Policy Anticipation Effects in New Keynesian Models*

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Central banks announcements of future monetary policy and future states of the economy make economic agents to react before the announced changes take place. We evaluate these anticipation effects in the context of a realistic dynamic economic model of central banking. In our experiments, we consider temporary and permanent anticipated changes in the model's parameters such as the inflation target, natural rate of interest and Taylor-rule coefficients, as well as anticipated switches in policy rules from a constant target interest rate to the standard interest rate rule leading to a time-varying target interest rate, and from inflation targeting to price-level targeting and average inflation targeting. We find that announced temporary and permanent changes have sizable anticipation effects on the real economy. Our methodological contribution is to develop novel perturbation-based framework suitable for analyzing anticipated changes.

Keywords: turnpike theorem; time-dependent models; nonstationary models; unbalanced growth; time varying parameters; anticipated shock; parameter shift; parameter drift; regime switches; stochastic volatility; technological progress; seasonal adjustments; Fair and Taylor method; extended path, extended function path

JEL Codes: C61, C63, C68, E31, E52

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1 Introduction

Many economic policies are announced ahead of being implemented. For instance, changes to taxes, tariffs, minimum wage, pension reforms, Social Security are frequently signed into law well before they are put in practice. Other notable examples include an announcement about a new member state's accession to the European Union (EU) or a member state's exit from the EU (i.e., Brexit) starting from a certain future date, an announcement of the presidential elections results before the new elected president comes to power. Moreover, nowadays, central banks increasingly rely on communication to implement monetary policy. Through their communications to the public, they indicate their future monetary policy and their evaluation of the future stance of the economy. For example, a central bank may promise to maintain a fixed interest rate for certain number of future periods before normalizing its policy (forward guidance) or it may make announcements about a future change in the inflation target. The common feature of the above examples is that economic agents start reacting to the announced future policies in the moment of announcement, before the given policy is implemented. Understanding the consequences of these anticipation effects is crucial from the policy evaluation perspective but they are usually ignored in economic literature.

This paper demonstrates the importance of anticipatory effects in the context of new Keynesian models. We explore the effect of several time dependent policies conjectured in the literature to be welfare improving. We consider both gradual and one-time anticipated changes in the economic environment. A distinctive feature of our analysis is that it is carried out in a realistic open-economy model. Our model contains 47 equations and unknowns, including 21 state variables and it is a smaller replica of the Terms of Trade Model (ToTEM) used by the Bank of Canada for projection and policy analysis, which in turn contains ?? equations and unknowns and ?? state variables. Importantly, our baby ToTEM model follows the full size central-banking model as close as possible and generates very similar impulse response functions; see Dorich et al. (2013) and Lepetuyk et al. (2020) for a description of the ToTEM and bToTEM models, respectively.

The policy experiments we consider include: (1) a gradual decline in the natural rate of interest; (2) a gradual change in the inflation target happening in the future with either certainty or with some probability; (3) normalization of monetary policy regarding future nominal interest rates, when the economy is initially at a zero lower bound (ZLB) on nominal interest rates; (4) a switch to a more aggressive Taylor rule; (5) a switch to price-level targeting instead of inflation targeting; (6) a switch to average inflation targeting instead of inflation targeting.

Our findings are as follows: (1) Being fully anticipated by agents, a gradual decline in the natural rate of interest from 3 percent to 2 percent over consecutive five years leads to a substantial expansion of the economy. (2) Postponing an increase in the inflation target has large expansionary effects on output over the transition to a new steady state. If this policy is implemented probabilistically (with some probability after being announced), there are still substantial anticipation effects both before and after uncertainty is resolved. (3) When an economy is at ZLB on nominal interest rates, the central bank uses policy announcements (forward guidance) about its future return to the standard interest rate rule to direct the economy's transition out of ZLB. The more it postpones such a return to the standard rule, the larger is output growth over the transition; an initial jump in output is however invariant to the horizon of this forward guidance policy. Therefore, this experiment informs policy makers on optimal horizons of monetary policy normalizations after ZLB periods. (4) Nevertheless, a more aggressive (but realistic) behavior of the central bank toward targeting inflation and output is not translated into important anticipatory effects on the side of economic agents. (5) Switching from inflation-level targeting to price-level targeting has smaller impacts with larger implementation lags. Price-level targeting was argued in the literature to be welfare improving. Therefore, a central bank that waits to implement the new policy in practice loses time, and the economy does not get earlier benefits from higher output. (6) Finally, a switch to average inflation targeting also has modest anticipation effects; this is because average inflation targeting is in a middle ground between inflating and price-level targeting. In sum, the model's implications about the importance of anticipation effects are mixed: there are substantial policy anticipation effects present in our experiments (1)–(3) but such effects are relatively small in experiments (4)–(6).

On the methodological side, we introduce a novel perturbation-based framework for analyzing models with anticipated effects. Such models represent a challenge to economic dynamics because they lead to time dependent optimal decision rules that change from one period to another. Conventional perturbation methods cannot be used for analyzing anticipatory effects since they construct time-invariant (stationary) solutions. To characterize anticipatory effects, we build on the turnpike analysis in Maliar, Maliar, Taylor and Tsener (2020, henceforth, MMTT) who show that if the time horizon is long enough, the trajectory of a finite-horizon economy provides an accurate approximation to the trajectory of the corresponding infinite-horizon economy. We modify the conventional perturbation analysis to facilitate the construction of time dependent-solutions characterizing anticipatory effects. Our ubiquitous perturbation method is implemented using a popular Dynare platform combined with user-friendly MATLAB software and can be easily adapted to other applications the reader may be interested in.

There are several methods in the related literature that can be used to analyze models with anticipatory effects; see MMTT (2020) for an discussion and literature review. First, an extended path (EP) method of Fair and Taylor (1983) constructs a path for variables under one specific realization of shocks by using certainty equivalence approximation. A shortcoming of EP are that the certainty equivalence approach can be insufficiently accurate in models with higher volatility and strong nonlinearity. Second, an extended function path (EFP) improves on that shortcoming by replacing the assumption of certainty equivalence with accurate deterministic and stochastic integration methods but it has a relatively high computational expense characteristic for global nonlinear projection methods. In contrast, our perturbation method deliver similar accuracy to the most accurate EFP solutions in the text problems but is tractable in problems with a high dimensionality, such as large-scale central-banking models. Finally, a perturbation method with news shocks by Schmitt-Grohé and Uribe (2012) also allows to analyze anticipatory effects but we show that the two methods have different ways of modeling anticipatory effects and deliver sufficiently different solutions: in our case, we assume a nonrecurrent event or a sequence of non-recurrent events meanwhile Schmitt-Grohé and Uribe (2012) consider a model in which the news comes with the same periodicity and duration following a given stationary process. To put things simple, in our analysis, we model a one-time accession of a new member to the EU while in their analysis, a country will move between accessing and exiting EU with some probability in every period of time.

The rest of the paper is organized as follows: Section 2 describes the large-scale central banking model and presents our perturbation methodology for analyzing anticipatory effects. Section 3 analyzes our six policy experiments and presents comparison results with news shocks framework, and finally, Section 4 concludes.

2 Methodology

In this section, we present the model and outline the methodology of our numerical analysis.

2.1 The model

Nowadays, the central banks, leading international organizations and government agencies, use large-scale macroeconomic models for projection and policy analysis. In this paper, we consider a scaled down version of the terms of trade economic model (ToTEM) of the Bank of Canada proposed in LMM (2020). The full-scale ToTEM model is the main projection and policy analysis model of the Bank of Canada, and it is very large: it contains 356 equations and unknowns, including 215 state variables; see Dorich et al. (2013). It includes several types of utility-maximizing consumers, several profit-maximizing production sectors, fiscal and monetary authorities, as well as a foreign sector. In turn, the scaled down version of ToTEM of LMM (2020) has 47 equations and unknowns, including 21 state variables.

Like the full-scale ToTEM model, the scaled-down version is a small open-economy model that features the new-Keynesian Phillips curves for consumption, labor and imports. As in ToTEM, we assume the rule-of-thumb price settlers in line with Galí and Gertler (1999). There is a quadratic adjustment cost of investment and a convex cost of capital utilization. There is bidirectional trade that consists of exporting domestic consumption goods and commodities, and importing foreign goods for domestic production.

Final-good production. Final consumption goods are produced in two stages. In the first stage, intermediate goods are produced competitively using labor, capital, commodities and imports. In the second stage, final goods are aggregated from differentiated goods that each produced by a monopolistically competitive firm from the intermediate goods and from the final goods. The final goods can be consumed be households. They can also be transformed using linear technologies into other types of goods, namely, investment goods and noncommodity exports goods.

In the first production stage, a representative perfectly competitive firm produces an intermediate good by solving the following profit maximization problem:

$$\max_{\left\{Z_t^g, Z_t^n, L_t, K_t, I_t, COM_t^d, M_t, u_t, d_t\right\}} E_0 \sum_{t=0}^{\infty} \mathcal{R}_{0,t} \left(P_t^z Z_t^n - W_t L_t - P_t^i I_t - P_t^{com} COM_t^d - P_t^m M_t\right)$$

$$\text{s.t. } Z_t^g = \left[\delta_l \left(A_t L_t\right)^{\frac{\sigma-1}{\sigma}} + \delta_k \left(u_t K_{t-1}\right)^{\frac{\sigma-1}{\sigma}} + \delta_{com} \left(COM_t^d\right)^{\frac{\sigma-1}{\sigma}} + \delta_m \left(M_t\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

$$\log A_t = \varphi_a \log A_{t-1} + (1-\varphi_a) \log \bar{A} + \xi_t^a, \tag{2}$$

$$K_t = (1-d_t) K_{t-1} + I_t, \tag{3}$$

$$d_t = d_0 + \bar{d} e^{\rho(u_t - 1)}, \tag{4}$$

$$Z_t^n = Z_t^g - \frac{\chi_i}{2} \left(\frac{I_t}{I_{t-1}} - 1\right)^2 I_t, \tag{5}$$

where Z_t^g and Z_t^n are gross production and net (of adjustment costs) production of final goods; L_t, K_t, I_t COM_t^d, M_t, u_t and d_t are labor, capital, investment, commodity inputs, imports, capital utilization and depreciation rate, respectively; A_t is the level of labor-augmenting technology; ξ_t^a is a normally distributed variable, and φ_a is an autocorrelation coefficient. The firm discounts nominal payoffs according to household's stochastic discount factor $\mathcal{R}_{t,t+j} = \beta^j \left(\lambda_{t+j}/\lambda_t\right) \left(P_t/P_{t+j}\right)$, where λ_t is household's marginal utility of consumption, and P_t is the final good price. Investment goods and noncommodity exports are assumed to be produced from the final goods according to linear technology, $P_t^i = \iota_i P_t$ and $P_t^{nc} = \iota_x P_t$, where P_t^i and P_t^{nc} are the price of investment goods and noncommodity exports goods, respectively.

In the second stage of production, monopolistically competitive firms produce a continuum of differentiated good. Then, these differentiated goods are aggregated into the final good by an aggregating firm that solves the following cost minimization problem

$$\min_{\{Z_{it}\}} \int_0^1 P_{it} Z_{it} di$$

$$\text{s.t. } Z_t = \left(\int_0^1 Z_{it}^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where Z_t and P_{it} are given; Z_{it} is a differentiated good i, which implies the following demand function for the differentiated good i:

$$Z_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} Z_t, \quad \text{with } P_t \equiv \left(\int_0^1 P_{it}^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}.$$

Each differentiated good is produced from the intermediate goods and from the final goods using technology featuring perfect complementarity,

$$Z_{it} = \min\left(\frac{Z_{it}^{n}}{1-s_m}, \frac{Z_{it}^{mi}}{s_m}\right), \tag{6}$$

where Z_{it}^{n} is an intermediate good and Z_{it}^{mi} is a final good input, and s_m is a Leontief parameter.

