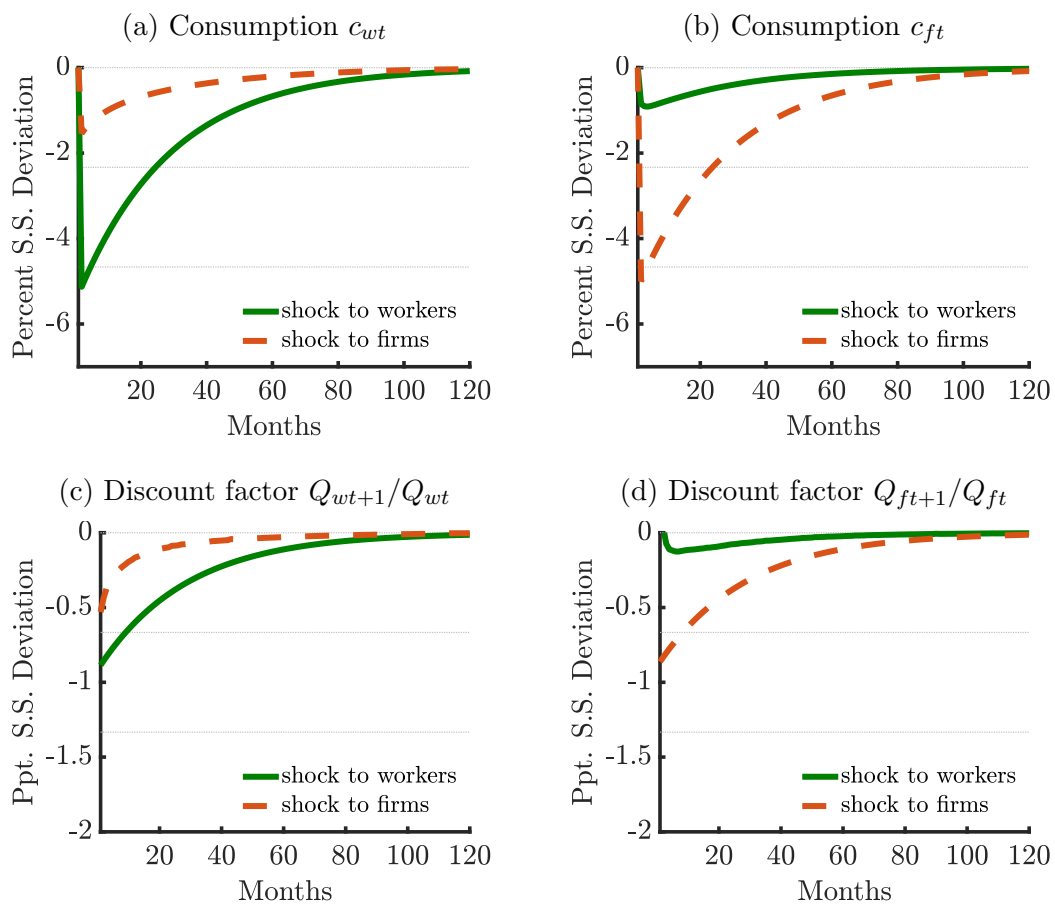


Figure 3: Responses to Deleveraging Shock to Workers or Firms

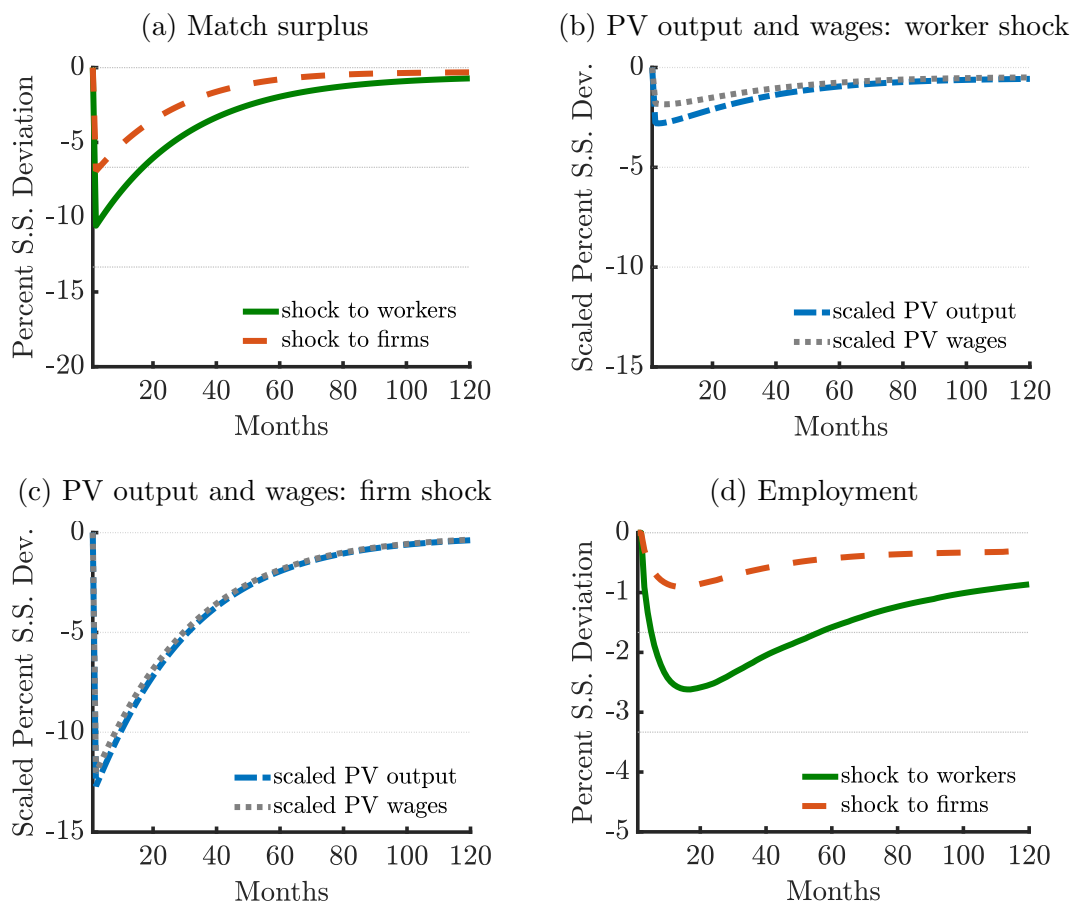


Figure 4: Responses to Deleveraging Shock to Both Workers and Firms

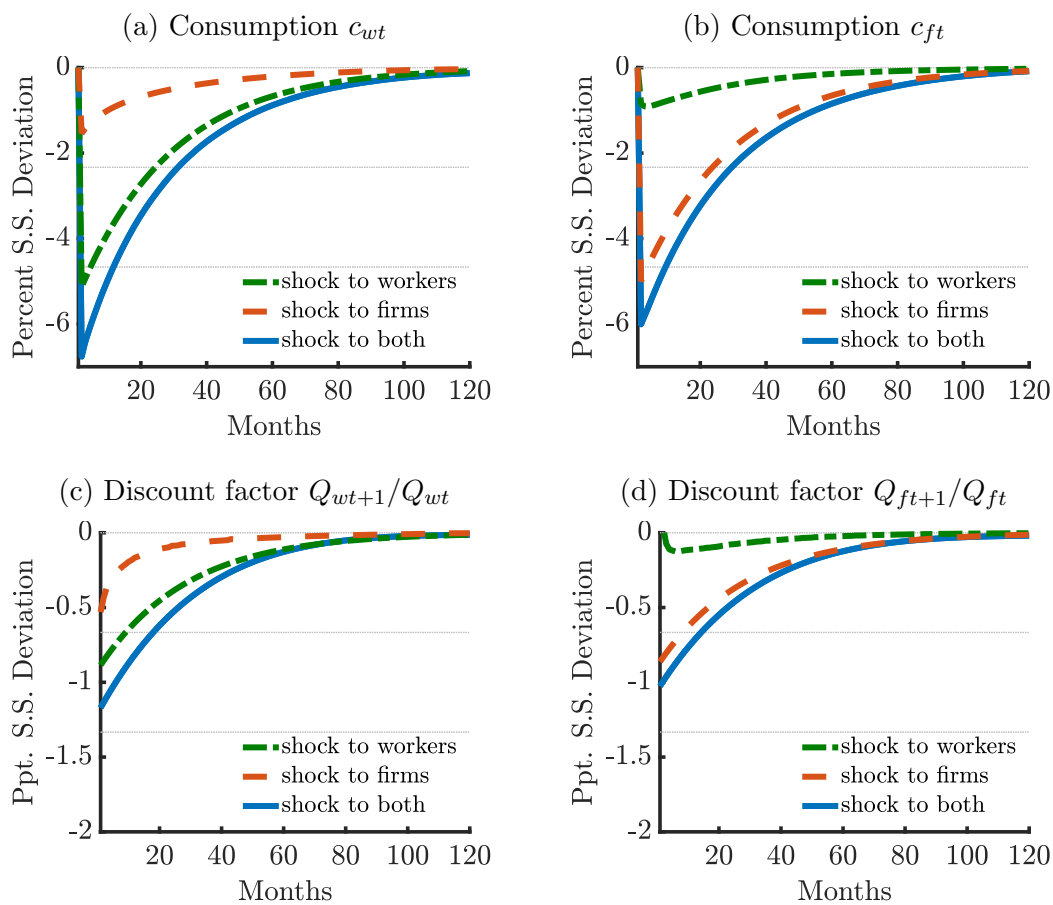


Figure 5: Responses to Deleveraging Shock to Both Workers and Firms

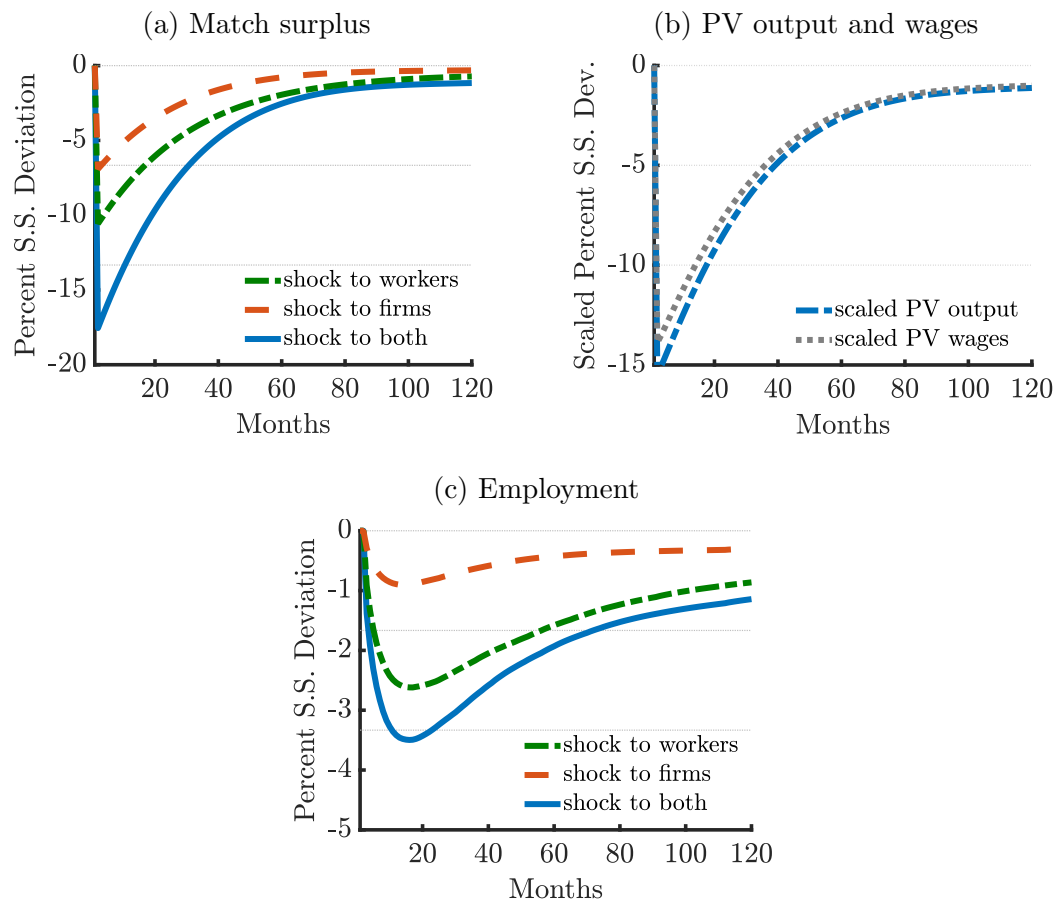


Figure 6: Responses to Deleveraging Shock to Workers with and without Human Capital