There are two types of the monopolistically competitive firms producing differentiated goods: rule-of-thumb firms of measure ω and forward-looking firms of measure $1-\omega$. Both rule-of-thumb firms and forward-looking firms index their price to the inflation target $\bar{\pi}_t$ with probability θ as $P_{it} = \bar{\pi}_t P_{i,t-1}$. The rule-of-thumb firms partially index their price to lagged inflation and target inflation with probability $1-\theta$,

$$P_{it} = (\pi_{t-1})^{\gamma} (\bar{\pi}_t)^{1-\gamma} P_{i,t-1}. \tag{7}$$

Forward-looking firms choose their prices P_t^* with probability $1-\theta$ to maximize profits generated when the price remains effective

$$\max_{P_t^*} E_t \sum_{j=0}^{\infty} \theta^j \mathcal{R}_{t,t+j} \left(\prod_{k=1}^{j} \bar{\pi}_{t+k} P_t^* Z_{i,t+j} - (1-s_m) P_{t+j}^{z} Z_{i,t+j} - s_m P_{t+j} Z_{i,t+j} \right) \tag{8}$$

$$\text{s.t. } Z_{i,t+j} = \left(\frac{\prod_{k=1}^{j} \bar{\pi}_{t+k} P_t^*}{P_{t+j}} \right)^{-\varepsilon} Z_{t+j}.$$

The production in the first stage Z_t^n and that in the second stages Z_t are related via price dispersion Δ_t,

$$Z_t^n = \int_0^1 Z_{it}^n di = (1-s_m) \int_0^1 Z_{it} di = (1-s_m) \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} Z_t di = (1-s_m) \Delta_t Z_t, \tag{9}$$

where $\Delta_t \equiv \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} di$.

Production of commodities. Commodities are produced by a domestic firm using final goods and land as inputs. They are sold domestically or exported to the rest of the world. The domestic firm solves

$$\max_{Z_t^{com}, COM_t} \{P_t^{com} COM_t - P_t Z_t^{com}\}$$

$$\text{s.t. } COM_t = (Z_t^{com})^{s_z} (A_t F)^{1-s_z} - \frac{\chi_{com}}{2} \left(\frac{Z_t^{com}}{Z_{t-1}^{com}} - 1 \right)^2 Z_t^{com}, \tag{10}$$

where Z_t^{com} is the final good input, and F is a fixed production factor, which may be considered as land. Similarly to production of final goods, the commodity producers incur quadratic adjustment costs when they adjust the level of final good input. The commodities are sold domestically (COM_t^d) or exported to the rest of the world (X_t^{com}), $COM_t = COM_t^d + X_t^{com}$. They are sold at the world price adjusted by the nominal exchange rate, $P_t^{com} = e_t P_t^{comf}$, where e_t is the nominal exchange rate (i.e., domestic price of a unit of foreign currency), and P_t^{comf} is the world commodity price; in real terms, the latter price is given by $p_t^{com} = s_t p_t^{comf}$, where $p_t^{com} \equiv P_t^{com}/P_t$ and $p_t^{comf} \equiv P_t^{comf}/P_t^f$ are domestic and foreign relative prices of commodities, respectively, P_t^f is the foreign consumption price level, and $s_t = e_t P_t^f / P_t$ is the real exchange rate.

Production of imports. The representative perfectly competitive firm produces the final imported good M_t from a continuum of intermediate imported goods M_{it} and solves the following cost-minimization problem,

$$\min_{\{M_{it}\}} \int_0^1 P_{it}^m M_{it} di$$
$$\text{s.t. } M_t = \left(\int_0^1 M_{it}^{\frac{\varepsilon_m - 1}{\varepsilon_m}} di \right)^{\frac{\varepsilon_m}{\varepsilon_m - 1}},$$

where M_{it} is an intermediate imported good i. The demand for an intermediate imported good i is given by

$$M_{it} = \left(\frac{P_{it}^m}{P_t^m} \right)^{-\varepsilon_m} M_t, \quad \text{with } P_t^m \equiv \left(\int_0^1 \left(P_{it}^m\right)^{1-\varepsilon_m} di \right)^{\frac{1}{1-\varepsilon_m}}.$$

Prices of the intermediate imported goods are sticky in a similar way as the prices of the differentiated final goods. A measure ω_m of the importers follows the rule-of-thumb pricing, and the others are forward looking. The optimizing forward-looking importers choose the price P_t^{m*} in order to maximize profits generated when the price remains effective

$$\max_{P_t^{m*}} E_t \sum_{j=0}^{\infty} (\theta_m)^j \mathcal{R}_{t,t+j} \left(\prod_{k=1}^{j} \bar{\pi}_{t+k} P_t^{m*} M_{i,t+j} - e_{t+j} P_{t+j}^{mf} M_{i,t+j} \right)$$
$$M_{i,t+j} = \left(\frac{\prod_{k=1}^{j} \bar{\pi}_{t+k} P_t^{m*}}{P_{t+j}^m} \right)^{-\varepsilon_m} M_{t+j},$$

where P_t^{mf} is the price of imports in the foreign currency. All importers face the same marginal cost given by the foreign price of imports.

Households. Households maximize the lifetime utility by choosing holdings of domestic and foreign-currency denominated bonds, labor and consumption, and they are subject to habits in consumption. Each household supplies a variety of differentiated labor service to the labor market, which is monopolistically competitive. The representative household of type h solves the following utility-maximization problem:

$$\max_{C_t, L_{ht}, B_t, B_t^f} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\mu}{\mu - 1} \left(C_t - \xi \bar{C}_{t-1} \right)^{\frac{\mu-1}{\mu}} \exp\left(\frac{\eta(1-\mu)}{\mu(1+\eta)} \int_0^1 \left(L_{ht}\right)^{\frac{\eta+1}{\eta}} dh \right) \eta_t^c \right\},$$
$$\text{s.t. } P_t C_t + \frac{B_t}{R_t} + \frac{e_t B_t^f}{R_t^f \left(1 + \kappa_t^f\right)} = B_{t-1} + e_t B_{t-1}^f + \int_0^1 W_{ht} L_{ht} dh + \Pi_t, \tag{11}$$
$$\log \eta_t^c = \varphi_c \log \eta_{t-1}^c + \xi_t^c, \tag{12}$$

where C_t, L_{ht}, B_t, B_t^f are consumption of final goods, labor service of type h, holdings of domestic and foreign-currency denominated bonds, respectively; $\bar{C}_t$ is the aggregate consumption, taken by the household as given; $\beta \in (0, 1)$ is a subjective discount factor; μ and η are the utility-function parameters; η_t^c is a consumption demand shock, ξ_t^c is a normally distributed variable, and φ_c is an autocorrelation coefficient; R_t and R_t^f are domestic and foreign nominal interest rate, respectively; κ_t^f is the risk premium on the foreign interest rate; W_{ht} is the nominal wage of labor of type h; Π_t is profits paid by the firms. The representative household supplies a variety of differentiated labor service to the labor market, which is monopolistically competitive.

Labor packer. A labor packer aggregates differentiated labor services by solving

$$\min_{\{L_{ht}\}} \int_0^1 W_{ht} L_{ht} dh$$
$$\text{s.t. } L_t = \left(\int_0^1 L_{ht}^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dh \right)^{\frac{\varepsilon_w}{\varepsilon_w - 1}},$$

where L_t is aggregated labor demanded by firms in the first stage of production. Cost minimization of the labor packer implies the following demand for individual labor:

$$L_{ht} = \left(\frac{W_{ht}}{W_t} \right)^{-\varepsilon_w} L_t, \quad \text{with } W_t \equiv \left(\int_0^1 W_{ht}^{1-\varepsilon_w} dh \right)^{\frac{1}{1-\varepsilon_w}}. \tag{13}$$

Labor unions. Labor unions set wages. There are two types of labor unions: rule-of-thumb unions of measure ω_w and forward-looking unions of measure $1 - \omega_w$. Within each type, with probability θ_w the labor unions index their wage to the inflation target $\bar{\pi}_t$ as follows $W_{it} = \bar{\pi} W_{i,t-1}$. The rule-of-thumb unions that do not index their wage in the current period follow the rule

$$W_{it} = \left(\pi_{t-1}^w \right)^{\gamma_w} \left(\bar{\pi}_t \right)^{1-\gamma_w} W_{i,t-1}. \tag{14}$$

A forward-looking unions that do not index its wage solves

$$\max_{W_t^*} E_t \sum_{j=0}^{\infty} (\beta \theta_w)^j \left\{ \frac{\mu}{\mu - 1} \left(C_{t+j} - \xi \bar{C}_{t+j-1} \right)^{\frac{\mu-1}{\mu}} \exp \left(\frac{\eta (1-\mu)}{\mu (1+\eta)} \int_0^1 \left(L_{ht+j} \right)^{\frac{\eta+1}{\eta}} dh \right) \eta_{t+j}^c \right\} \tag{15}$$

$$\text{s.t. } L_{h,t+j} = \left(\frac{\prod_{k=1}^{j} \bar{\pi}_{t+k} W_t^*}{W_{t+j}} \right)^{-\varepsilon_w} L_{t+j}, \tag{16}$$

$$P_{t+j} C_{t+j} = \prod_{k=1}^{j} \bar{\pi}_{t+k} W_t^* L_{h,t+j} dh + \Psi_{t+j}, \tag{17}$$

where Ψ_{t+j} includes terms in budget constraints (11) other than C_{t+j} and $L_{h,t+j}$.

Monetary authority. The central bank uses a Taylor rule to set the short-term nominal interest rate,

$$R_t = \rho_r R_{t-1} + (1 - \rho_r) \left[\bar{R} + \rho_\pi \left(\pi_t - \bar{\pi}_t \right) + \rho_Y \left(\log Y_t - \log \bar{Y}_t \right) \right] + \eta_t^r, \tag{18}$$

where ρ_r measures the degree of smoothing of the interest rate; $\bar{R}$ is the nominal neutral interest rate; ρ_π measures a response to the inflation gap; $\bar{\pi}_t$ is the inflation target; ρ_Y measures a response to the output gap; $\bar{Y}_t$ is potential output; η_t^r is an interest rate shock following a process

$$\eta_t^r = \varphi_r \eta_{t-1}^r + \xi_t^r,$$

where ξ_t^r is a normally distributed variable, and φ_r is an autocorrelation coefficient. Potential output changes with productivity according to

$$\log \bar{Y}_t = \varphi_z \log \bar{Y}_{t-1} + (1 - \varphi_z) \log \left(\frac{A_t \bar{Y}}{\bar{A}} \right).$$

Foreign demand for noncommodity exports. By analogy with the demand for imports, the foreign demand function for noncommodity exports is assumed to be

$$X_t^{nc} = \gamma^f \left(\frac{P_t^{nc}}{e_t P_t^f} \right)^{-\phi} Z_t^f, \tag{19}$$

where P_t^{nc} is a domestic price of noncommodity exports; γ^f is the demand-function parameter. In real terms, we have

$$X_t^{nc} = \gamma^f \left(\frac{s_t}{p_t^{nc}} \right)^{\phi} Z_t^f. \tag{20}$$

Balance of payments. The balance of payments in nominal terms is given by

$$\frac{e_t B_t^f}{R_t^f \left(1 + \kappa_t^f\right)} - e_t B_{t-1}^f = P_t^{nc} X_t^{nc} + P_t^{com} X_t^{com} - P_t^m M_t, \tag{21}$$

where B_t^f is domestic holdings of foreign-currency denominated bonds, and R_t^f is the nominal interest rate on the bonds. By normalizing the bonds holdings as $b_t^f \equiv \frac{e_t B_t^f}{\pi_{t+1}^f P_t \bar{Y}}$, the balance of payments in real terms becomes

$$\frac{b_t^f}{r_t^f \left(1 + \kappa_t^f\right)} - b_{t-1}^f \frac{s_t}{s_{t-1}} = \frac{1}{\bar{Y}} \left(p_t^{nc} X_t^{nc} + p_t^{com} X_t^{com} - p_t^m M_t \right), \tag{22}$$

where r_t^f is the real interest rate on the foreign-currency denominated bonds.

Rest-of-the-world economy. The rest of the world is specified by three exogenous processes that, respectively, describe the evolution of foreign output Z_t^f, the foreign real interest rate r_t^f, a foreign commodity price p_t^{comf},

$$\begin{aligned}
\log Z_t^f &= \varphi_{zf} \log Z_{t-1}^f + \left(1 - \varphi_{zf}\right) \log \bar{Z}^f + \xi_t^{zf}, & (23)\\
\log r_t^f &= \varphi_{rf} \log r_{t-1}^f + \left(1 - \varphi_{rf}\right) \log \bar{r} + \xi_t^{rf}, & (24)\\
\log p_t^{comf} &= \varphi_{comf} \log p_{t-1}^{comf} + \left(1 - \varphi_{comf}\right) \log \bar{p}^{comf} + \xi_t^{comf}, & (25)
\end{aligned}$$

where ξ_t^{zf}, ξ_t^{rf} and ξ_t^{comf} are normally distributed random variables, and φ_{Zf}, φ_{rf} and φ_{comf} are autocorrelation coefficients.

Uncovered interest rate parity. We impose an augmented uncovered interest rate parity condition

$$e_t = E_t \left[\left(e_{t-1}\right)^{\varkappa} \left(e_{t+1} \frac{R_t^f \left(1 + \kappa_t^f\right)}{R_t} \right)^{1-\varkappa} \right]. \tag{26}$$

Stationarity condition for the open-economy model. The risk premium κ_t^f is a decreasing function of foreign assets

$$\kappa_t^f = \varsigma \left(\bar{b}^f - b_t^f \right), \tag{27}$$

where $\bar{b}^f$ is the steady state level of the normalized bond holdings. This assumption ensures a decreasing rate of return to foreign assets.

Summary of the model's variables. In sum, there are the following four types of variables in this model: 6 exogenous state variables

$$z_t \equiv \left\{ A_t, \eta_t^R, \eta_t^c, p_t^{comf}, r_t^f, Z_t^f \right\};$$

15 endogenous state variables

$$x_t \equiv \left\{ C_{t-1}, R_{t-1}, s_{t-1}, \pi_{t-1}, \Delta_{t-1}, w_{t-1}, \pi_{t-1}^w, \Delta_{t-1}^w, p_{t-1}^m, \pi_{t-1}^m, I_{t-1}, Z_{t-1}^{com}, b_{t-1}^f, \bar{Y}_{t-1,} K_{t-1} \right\},$$

where π_{t-1}^w, Δ_{t-1}^w, p_{t-1}^m, π_{t-1}^m are wage inflation, wage dispersion, and price and inflation of imports, respectively; 47 non-state (or non-predetermined) variables,

$$y_t \equiv \left\{ \begin{array}{c} F_{1t}, F_{2t}, F_{1t}^w, F_{2t}^w, F_{1t}^m, F_{2t}^m, q_t, \lambda_t, s_t, \\ L_t, K_t, I_t, COM_t^d, M_t, u_t, d_t, Z_t^g, Z_t^n, Z_t, C_t, Y_t, \pi_t, rmc_t, \Delta_t, \pi_t^m, \bar{\pi}_t, p_t^m, R_t, p_t^z, w_t, \\ MPK_t, R_t^k, p_t^i, \kappa_t^f, b_t^f, X_t^{nc}, X_t^{com}, COM_t, Z_t^{com}, \pi_t^w, w_t^*, \Delta_t^w, \bar{Y}_t, p_t^{com}, p_t^{nc}, p_t^{mf}, p_t^y, \end{array} \right\},$$

where $\{F_{1t}, F_{2t}\}$, $\{F_{1t}^w, F_{2t}^w\}$, $\{F_{1t}^m, F_{2t}^m\}$ are supplementary variables in Phillips curves for prices, wages and imports, respectively; q is Tobin's q; rmc_t and MPK_t are real marginal cost and marginal productivity of capital, respectively; p_t^{nc} and p_t^y are prices of noncommodity goods and output, respectively. In Appendix A, we describe our calibration procedure, which closely follows LMM (2020).

2.2 Methodology of our numerical analysis

In the paper, we consider announcements about economic policies that will be implemented at some future dates and we will analyze a reaction of economic agents to such future announcements. Assuming that we are at $t = 0$ and that the policy will be implemented at $T > 0$, we obtain a solution in the form of a sequence of optimal decision functions for periods $t = 0, 1, ..., T$ that characterize anticipatory effects (obviously, the optimal decision functions will differ depending on how far the economy is from the moment the policy is introduced).

Below, we describe a perturbation-based framework that approximate the sequence of time-dependent decision functions that characterize the response to anticipatory effects. Let us consider an infinite-horizon nonstationary equilibrium problem in which a solution is characterized by a set of equilibrium conditions for $t = 0, 1, \ldots,$

$$E_t \left[G_t \left(z_t, x_t, y_t, z_{t+1}, x_{t+1}, y_{t+1} \right) \right] = 0, \tag{28}$$

$$z_{t+1} = Z_t \left(z_t, \epsilon_{t+1} \right), \tag{29}$$

where (z_0, x_0) is given; E_t denotes the expectations operator conditional on information available at t; $z_t \in \mathbb{R}^{d_z}$ is a vector of exogenous state variables at t, and Z_t is a time-dependent law of motion for z_t; $x_t \in \mathbb{R}^{d_x}$ is a vector of endogenous (random) state variables at t; $y_t \in \mathbb{R}^{d_y}$ is a vector of non-state variables – prices, consumption, labor supply, etc. – also called non-predetermined variables; $\epsilon_{t+1} \in \mathbb{R}^p$ is a vector of shocks; G_t is a continuously differentiable vector function. Note that the latter function is time-dependent because the model is nonstationary (due to, for example, time-dependent parameters in policy rules, production function, utility function). A solution is given by a set of time-dependent equilibrium functions $x_{t+1} = X_t(z_t, x_t)$, and $y_t = Y_t(z_t, x_t)$ that satisfy (28), (29) in the relevant area of the state space.

Our perturbation analysis proceeds in the following two steps:

Step I: solving a T-period stationary economy Assume that in a very remote period T, the economy becomes stationary, i.e., $G_t(\cdot) = G(\cdot)$ and $Z_t(\cdot) = Z(\cdot)$ for all $t \geq T$. Therefore, the system (28), (29) becomes

$$E_t \left[G \left(z_t, x_t, y_t, z_{t+1}, x_{t+1}, y_{t+1} \right) \right] = 0, \tag{30}$$
$$z_{t+1} = Z \left(z_t, \epsilon_{t+1} \right). \tag{31}$$

Solving (30), (31) allows us to find the solution $x_{T+1} = \widehat{X}_T \left(z_T, x_T \right)$, and $y_{T+1} = \widehat{Y}_T \left(z_T, x_T \right)$.

Step II: constructing a function path Using a T-period solution $y_{T+1} = \widehat{Y}_T \left(z_T, x_T \right)$ as a terminal condition, iterate backward for $T-1, ..., 1$ on the corresponding equilibrium conditions to construct a sequence (path) of time-dependent value and decision functions $\{X_{T-1}(\cdot), ..., X_1(\cdot)\}$ and $\{Y_{T-1}(\cdot), ..., Y_1(\cdot)\}$. For example, for period t, the system on which we iterate backward is

$$E_t \left[G_t \left(z_t, x_t, y_t, z_{t+1}, x_{t+1}, Y_t \left(z_{t+1}, x_{t+1} \right) \right) \right] = 0,$$
$$z_{t+1} = Z_t \left(z_t, \epsilon_{t+1} \right),$$

here we solve for today's endogenous variables y_t and x_t, given tomorrow's functions $z_{t+1} = Z_t \left(z_t, \epsilon_{t+1} \right)$ and $y_{t+1} = Y_t \left(z_t, x_t \right)$.

In both steps, we use perturbation to find numerical approximations of decision functions. Note that for our central banking model, the variables z_t, x_t, y_t are defined in Section 2.1. The variables y_{t+1} in (28) are given by

$$y_{t+1} \equiv \left\{ \begin{array}{c} F_{1t+1}, F_{2t+1}, F^w_{1t+1}, F^w_{2t+1}, F^m_{1t+1}, F^m_{2t+1}, \lambda_{t+1}, q_{t+1}, u_{t+1}, I_{t+1}, \\ Z^{com}_{t+1}, \pi_{t+1}, \bar{\pi}_{t+1}, \pi^m_{t+1}, \pi^w_{t+1}, p^{com}_{t+1}, p^z_{t+1}, MPK_{t+1}, s_{t+1}, \end{array} \right\}.$$

When nonstationary and unbalanced growth models have no deterministic steady state, it is unclear around what point(s) decision rules must be approximated. To deal with this issue, we augment the model's equations with a time-varying growth rate γ_{xt} that captures how much endogenous state variables grow from period t to $t+1$ due to the time trend or the parameter change. We first assume that growth rates $\{\gamma_{xt}\}_{t=1}^T$ are given and then find those growth rates iteratively.

One objective of this paper is to make the proposed perturbation framework ubiquitous and portable to other applications that the reader can be interested in. To this purpose, we show how to construct time-dependent decision functions using Dynare. To explain how to solve a nonstationary model in Dynare, let's first consider a standard second-order perturbation solution to a stationary model around a deterministic steady state $(\bar{v}; 0)$,

$$g(v;\sigma) \approx \underbrace{g(\bar{v};0) + g_x(\bar{v};0)(v-\bar{v})}_{\text{1st-order perturbation solution}} + \underbrace{\frac{1}{2} g_{xx}(\bar{v};0)(v-\bar{v})^2 + \frac{1}{2} g_{\sigma\sigma}(\bar{v};0)\sigma^2}_{\text{2nd-order terms}}, \tag{32}$$

where $g(v;\sigma)$ is a decision function to be approximated; $v = (x, z)$ is a vector of endogenous and exogenous state variables; σ is a perturbation parameter that scales volatility of shocks; $(\bar{v}; 0)$ is a deterministic steady state; $g(\bar{v}; 0)$, $g_x(\bar{v}; 0)$ and $g_{xx}(\bar{v}; 0)$ are, respectively, steady state values, Jacobian and Hessian matrices of g; $(v - \bar{v})$ is a deviation from a steady state; $(v - \bar{v})^2 \equiv (v - \bar{v}) \otimes (v - \bar{v})$ is a tensor product of the deviations. Three observations are in order: First, a constant term of the policy function is given by $g(\bar{v}; 0) + \frac{1}{2} g_{\sigma\sigma}(\bar{v}; 0)\sigma^2$ and hence, is affected by variances of shocks. Second, the first-order perturbation solution does not depend on the degree of volatility σ, i.e., $g_\sigma(\bar{v}; 0) = 0$. Finally, the term $g_{\sigma x}(\bar{v}; 0)$ is omitted as well because it is equal to zero; see Schmitt-Grohé and Uribe (2004).

In Step I, a Taylor expansion of the policy functions in a stationary model is found around the deterministic steady state $\bar{v}$ of the model. In Step II, we consider two alternative options. The first option is to find solutions for v_{t+1} and v_t around v_t and v_{t-1}, respectively, such that $v_t = v_{t-1} \equiv \bar{v}_t$; in Dynare, it can be implemented by coding v_t and v_{t+1} using the same variable names. The other option is to consider v_{t+1} and

v_t perturbed around $\bar{v}_t$ and $\bar{v}_{t-1}$, respectively, such that $\bar{v}_t = \bar{v}_{t-1}\gamma_{v,t-1}$, where $\gamma_{v,t-1}$ is a time-dependent growth rate; in Dynare, it can be implemented by coding v_t and v_{t+1} with different variable names. In Appendix A, we illustrate how to find growth rates iteratively on a toy example of a neoclassical stochastic growth model with labor augmenting technological progress. This model has a useful property of balanced growth: As perturbation solutions are obtained without exploiting such a property, we are able to better evaluate the performance of the solution method by comparing the approximate perturbation solutions to almost exact solutions that rely on that property. In the main text, we focus on the analysis of anticipatory effects in the large-scale open economy model.

3 Analyzing nonstationary large-scale central banking model

In this section, we present the results for a series of policy experiments in which there are anticipated changes in some model's parameters. In all the figures, the variables are shown in percentage deviations from the initial risky steady state, except for the interest rate and the inflation rate, which are both shown in percentage point deviations from the risky steady state and expressed in annualized terms.

3.1 A decline in the real neutral interest rate

A central bank's announcements reveal information not just about policy, but also about its assessment of the economic outlook. In particular, Nakamura and Steinsson (2018) find that Fed announcements contain information about the path of the natural rate. In this experiment, we model an anticipated gradual decline in the natural rate of interest. Namely, we assume that initial value of the real neutral interest rate is 3 percent and that it starts gradually to go down to 2 percent over 20 quarters. The real neutral interest rate remains at the new level forever. To model a decrease in the neutral rate, we exploit the fact that in steady state, this rate is equal to the inverse of the discount factor, and we translate the assumed decrease in the neutral interest rate into an increase in the discount factor.