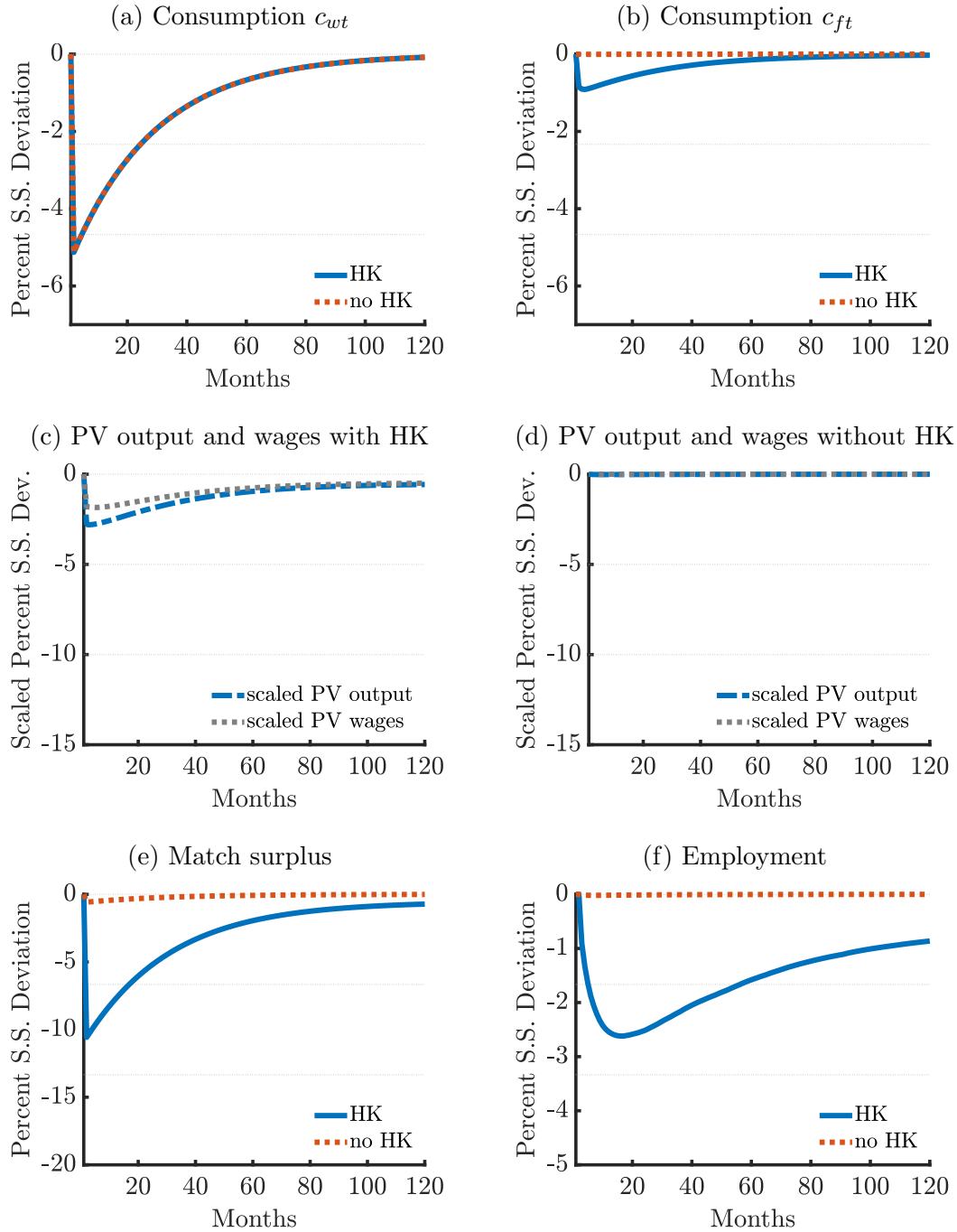




Figure 7: Responses to Deleveraging Shock to Firms with and without Human Capital

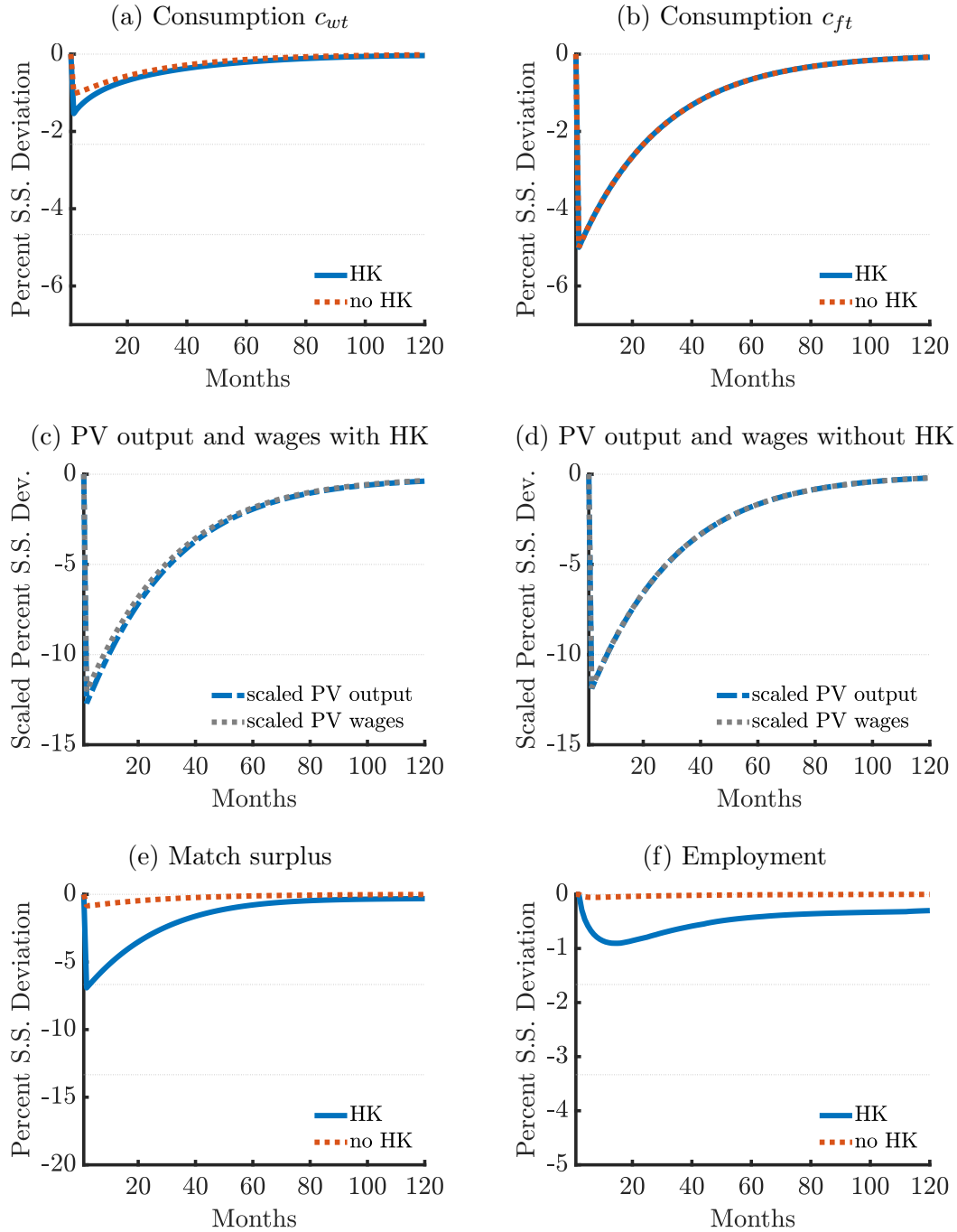


Figure 8: Responses to Deleveraging Shock to Both Workers and Firms with and without Human Capital

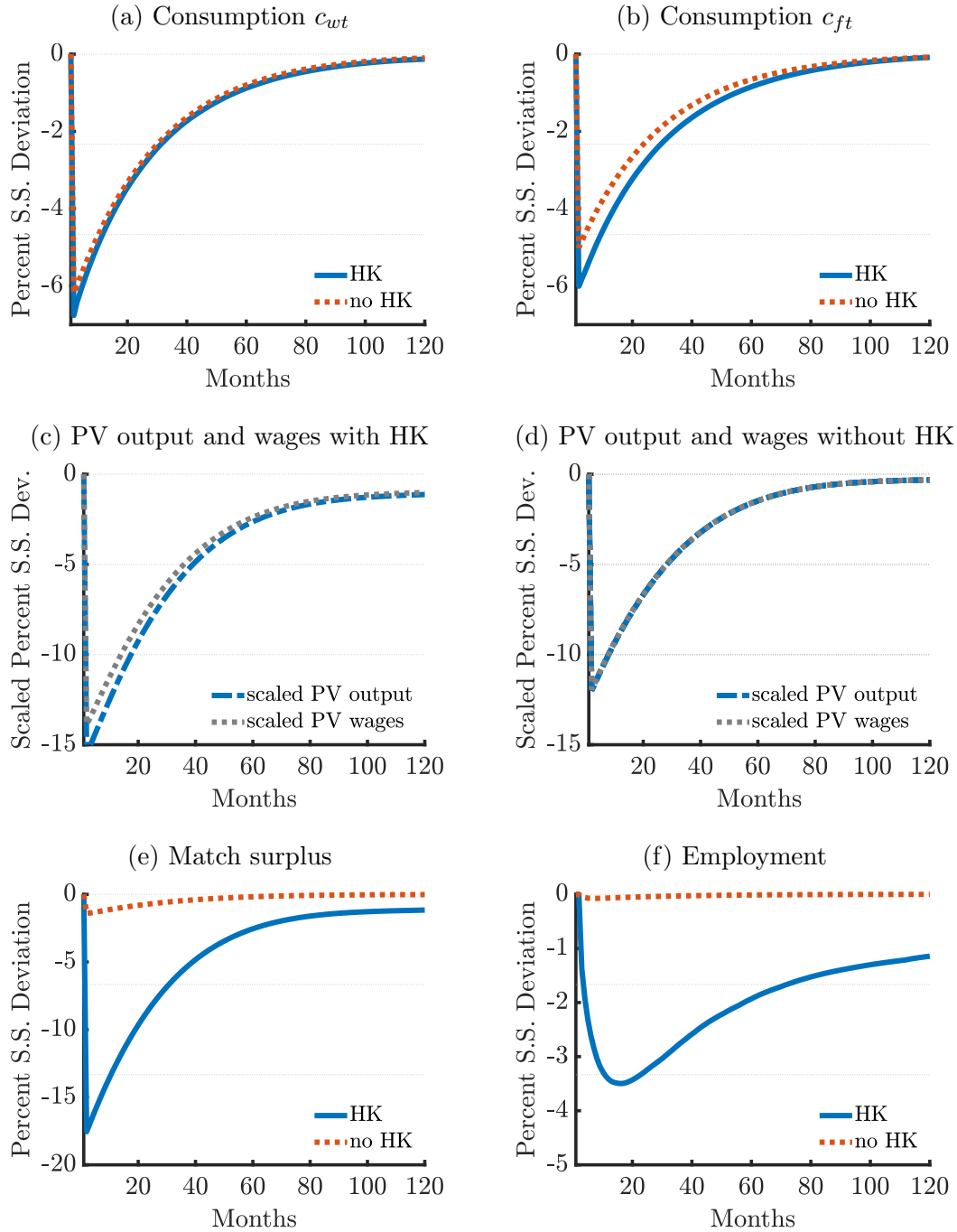


Figure 9: Responses to Deleveraging Shock to Workers with Intervention

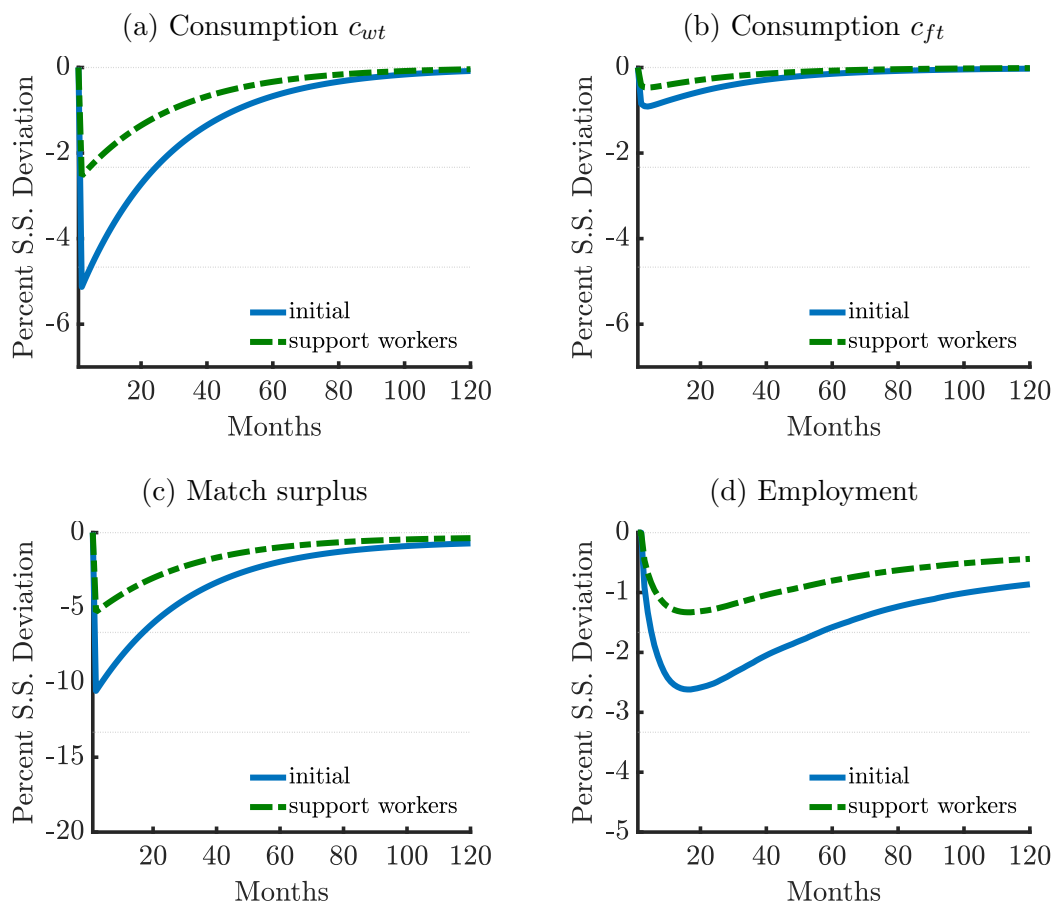