Empirical evidence indicates that long-term rates declined from the early 1960s through the mid-1970s, increased until the late 1980s, and declined again from that point on; see, e.g., Yi and Zhang (2019). Moreover, such a decline is not related to the Great Recession. The factors that are responsible for declining long-run rates include lower TFP growth, lower working-age population growth, long-run trends in marginal productivity of capital and risk premium. With this experiment, we investigate how a gradual and permanent reduction in the real interest rate affects the economy.

As is seen from Figure 1, a long-run gradual decrease in the real interest rate results in gradual increase in consumption, investment, labor, capital, and imports. For example, investment increases by 5 percent at the peak. Because of significantly higher investment and labor, output grows by more than 1 percent. The changes in inflation are so small that the nominal and real interest rates behave almost identically. The anticipation effects are the largest in the commodity export and exchange rate which fall by 2 percent and 1.8 percent, respectively, when it became known that the real neutral interest rate will gradually decrease. The results for this experiment indicate that recent trends in the real rate of interest are beneficial for growth.

To compute the solution for this case, we need to take into account that steady state is different in every period of time and that we are to find a path of time-varying growth rates. Since the latter growth-rate path is unknown, we are to iterate on both the growth-rate path and time-varying decision rules. For details on this method, see descriptions of Methods 3-5 in Appendix A where these methods are illustrated for the optimal growth model with labor-augmenting technical change.

3.2 A change in the inflation target

In this experiment, we consider a change in the inflation target that appears in the Taylor rule (18). In particular, we assume that the central bank announces in advance that it will increase the inflation target $\bar{\pi}_t$ and that everyone considers the announcement to be fully credible. Why is it a relevant policy experiment?

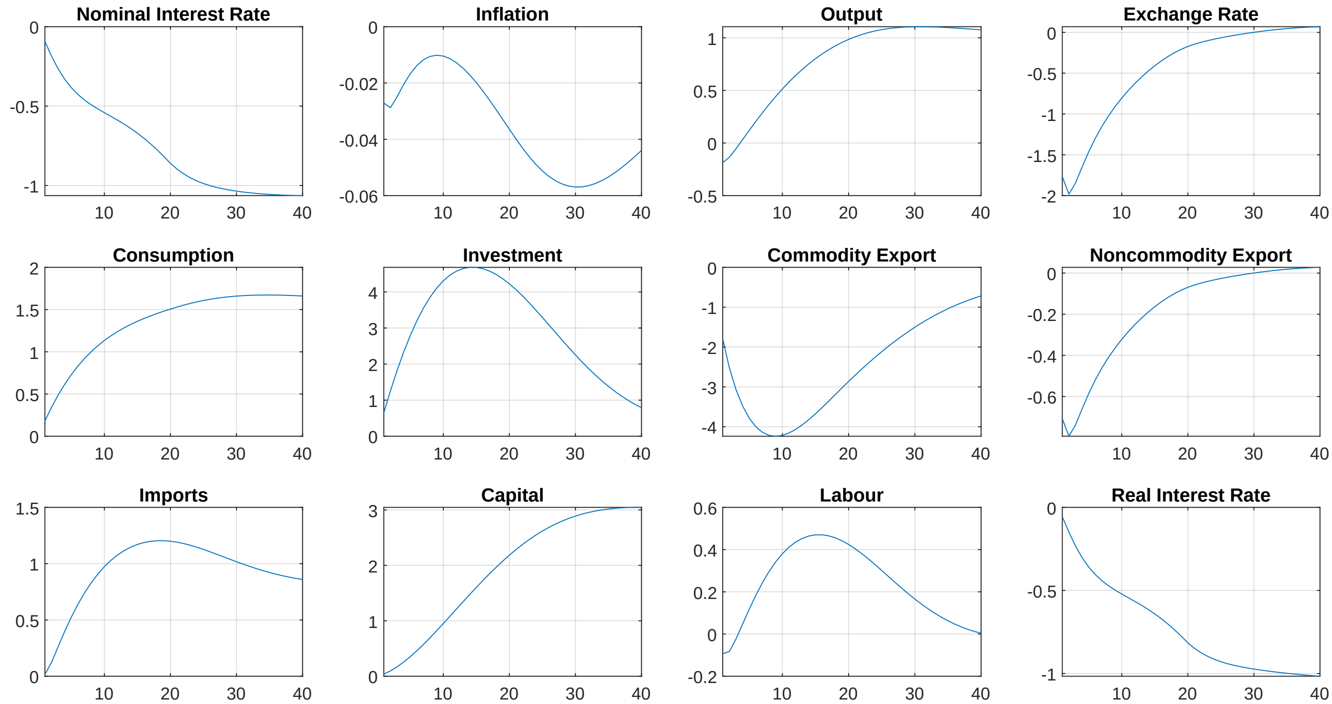

Figure 1: A gradual decline in the real neutral interest rate

During the Great Recession of 2007-2009, central bank's nominal policy rates across a number of countries fell to a zero lower bound (ZLB) on nominal interest rates. There is ample literature arguing that the inflation target is a good policy instrument for dealing with ZLB episodes. For example, Krugman (1998) proposes to use a 4 percent inflation target in the Japanese economy to deal with persisting deflation. Summers (1991) and Fischer (1996) suggest to keep an inflation target in the range of $1-3$ percent if the economy hits ZLB. Furthermore, Blanchard, Dell'Arriccia and Mauro (2010), Williams (2009) and Ball (2013) argue that a higher inflation target would have prevented the interest rate from falling to the ZLB.

In Canada, inflation-targeting framework was adopted in 1991, and since 1995, the inflation target was maintained at the level of 2 percent. The inflation target is reviewed and renewed every five years. In particular, the last review was in October of 2016, when the Bank of Canada decided to keep the target at the same level; this renewal covers the period from January 1st, 2017 to December 31st, 2021. There are two types of possible anticipation effects here. First, we would have had a policy implementation lag leading to anticipation effects if the Bank of Canada decided to change the target in October 2016. Second, in spite of the fact that the inflation target was not changed in 2016, anticipation effects were still present as there were some chances that it would be changed given that Canada was close to the ZLB at that time and policymakers were seriously discussing this possibility.

Figure 2 displays dynamics of the main model's variables. We present the results for the method that finds a perturbation solution obtained around a deterministic steady state (labeled as Method 2 in Appendix B; our sensitivity results for other methods predict similar patterns of behavior).

We consider two cases: first, at $t=1$, the central bank makes an announcement that starting from $t=1$, it will gradually increase the inflation target $\bar{\pi}_t$ from 2 percent to 3 percent during a period of 8 quarters, and second, the same change takes place but starting from $t=5$ (i.e., in one year); the inflation target remains at the new (higher) level forever.

When the inflation-target change begins at $t=1$, inflation follows the same pattern as the target. What is the reason for such behavior of inflation? In our experiment, we assume full credibility of the inflation-target policy. The agents who are not optimizers and index their price by inflation target determine the behavior of inflation, so that inflation repeats the pattern of the inflation target. As a result, the nominal

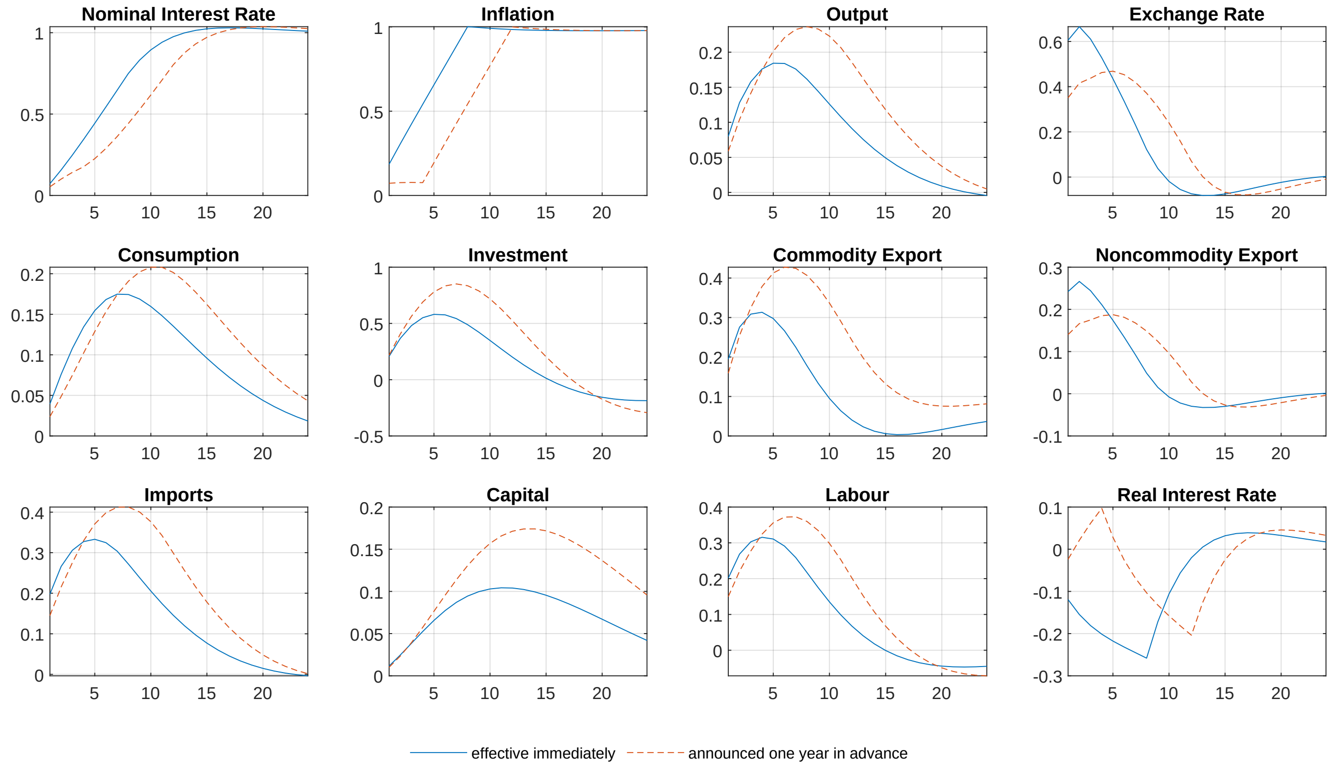

Figure 2: A gradual increase in the inflation target

interest rate gradually increases over the first fifteen periods by 1 percent, and it stays at the new level forever (see Figure 2; note that the real neutral rate is the same as before). Following the announcement, output, investment and commodity exports jump up, and over the transition, the economy experiences an investment- and export-driven growth with the peak increase of output of 0.2 percent. Output begins to descend toward its original level after the eighth period. Consequently, there is only a temporary expansionary effect on the economy due to a higher inflation target.

When the inflation-target change is delayed for one year, the variables behave qualitatively similar. One visible difference from the previous case is that most variables in the figure experience larger increases at the peak (the exchange rate and noncommodity export are exceptions). Therefore, it pays for the central bank to announce this type of policy in advance as output increases more during the transition. The larger jumps in such variables as output, consumption, investment, capital are entirely due to anticipatory effects. That is, agents expect the real interest rate to be lower in the near future, and they accumulate more capital in advance of the more favorable environment which has positive effects on the economy today.

Table 1 contains the mean and maximum residuals in the model's equations used for computing the corresponding variables in the table. As we can see, the maximum residuals range between $10^{-3.13}$ and $10^{-5.84}$, i.e., between .07 percent and .0001 percent, which are very low.[1] For the remaining experiments, we do not report residuals in equations as they are of comparable size.

In the second experiment, we model a probabilistic setting in which agents rationally expect that the inflation target might change in the future. Specifically, we assume that there is a 50-percent chance that starting from $t = 5$ the inflation target $\bar{\pi}_t$ gradually increases from 2 to 3 percent during 8 quarters; otherwise, the inflation target remains the same. Our computational method allows us relatively easily to adapt for modeling of this experiment. When computing policies in period $t = 4$, we specify explicitly using the Dynare macro language that period-4-expectations are weighted sums of expectations over the

[1] In LMM (2020), we reported larger residuals when the economy was hit by a large negative demand shock and the ELB was reached.

	R_t	π_t	Y_t	C_t	I_t	X_t^{nc}	X_t^{com}	M_t	L_t	K_t
Average	-4.66	-5.25	-4.09	-4.47	-5.11	-4.10	-3.17	-5.10	-4.42	-5.91
Maximum	-4.30	-5.17	-4.03	-4.38	-5.03	-4.08	-3.13	-4.98	-4.37	-5.84

Table 1: Residuals in the model's equations on the simulated path, log 10 units. R_t, π_t, Y_t, C_t, I_t, X_t^{nc}, X_t^{com}, M_t, L_t, K_t are the nominal interest rate, inflation, output, consumption, investment, noncommodity export, commodity export, imports, labor and capital, respectively.

two possible realizations in period $t = 5$.