Figure 10: Responses to Deleveraging Shock to Firms with Intervention

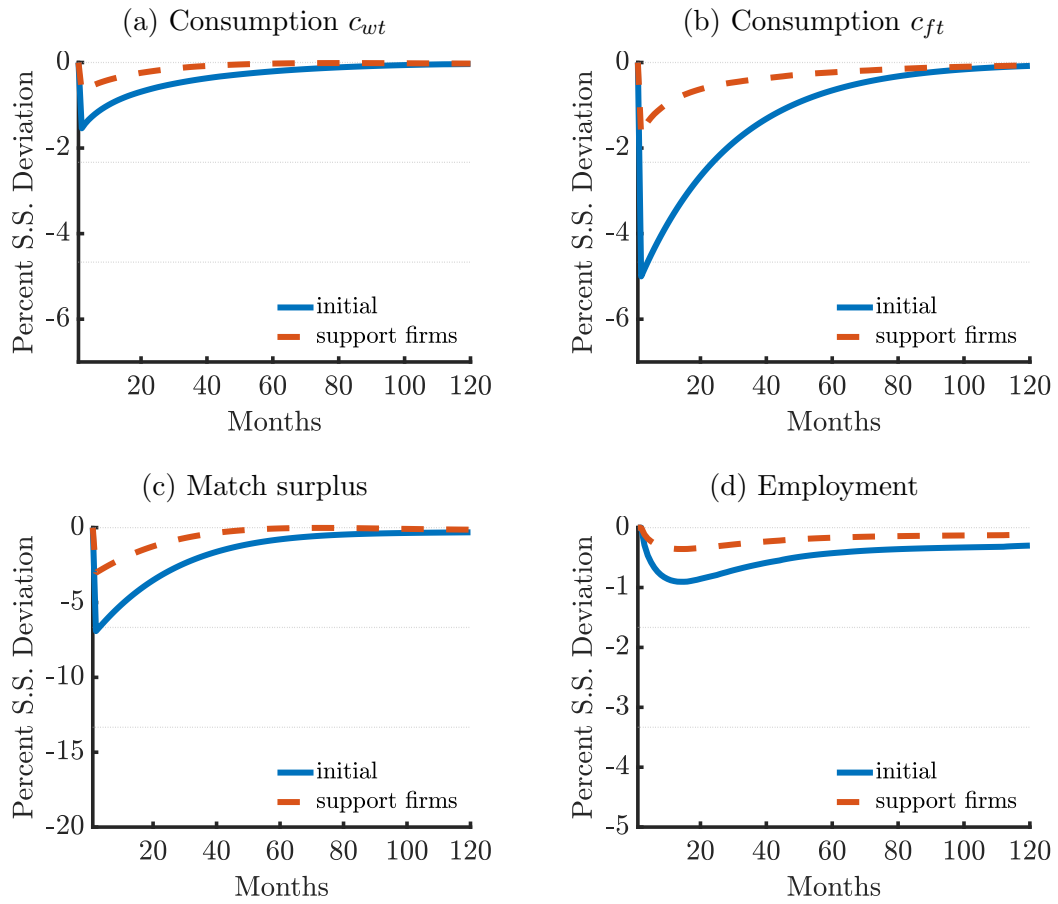


Figure 11: Responses to Deleveraging Shock to Both