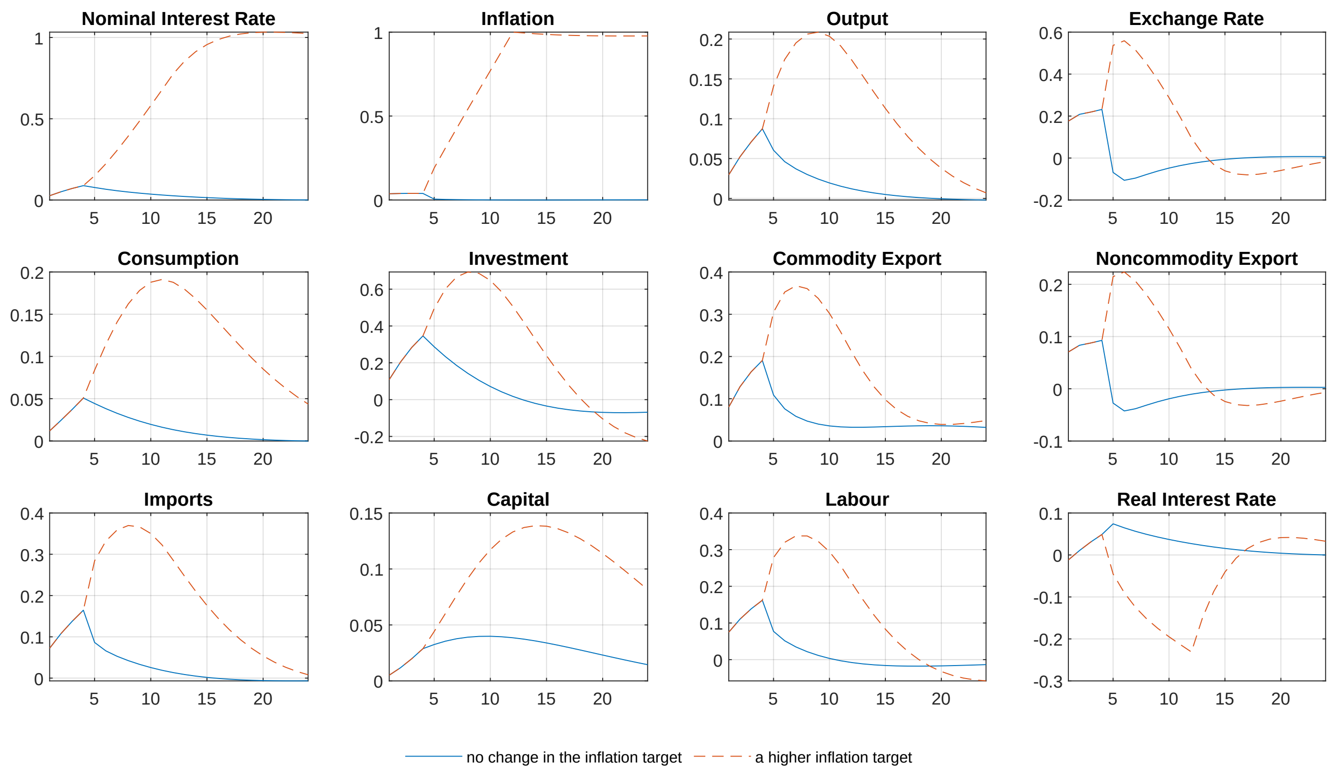

Figure 3: A gradual increase in the inflation target (50% probability)

This experiment is plotted in Figure 3. There are two alternative transition paths differing from period 5 onwards, one per each scenario, i.e., with and without an increase in $\bar{\pi}_t$. Similar to the first experiment, inflation mimics the behavior of the inflation target: it gradually rises to the new steady state level. Starting from the risky steady state at $t = 1$, all the variables experience mild increases, which are due to anticipatory effects on the side of economic agents. Once it becomes known whether the target will go up or not, all the variables quickly return to the original steady state in case of no increase, and they experience a more pronounced hump-shape behavior and return to a new steady state in case of the target increase. In Appendix B, we extend the latter experiment to vary the probability of switching to a higher inflation target at $t = 5$, namely, it is either 25 percent or 75 percent (instead of 50 percent). As those figures show, in case of no inflation-target change, the transition back to the old steady state is significantly faster for the 25-percent case than for the 75-percent case.

3.3 Monetary policy normalization

During the Great Recession of 2007–2009, the nominal interest rate hit the ZLB on nominal interest rates. As a result, central banks could not rely on Taylor rules to conduct their monetary policy and resorted to forward guidance – an unconventional monetary policy consisting in announcing future interest-rate changes. As emphasized by the literature, central bank's communication of the policy-rate future path is the main channel through which forward guidance policy affects the economy. Eggertsson and Woodford (2003) demonstrate that a central bank's promises to keep low interest rates for longer periods helps alleviate negative consequences of binding ZLB. As agents expect future interest rates to be lower than otherwise would be the case without forward guidance, they increase today's investment and consumption, which stimulates the today's economy. Campbell et al. (2012) name this form of forward guidance Odyssean. Another form is Delphic forward guidance: a central bank may have better information about the state of the shocks that hit the economy, and it communicates a forecasted path of policy rates.[2]

In this paper, we assume that a central bank uses forward guidance to convey a policy change when lifting off from the ZLB (rather than to communicate a future path of the interest rate). To model forward guidance in our model, we assume that initially, the economy is at ZLB so that the central bank cannot follow a Taylor rule, and that at $t = 1$, the central bank announces that it will return to the standard Taylor rule (18) at T. To model the ZLB period, we assume that the nominal interest rate is given by

$$R_t = R^{elb},$$

where R^{elb} is an effective lower bound (ELB) on nominal interest rates. When the interest-rate policy is normalized at T, the Taylor rule's coefficients return back to normal values, and the policy is described by the rule (18).

Figure 4 presents the results for this experiment when the solutions are approximated around a deterministic steady state (when computing a point around which solutions are perturbed, taking into account uncertainty leads to similar plots). The change in the interest rate rule announced at $t = 1$ is anticipated by agents. We compare three cases, depending on whether the interest-rate policy returns to normal (i) immediately ($T = 1$), (ii) in one year ($T = 4$), (iii) in two years ($T = 8$). In all the cases, the initial interest rate is below its risky steady state, however, it eventually returns to the steady state.

When the policy change is announced, all the other variables jump up above the steady state. Local currency depreciation makes domestic exports more competitive, which leads to an increase in exports of both commodity (oil) and noncommodity goods. Domestic firms benefit from increased sales, which leads to immediate increases in output, labor, investment and capital. On the other hand, as households work more, they demand more of imported goods, so that imports go up as well. Note that an output increase is the largest when the announced policy change is postponed for the longest period of eight periods. Therefore, we conclude that there are large anticipatory effects in the model. The differences in output dynamics across the considered cases are only present over the transition but not in the initial period – in all three cases, an initial output jump is of equal size. The dependence of the initial reaction in output on the horizon of the forward guidance policy is known in the literature as a forward guidance puzzle. Even though there is no such a dependence in the figure, we do still see that the policy horizon matters for the total effect on output: it reacts more if the actual change in policy rates is postponed further away in the future.

Our above experiment answers a question whether a central bank should communicate to the public its future monetary policy. After the Great Recession of 2007–2009, when the economic conditions improved, an important policy question was how and when to normalize the monetary policy, where normalizing means switching back to some Taylor rule; see Yellen (2015). In particular, the following questions arose after the end of the crisis: (1) Should the central bank normalize policy now or later? (2) Should the central bank do it gradually or all at once? (3) Should the regime shift be announced in advance? (4) Should the policy normalization be time or state dependent? All these questions are hard to address in the context of standard stationary new Keynesian framework because monetary policy normalization is a nonstationary problem. Nevertheless, the technique developed in this paper enables us to study these

[2] Marinkov (2020) argues that during the ZLB period, agents may misjudge a central bank's reaction function and bias their expectations. In this case, the central bank may want to use forward guidance as a guiding tool to correct agents' beliefs.

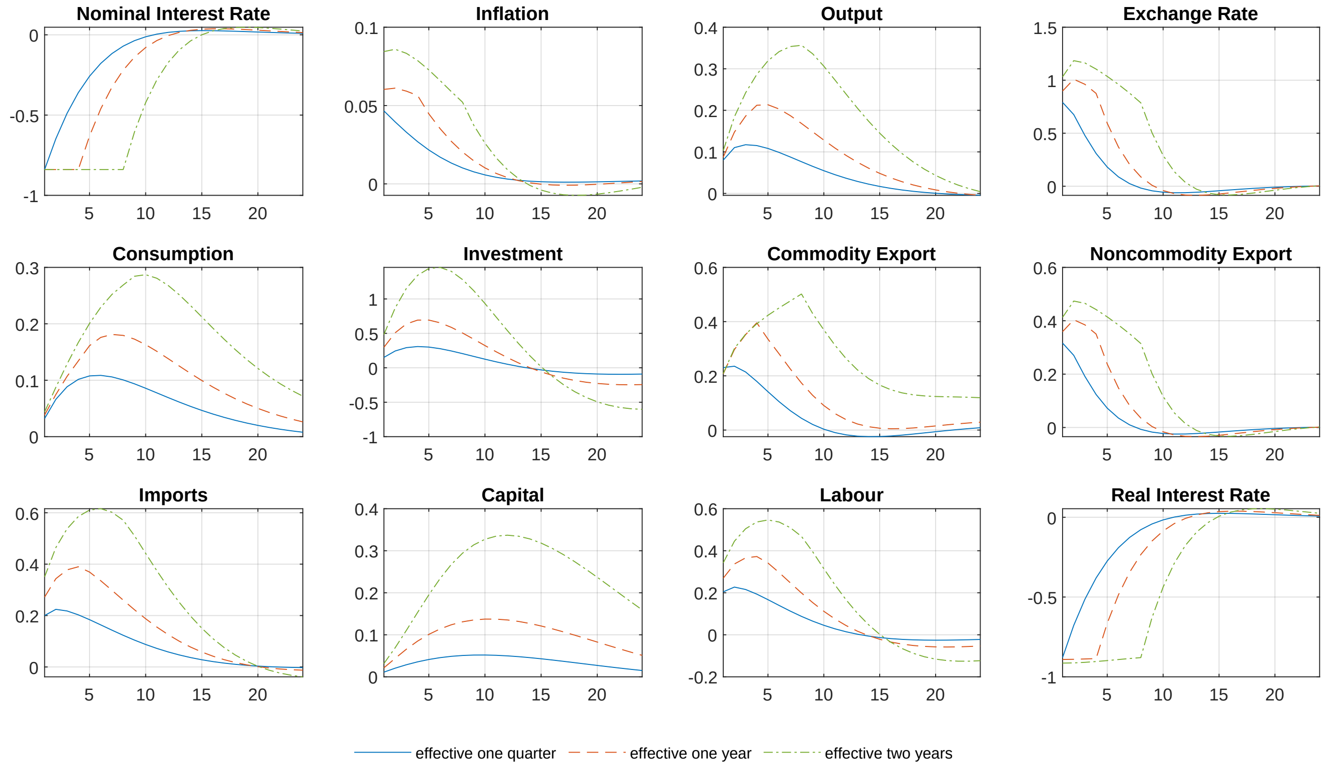

Figure 4: Forward guidance

questions easily. In our above experiment, we compare the economy's behavior under policies that differ in time periods of return to normal values, which correspond to questions (1) and (3). We conclude that there are gains from postponing policy normalization and announcing future lift-off in advance. Similarly, questions (2) and (4) can be answered using our techniques; we leave them for future research.

3.4 Switching to a more aggressive Taylor rule

In this experiment, we consider a one-time change in the sensitivity of the policy rate to inflation and the output gap, as measured by ρ_π and ρ_Y, respectively, in the Taylor rule (18). It differs from the previous experiments where we considered gradual anticipated changes in the model's parameters. Figure 5 plots the economy's responses to two-time increases in either ρ_π or ρ_Y or both, relative to benchmark parameterization. Note that this change in the coefficient values is quite large relative to what a central bank would typically consider. Switching to more aggressive Taylor rules is anticipated at $t = 1$ but occurs at $t = 2$, so that there are immediate anticipatory effects in all the model's variables.

As we can see, both policies – a higher ρ_π and a higher ρ_Y – are inflationary. However, a double increase in the sensitivity to inflation ρ_π is more effective in expanding the economy: output, consumption, investment, capital, labor are visibly higher both at peak and in the new steady state than in the old steady state; commodity production slightly drops, which is related a lower commodity exports. A double increase in the sensitivity to the output gap has more modest effects however. When there is a stronger response to both inflation and the output gap, the quantitative expressions of the effects are roughly in between the other two cases. That is, given that there is a trade off between inflation and the output gap in the policy rule, responding stronger to the output gap undoes the effects of stronger responses to inflation. Overall, total effects are not quantitatively important in our experiment: switching to significantly more aggressive Taylor rules has minor effects on the economy's behavior when the economy is not hit by any shocks. Possibly, more significant changes in the Taylor rule's coefficients would imply quantitatively more

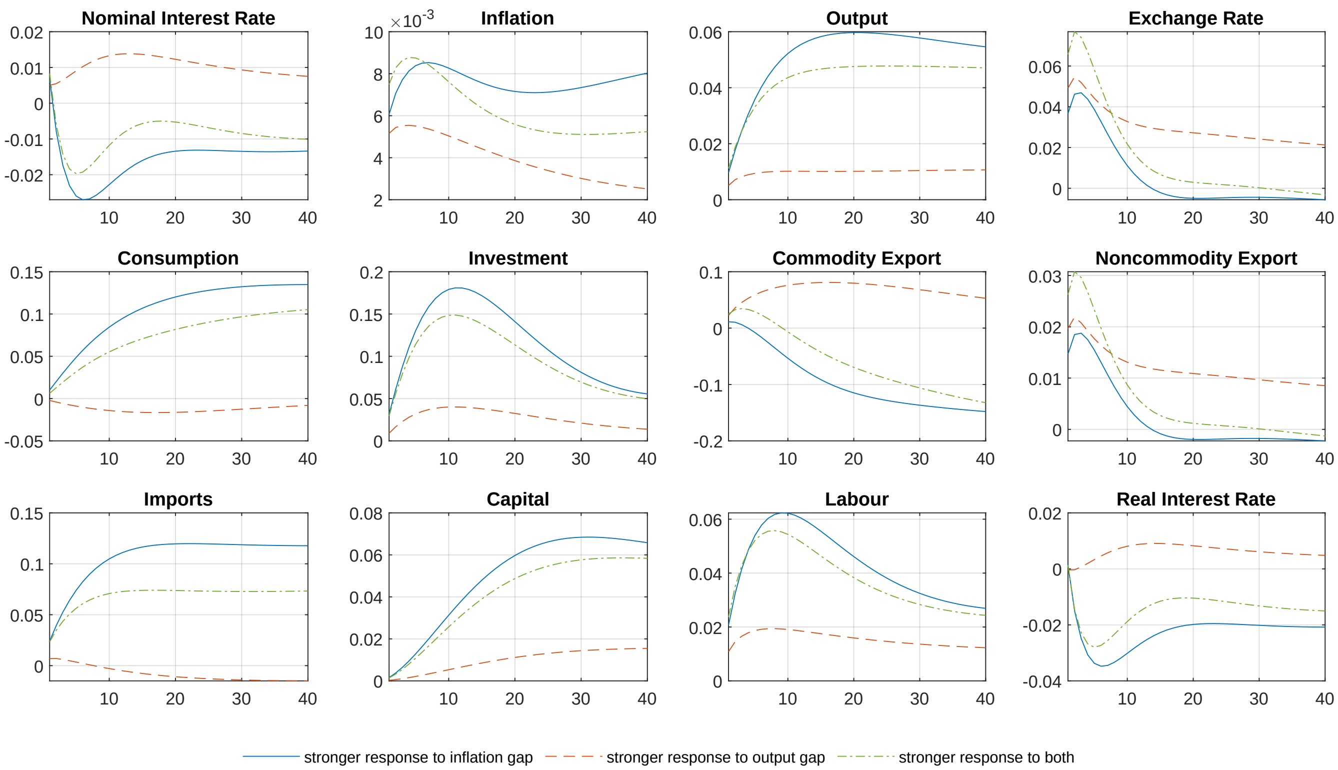

Figure 5: A switch to more aggressive Taylor rules

important effects.[3]

3.5 Switching from inflation targeting to price-level targeting

In this experiment, the central bank switches from the standard Taylor rule (18) targeting inflation to the one targeting a price-level gap,

$$R_t = \rho_r R_{t-1} + (1-\rho_r)\left\{\bar{R} + \rho_\pi\left(\log P_t - \log \bar{P}_t\right) + \rho_Y\left(\log Y_t - \log \bar{Y}_t\right)\right\} + \eta_t^r, \tag{33}$$

where P_t is the actual price level, and $\bar{P}_t$ is the target price level that grows at the rate of inflation target $\bar{P}_t = \bar{P}_{t-1}\bar{\pi}_t$. Therefore, price-level targeting does not suggest that policymakers pursue a constant price level but set a target for the price level that rises over time.

An inflation-targeting central bank does not pay attention to temporary changes in inflation as long as inflation comes back to target after some time ("lets bygones to be bygones"). In contrast, price-level-targeting central bank aims at reversing temporary deviations of inflation from target each time it misses it (e.g., a central bank increases inflation when inflation falls below target). As a result, under inflation targeting, an inflation shock permanently shifts price path to a different level, while under price-level-targeting, any movement in inflation above target is matched with an equal and opposite movement in inflation below target, so that the economy stays on a predetermined price path. As a consequence, with standard inflation targeting, agents will face considerable amount of uncertainty about the future price level (the central bank treats past target misses as bygones and returns inflation to the target level gradually, without taking into account any impact on the price level), while with price-inflation targeting, agents will be confident where inflation will be in the future, even with positive average inflation.

[3] For example, Taylor (1999) argues that the Taylor rule with $\rho_\pi = 0.5$ and $\rho_Y = 1$ is more reasonable than the one advocated in Taylor (1993) when $\rho_\pi = 0.5$ and $\rho_Y = 0.5$.

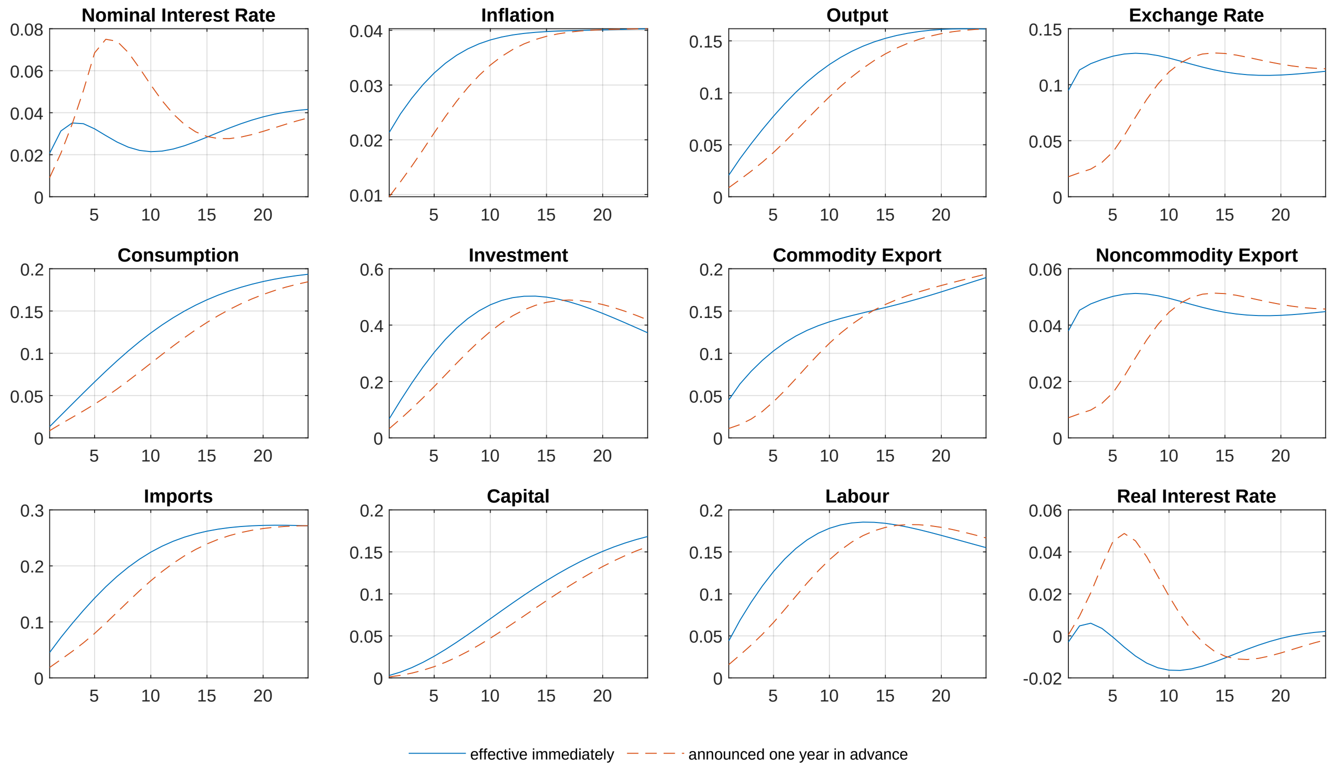

Figure 6: A switch to price-level targeting

In Figure 6, we present the results for two policy experiments in which the policy change becomes effective either immediately (at $t = 1$) or in one year after being announced (at $t = 5$). The new interest-rate rule is associated with higher steady state levels for all the model's variables in the figure. Therefore, switching to price-level targeting has expansionary effects on the economy. Moreover, for all of the variables (except of the nominal interest rate), the immediately implemented policy gives larger benefits than the policy announced one year in advance. That is, if the central bank postpones to implement the switch, the economy reaches the new steady state almost at the same time as the immediate policy but over the transition, the effects are smaller.

Since the seminal paper of Svensson (1999), the literature argues that price-level targeting is a "free lunch" in a sense that it positively affects a short-run trade off between inflation and output variability (namely, it reduces inflation variability without an increase in output variability); see Hatcher and Minford (2016) and Ambler (2009) for surveys. Bernanke (2017) proposes to use a temporary price-level target when short-term interest rates are at (or near) ZLB. When ZLB prevents policymakers from providing adequate stimulus, inflation is below target. Price-level-targeting policymakers compensate for periods of low inflation below target by following a temporary surge in inflation. Bank of Canada has seriously considered the use of price-level targeting; see Kahn (2009) and Bank of Canada (2011).

In the next experiment, we shock the economy, so that there appears a large output gap. In particular, we consider a permanent negative demand shock – a decrease in foreign demand shock, modeled as a negative innovation in the random-walk process for this shock. (A version of this experiment with a permanent decrease in productivity is presented in Appendix B.) In response, the central bank can either continue using a policy rule with inflation targeting or can switch to price-level targeting, which, as we saw, leads to higher steady state output. As a result, switching to price-level targeting can be viewed as an attempt to revive the economy.