Workers and Firms with Intervention

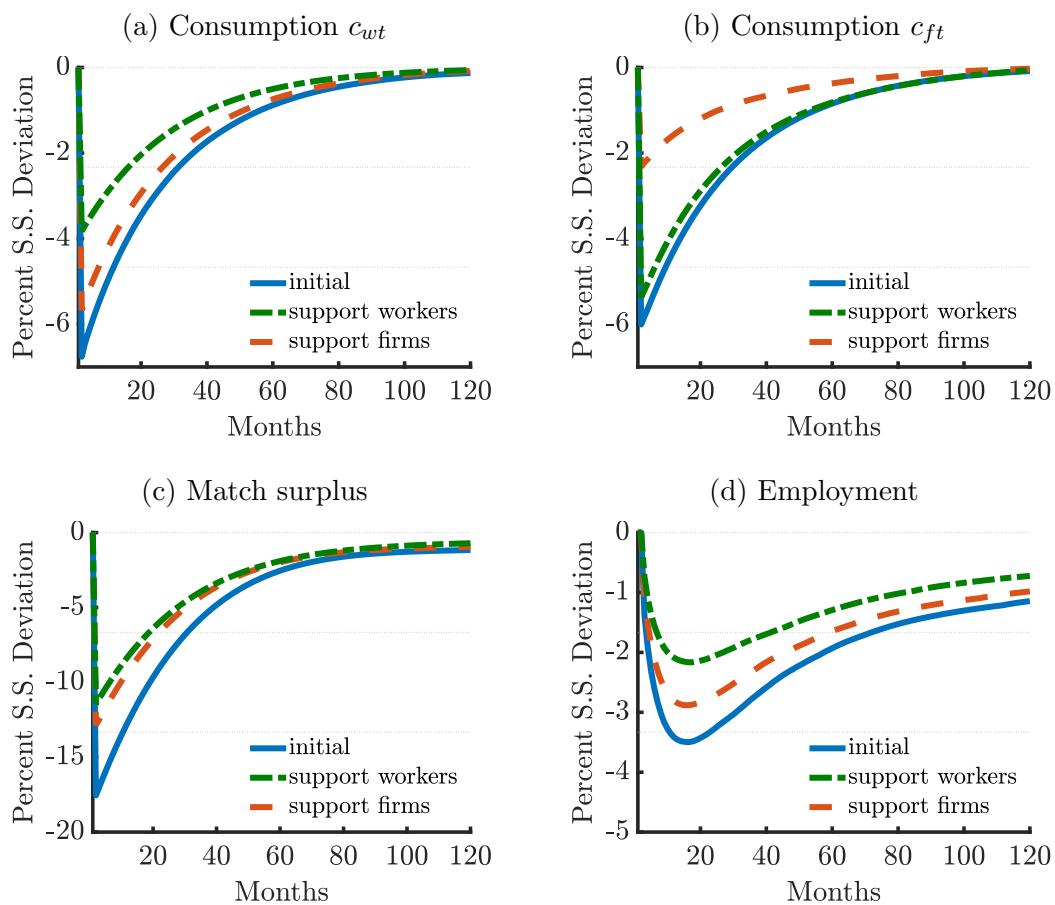
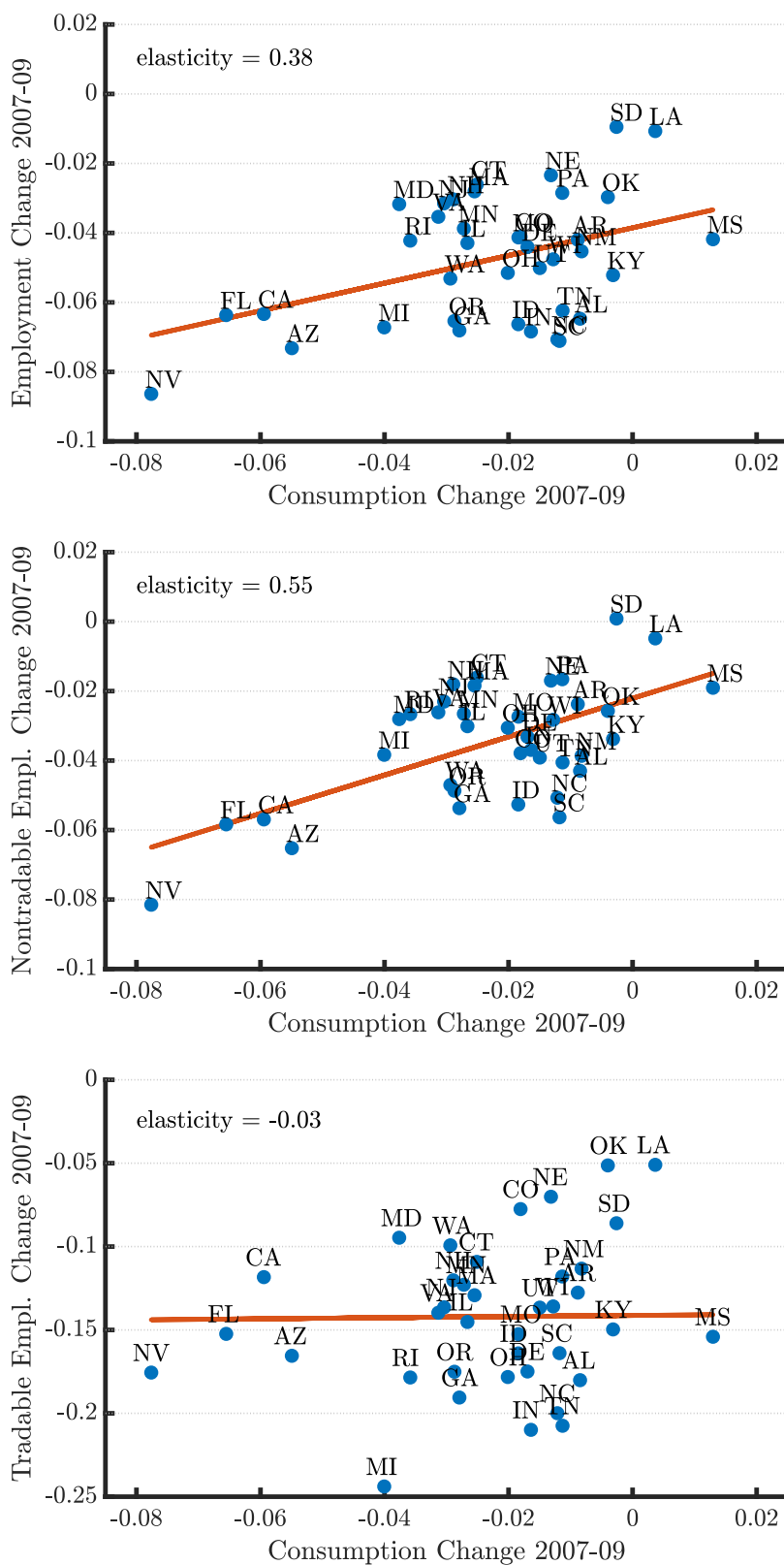
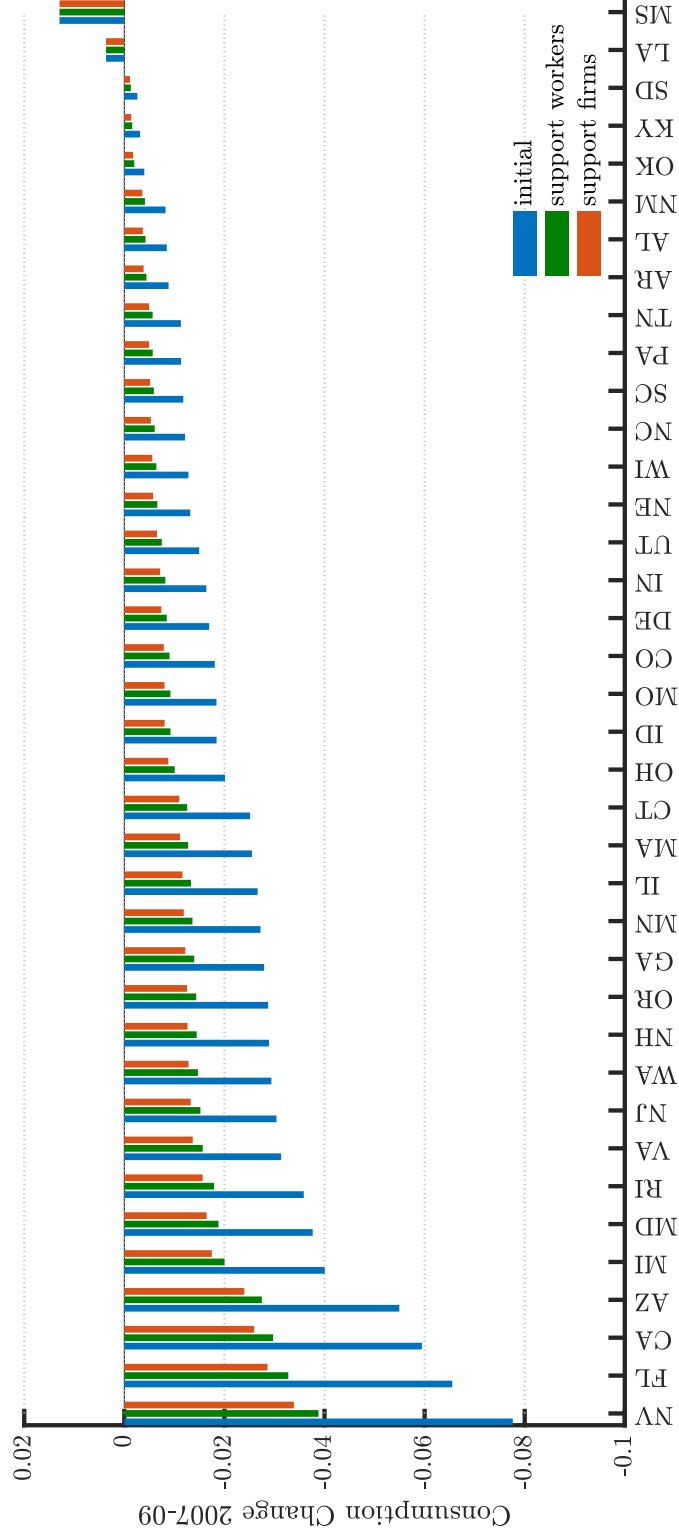