Figure 7 presents the results of this experiment when the switch is either implemented immediately or is delayed for one year (but it is still announced today). As we see, there is barely any difference between

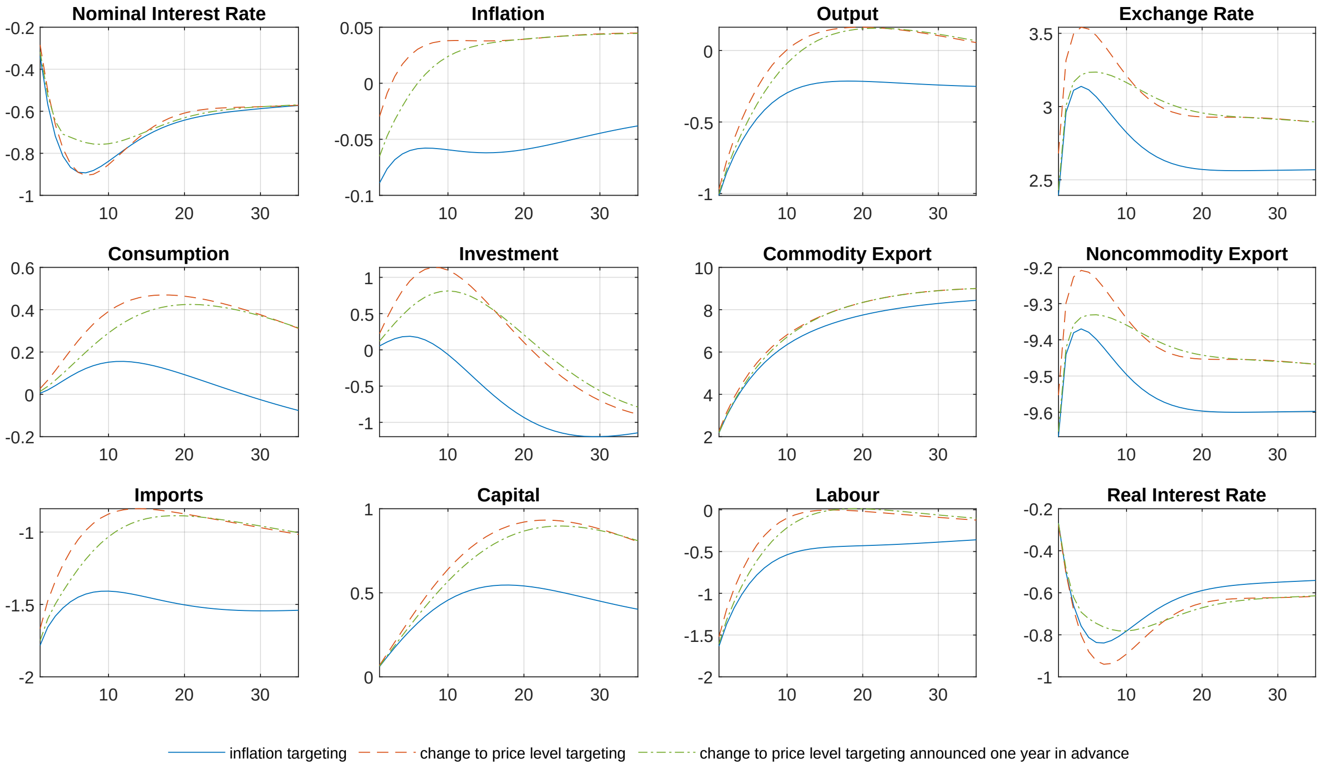

Figure 7: A negative foreign demand shock and a switch to price-level targeting

the immediate and delayed changes in monetary policy for such variables as commodities and commodity export. For all other variables, the immediate policy change has larger impacts than the delayed policy change, which is in line with the previous experiment in Figure 6. Therefore, the anticipation effects work in the direction of softening the effects of the negative demand shock, with impulse responses lying between the cases of no-change and immediate change. In particular, the dynamics of the nominal interest rate is smoother in case of anticipated policy, which leads to smoother behavior of the remaining variables. The initial impact on the economy is significant, e.g., output and labor fall by 1 and 1.5 percent in the three scenarios considered. With no switch in monetary policy, output recovers a bit but its new steady state is still below old steady state. With the policy switch, output is nearly the same as before the shock, and consumption is even higher. Note that each considered policy implies that the central bank tightens the monetary policy, even in the economic downturn.

3.6 Switching from inflation targeting to average inflation targeting

In this experiment, we consider a switch from the standard Taylor rule (18) targeting inflation to a rule that incorporates an average of the past inflation (and not actual inflation),

$$R_t = \rho_r R_{t-1} + (1-\rho_r)\left[\bar{R} + \rho_\pi \left(\frac{1}{M+1}\sum_{j=0}^{M} \pi_{t-j} - \bar{\pi}_t\right) + \rho_Y \left(\log Y_t - \log \bar{Y}_t\right)\right] + \eta_t^r. \tag{34}$$

Average inflation targeting shares many of the properties of price-level targeting. As was suggested by the previous literature, average inflation targeting is a middle ground between price-level targeting and inflation targeting; see Nessén and Vestin (2005). Under average inflation targeting, a central bank tries to achieve today's inflation such that, when averaged with previous inflation, it is equal to target inflation. For example, if inflation target is 2 percent over 3 years, and if in the third year, inflation deviates is 3 percent, the central bank will need to achieve policy-induced inflation of 1 percent in the next year. As a result, inflation will oscillate around average inflation target and the average inflation target is always achieved. The price level will stay close to its path, even though the level will sometimes deviate from the fixed path.

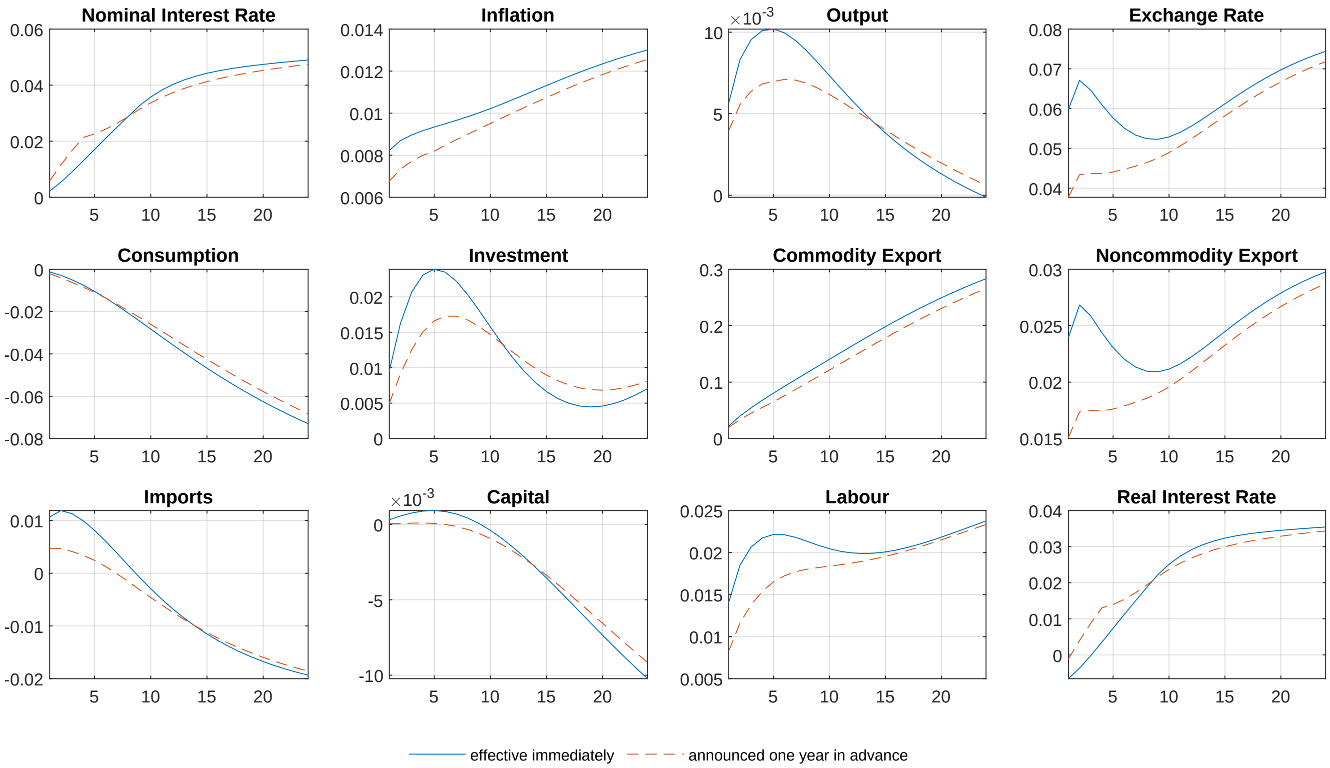

Figure 8: A switch to average inflation targeting

Figure 8 displays the results for two cases: one is when the switch happens immediately and the other when it is implemented with a lag of one year after it was announced. Amano et al. (2020) study optimal history dependence under average inflation targeting in the context of the standard new Keynesian model accounting for the ELB, and they find that optimal M ranges from 2 to 8. We assume $M = 8$ which is the largest number of lags found by Amano et al. (2020).

In contrast to our expectations, this policy change has very modest anticipation effects on the economy in the absence of any shocks. In fact, when the policy becomes effective immediately, there are larger responses in such variables as output, labor, imports, and noncommodity exports. That is, reacting to average inflation rather than inflation smooths out dynamics to a new steady state. Therefore, we would not expect the Canadian economy to experience any drastic changes in the course of transition to average inflation targeting.[4]

4 Are news shocks the same as anticipation effects?

News shocks have long been thought as a potential driving force of business cycles; see, e.g., Barro and King (1984), Beaudry and Portier (2006), Beaudry and Portier (2007), Jaimovich and Rebelo (2009). Schmitt-Grohé and Uribe (2012) propose a tractable perturbation framework for dealing with anticipated changes. In particular they model such changes as periodic shocks following a Markov process. That is,

[4] On August 31st 2020, the Fed's Chair Jeromy Powell announced that Fed will switch from inflation targeting to average inflation targeting. However, one month prior to that, there was evidence that Fed will evolve that framework, and as a result, there were substantial anticipatory price moves in the U.S. economy.

in their framework, an anticipated shock happens with some specified periodicity. An interestin question is how the anticipation effects produced by our perturbation-based method compare to those produced by Schmitt-Grohé and Uribe's (2012) method analyzing news shocks.

To study this question, we consider a version of the experiment of Section 3.4 in which the central bank switches to a more aggressive Taylor rule, namely, we assume that the sensitivity to inflation ρ_π in the Taylor rule (18) is doubled relative to its benchmark value. More specifically, we consider a permanent change in ρ_π, which is announced at $t = 1$ and implemented at $t = 2$. Our analysis provides a natural way of modeling this situation. Namely, we costruct a stationary solution for period $t = 2$ and we find a solution for period $t = 1$ that matches a given terminal condition (decision rule) constructed for period $t = 2$.

The analysis of Schmitt-Grohé and Uribe (2012) does not specify how their perturbation method can be used for analyzing non-recurrent anticipated events like the one we describe above. We tried out two ways of adapting periodic news shocks to our experiment: First, we consider a unit-root process for $\rho_{\pi,t}$, i.e., $\rho_{\pi,t} = \rho_{\pi,t-1} + \varepsilon_{t-1}$, in which initially $\rho_{\pi,0} = \rho_{\bar{\pi}}$. In this specification, the shock innovation, ε_t captures news that become known at period t and that have a direct impact at $t+1$. In our experiment, $\varepsilon_1 = \rho_{\bar{\pi}}$ at period $t = 1$, and at all other periods the shock innovation is zero. It implies that $\rho_{\pi,1} = \rho_{\bar{\pi}}$ and $\rho_{\pi,t} = 2\rho_{\bar{\pi}}$ for all $t \geq 2$. Second, we consider the news shock to be temporary, i.e., $\rho_{\pi,t} = \rho_{\bar{\pi}} + \varepsilon_{t-1}$. For the same shock innovations as above, we get $\rho_{\pi,2} = 2\rho_{\bar{\pi}}$ in period $t = 2$ and $\rho_{\pi,t} = \rho_{\bar{\pi}}$ in all other periods.

Figure 9 presents the impulse responses for the three considered solutions of the second order. The volatility of the news shock is assumed to be zero, so the initial risky steady state is the same for all three solutions. The following observations are in order: First, it appears that our solution with anticipation effects is situated in between the two news-shock solutions. Second, both our solution and permanent news-shock solution converge to new (although different) risky steady states, while the temporary news-shock solution converges to old steady state, given the temporary nature of the shock. Third, in the three cases, all the variables behave qualitatively similar: a more aggressive central bank leads to an increase in inflation and a decrease in nominal and real interest rates, which raises output, investment, capital and imports. Fourth, the gap between our solution and permanent news shock solution depends on the initial condition: we observe in our sensitivity experiments (these experiments are not reported) that the gap is smaller if we start below steady state. This is because the anticipation effects are mixed up with transition-dynamics effects. Finally, the difference between the two news-shock solutions in our second-order perturbation solution come from the differences in slopes of the decision rules (like the term $g_{xx}(\bar{v}; 0)$ in (32)). If we were to consider the first-order perturbation, the economy would remain at the deterministic steady state in the two news-shock solutions.