Figure 12: Employment and Consumption Across States 2007-2009



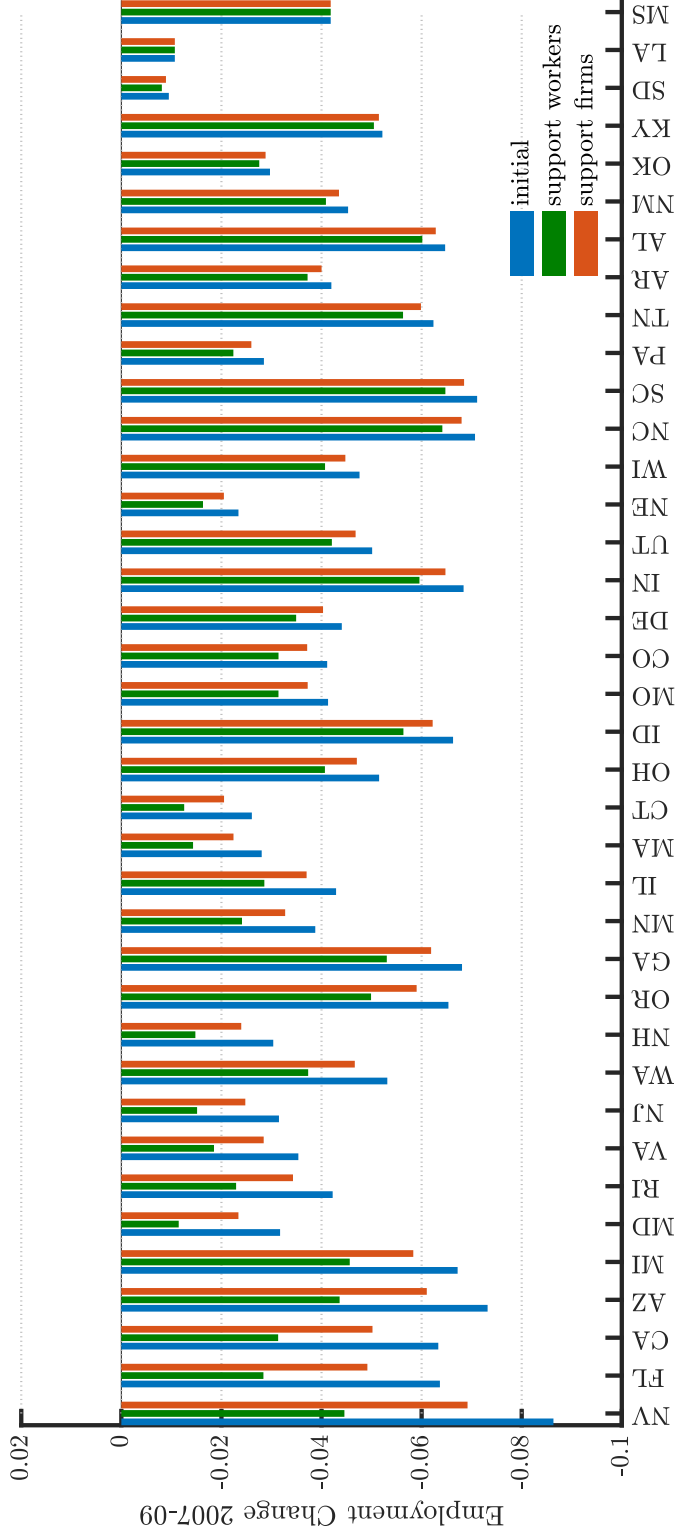
From top to bottom: (a) Employment versus consumption. (b) Nontradable employment versus consumption. (c) Tradable employment versus consumption.

Figure 13: State-Level Consumption with Intervention 2007-2009



Data (blue), intervention on workers (green), and intervention on firms (orange). The intervention consists of lump-sum transfers of equal amounts to workers and entrepreneurs starting in 2007 and reverting back to zero with a quarterly persistence of  $0.90^{1/3}$ . The size of transfers is such that the resulting state-level consumption change between 2007 and 2009 is 50% of the observed one when the observed one is negative and zero otherwise.

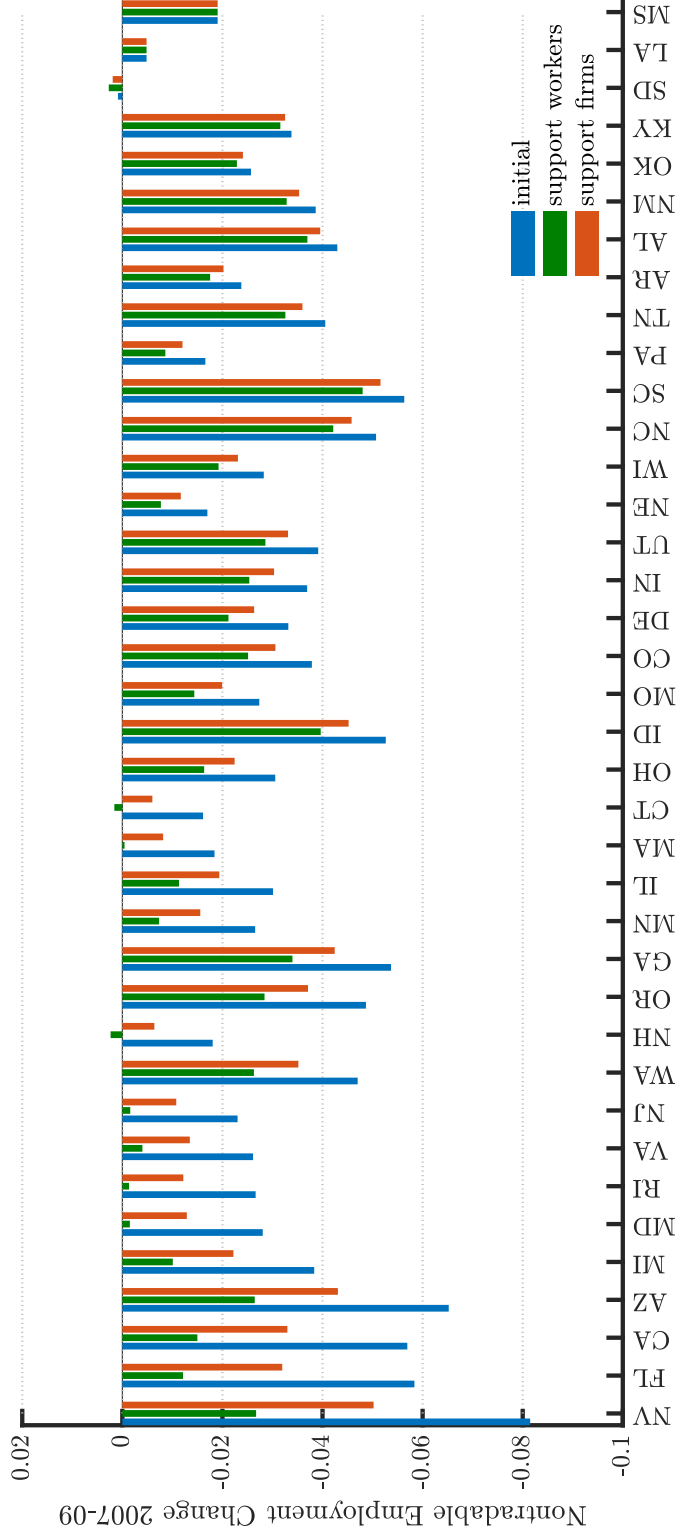
Figure 14: State-Level Employment with Intervention 2007-2009



Data (blue), intervention on workers (green), and intervention on firms (orange). The intervention consists of lump-sum transfers of equal amounts to workers and entrepreneurs starting in 2007 and reverting back to zero with a quarterly persistence of  $0.90^{1/3}$ . The size of transfers is such that the resulting state-level consumption change between 2007 and 2009 is 50% of the observed one when the observed one is negative and zero otherwise.

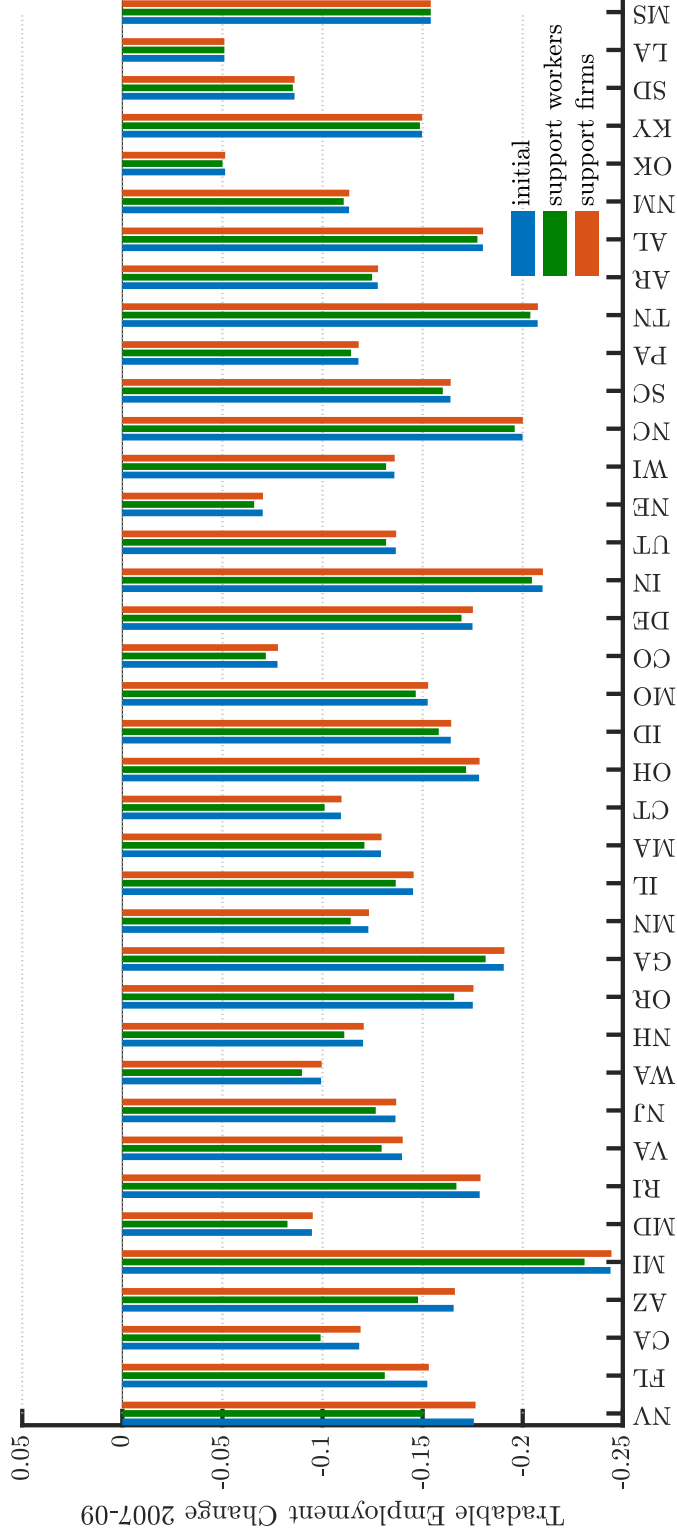


Figure 15: State-Level Nontradable Employment with Intervention 2007-2009



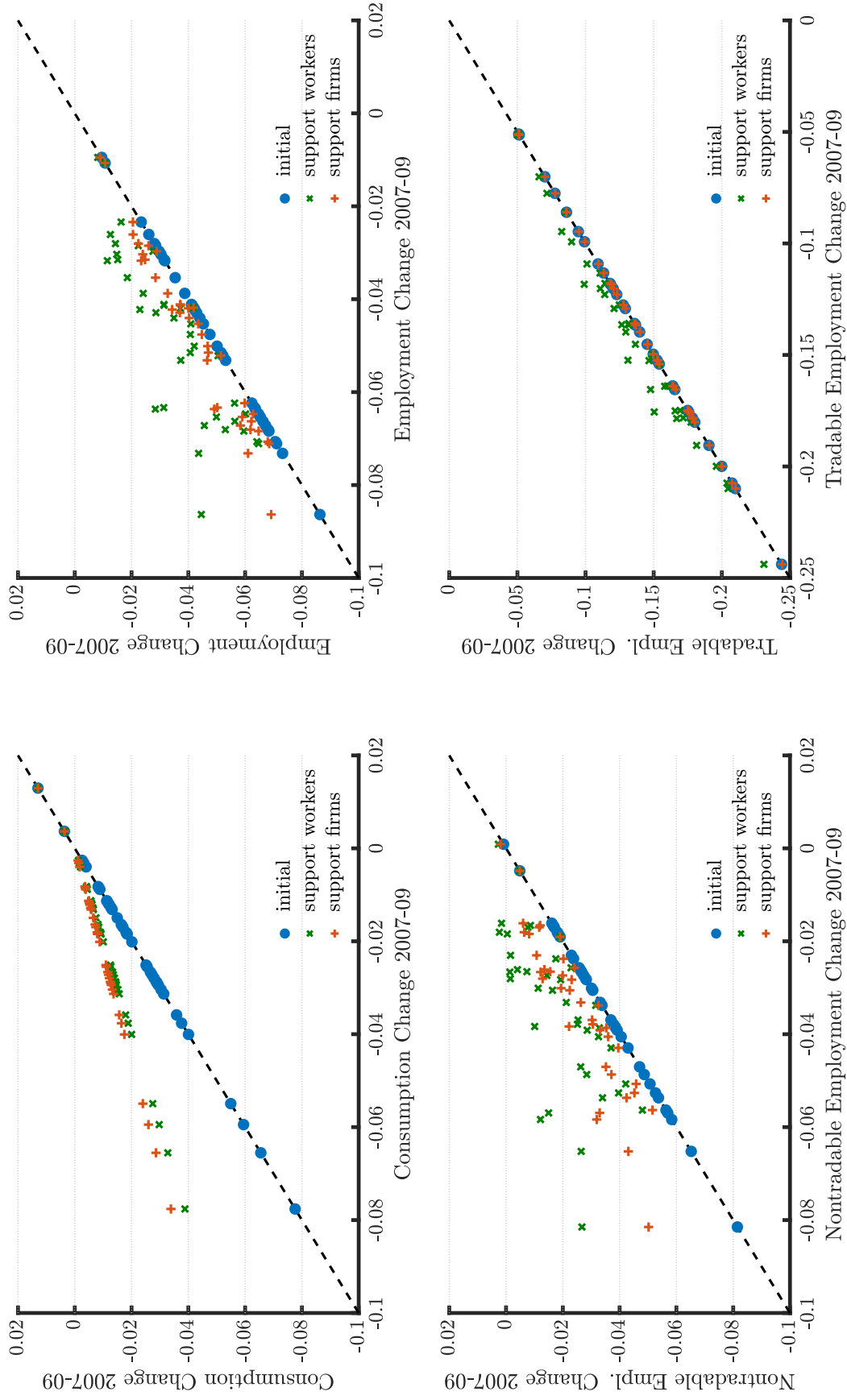
Data (blue), intervention on workers (green), and intervention on firms (orange). The intervention consists of lump-sum transfers of equal amounts to workers and entrepreneurs starting in 2007 and reverting back to zero with a quarterly persistence of  $0.90^{1/3}$ . The size of transfers is such that the resulting state-level consumption change between 2007 and 2009 is 50% of the observed one when the observed one is negative and zero otherwise.

Figure 16: State-Level Tradable Employment with Intervention 2007-2009



Data (blue), intervention on workers (green), and intervention on firms (orange). The intervention consists of lump-sum transfers of equal amounts to workers and entrepreneurs starting in 2007 and reverting back to zero with a quarterly persistence of  $0.90^{1/3}$ . The size of transfers is such that the resulting state-level consumption change between 2007 and 2009 is 50% of the observed one when the observed one is negative and zero otherwise.

Figure 17: Employment and Consumption Across States with Intervention 2007-2009



From top to bottom: (a) Predicted consumption versus observed consumption. (b) Predicted employment versus observed employment. (c) Predicted nontradable employment versus observed nontradable employment. (d) Predicted tradable employment versus observed tradable employment. The intervention consists of lump-sum transfers of equal amounts to workers and entrepreneurs starting in 2007 and reverting back to zero with a quarterly persistence of  $0.90^{1/3}$ . The size of transfers is such that the resulting state-level consumption change between 2007 and 2009 is 50% of the observed one when the observed one is negative and zero otherwise.