Furthermore, it is important to emphasize that the permanent news-shock approach predicts dramatically larger effects associated with the switch to a more aggressive Taylor rule than our approach (except for consumption). This is true both for the anticipation effects and for differences in steady states. For example, anticipation effects in investment are five times larger at peak for the news shocks than for our perturbation solutions. We conclude that, the two approaches are not equivalent: the news-shock approach significantly overstates the importance of anticipated shocks. This is because we assume a unit-root process for $\rho_{\pi,t}$, which implies that once a news shock happens, its effects persist forever. Moreover, the implications for news shocks crucially depend on the stochastic process assumed, which is not the case of our approach.

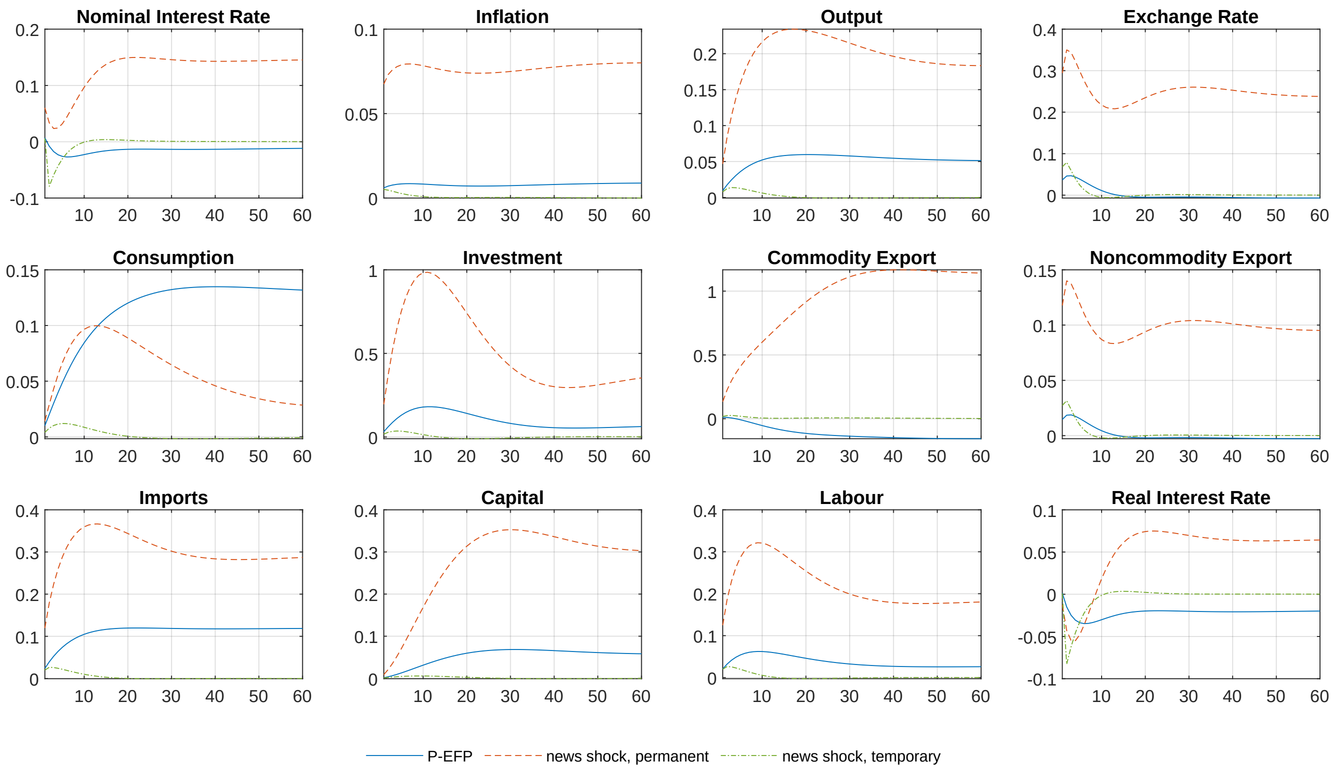

Figure 9: A switch to a more aggressive Taylor rule at $t = 2$ announced at $t = 1$.

5 Conclusion

The literature recognizes that private sector's expectations are important for policy outcomes. The empirical literature extensively investigates how agents' expectations impact policy outcomes, however, little work has been done on evaluating correctly such impacts within a DSGE framework. This paper fills in this gap. In this paper we focus on anticipation effects of monetary policy in a context of a realistic DSGE model of central banking. The anticipation effects are the strongest for such time dependent economic changes as a gradual change in the natural rate of interest, policy-rate normalization in the aftermath of the ZLB crisis, a gradual change in the inflation target level. The other time dependent policy changes like a switch to a more aggressive policy rate rule, a switch to price-level or average inflation targeting lead to more modest anticipation effects. Our perturbation-based framework for solving, calibrating, simulating and estimating of parameters provides a simple and tractable way of analyzing nonrecurrent transitions associated with a policy change. Literally, our analysis makes it possible to construct a precise model-consistent path of real-world economies.

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Appendix A

In this section, we illustrate the implementation of perturbation-based method on a toy example – a neoclassical stochastic growth model with labor augmenting technological progress. We consider a version of the model that allows for balanced growth. To solve this model, we proceed as if growth was unbalanced, and then compare our solutions to those obtained by an accurate projection method that solves detrended (stationary) model.

The growth model with labor-augmenting technological progress. We consider the following neoclassical stochastic growth model with labor augmenting technological progress:

$$\max_{\{c_t, k_{t+1}\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right] \tag{35}$$

$$\text{s.t. } c_t + k_{t+1} = (1-\delta) k_t + z_t f(k_t, A_t), \tag{36}$$

$$\ln z_{t+1} = \rho \ln z_t + \sigma \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim \mathcal{N}(0,1), \tag{37}$$

where (k_0, z_0) is given; E_t is an operator of conditional expectation; $c_t \geq 0$ and $k_t \geq 0$ are consumption and capital, respectively; $A_t = A_0 \gamma_A^t$ is labor augmenting technological progress with the rate $\gamma_A \geq 1$; u and f are utility and production functions, respectively; and $\beta \in (0,1)$; $\delta \in [0,1]$; $\rho \in (-1,1)$; $\sigma \in (0,\infty)$.

Discussion. Why cannot we solve a nonstationary model with conventional solution methods? For (35)–(37), the Euler equations is given by

$$u(c_t) = \beta E_t \left[u'(c_{t+1}) (1 - \delta + z_{t+1} f_k (k_{t+1}, A_{t+1})) \right].$$

The mainstream of economic literature considers stationary models. We can make our model stationary by setting at $A_t = A$ for all t. The solution to such model is characterized by time-invariant (stationary) decision functions. Conventional solution methods iterate on the Euler equation until a fixed-point decision function for consumption $c_t = C(k_t, z_t)$ is found.[5] However, if A_t grows over time, then the optimal decision function changes over time $C_t(\cdot) \neq C_{t+1}(\cdot)$, then there is no fixed-point solution $C(\cdot)$ so that the conventional methods are not applicable. Under additional restrictions on preferences and technology, the model with labor augmenting progress has balanced growth and can be converted into stationary; see King et al. (1988). We will not focus on this special case but will approximate a sequence (path) of time-dependent functions $\{C_0(\cdot), C_1(\cdot), ...\}$.

In doing this, we exploit a turnpike theorem; see Majumdar and Zilcha (1987) and Mitra and Nyarko (1991) for examples of turnpike theorems for nonstationary models. A turnpike theorem studies the convergence of finite-horizon economies to infinite horizon economies as time horizon increases. For this model, the turnpike theorem is proven in MMTT (2020). Two important consequences of the turnpike theorem help us compute solutions in the nonstationary economy: First, the infinite- and finite-horizon solutions follow closely one another for a long time and diverge only when the economy approaches to a terminal condition. Second, two terminal conditions $k_T = k'$ and $k_T = k''$ that are close to the solution to nonstationary model make the finite-horizon path closer to the infinite-horizon path. The former allows us to approximate infinite-horizon solutions by finite-horizon solutions, while the latter tells us that it is important to select a good terminal condition, the one close to the infinite-horizon equilibrium path.

To implement the perturbation procedure described in the main text, in Step I, we construct a stationary (time-invariant) model of period T and construct the corresponding Markov decision rule for consumption $C_T(\cdot)$, and in Step II, we use C_T to iterate backward on Euler equations in order to construct a sequence (path) of time-dependent value and decision functions $\{C_{T-1}(\cdot), C_{T-2}(\cdot), ..., C_0(\cdot)\}$, respectively. As a final step, we check the turnpike theorem by verifying that the constructed finite-horizon solution for initial τ periods converges periodwise to a limiting $\{C_0^*(\cdot), C_1^*(\cdot), ..., C_\tau^*(\cdot)\}$ as time horizon T increases. We elaborate on Steps I and II in details below.

First of all note that the model with growth has no natural steady state. To deal with this issue, we introduce time-varying growth rates of capital γ_{kt} that capture how much this state variable grows from period t to $t+1$ due to the time trend or the parameter change.

Step I: Solving for terminal decision functions. In Step I, we aim to construct stationary Markov terminal condition in the form of a decision function for consumption $c_T = C_T(k_T, z_T)$ which is as close as possible to unknown decision function of the infinite horizon model. We assume balanced growth $\gamma_{kT} = \gamma_{cT} = \gamma_A$, and feed the resulting two equations to a Dynare perturbation. Assuming $c_T = C_T(k_T, z_T)$ and $c_{T+1} = C_T(k_{T+1}, z_{T+1})$, we obtain the usual stationary solution to

$$
\begin{gathered}
u'(c_T) = \beta E_T\left[u'(c_{T+1}\gamma_A)\left(1-\delta + z_{T+1} f_k\left(k_{T+1}\gamma_A, A_T\gamma_A\right)\right)\right],\\
c_T = (1-\delta)\,k_T + z_T f\left(k_T, A_T\right) - k_{T+1}\gamma_A.
\end{gathered}
$$

Unless $\gamma_A = 1$, the model does not have a balanced growth and our approximation does not coincide with the infinite horizon solution at T. But the turnpike theorem implies that the specific terminal condition assumed at T does not affect significantly the solution up to τ provided that $\tau \leq T$. There are ways of constructing more accurate terminal conditions at additional costs.[6]

Step II: Finding a path of decision functions. In Step II, we start from the constructed terminal condition for T and proceed backward to compute the path of the decision functions for $t = T-1, T-2, ..., 0$

[5] Solution to the growth model could equally well be expressed by a decision function for next period capital $k_{t+1} = K(k_t, z_t)$.

[6] MMTT (2020) offer an alternative way of constructing a terminal condition. Namely, they assume that the solution is stationary in periods T, $T+1$ and $T+2$ provided that it is adjusted to growth. This gives 4 equations (Euler equation and constraint) for T and $T+1$, which can be solved with respect to steady state k_T^*, c_T^* and growth rates γ_{kT} and γ_{cT}.

by iterating backward on

$$u'(c_t) = \beta E_t \left[u'(C_{t+1}(k_{t+1}, z_{t+1}))(1-\delta + z_{t+1} f_k(k_{t+1}, A_{t+1}))\right], \tag{38}$$

$$k_{t+1} = (1-\delta) k_t + z_t f(k_t, A_t) - c_t. \tag{39}$$

In particular, for period $T-1$, given $c_T = C_T(k_T, z_T)$, Dynare produces the decision function for $c_{T-1} = C_{T-1}(k_{T-1}, z_{T-1})$, in period $T-2$, given $c_{T-1} = C_{T-1}(k_{T-1}, z_{T-1})$ we find $c_{T-2} = C_{T-2}(k_{T-2}, z_{T-2})$ and so on until the entire solution path is constructed.

Perturbation solutions we construct are obtained around a deterministic growth path. We consider five alternative methods for constructing such a path. We either assume some exogenous growth rates or precompute the growth rates endogenously by shutting down uncertainty in the model. Also, our methods differ in a way the policy functions are specified. In particular, for each deterministic growth-path specification, we have two versions of the algorithm: one in which a next-period policy function takes into account the volatility of uncertainty σ, and the other in which it does not setting $\sigma = 0$. Why might we want to handle the volatility differently? Perturbation policy functions of second and higher orders of approximation are not passing in general through a deterministic steady state of the model. Even in the balanced growth model, if true policy functions for period $t+1$ are combined with the model's equations written for period t, the deterministic steady state would not be a solution of the deterministic version of the combined system of equations. This feature can be overcome by recognizing explicitly that $C_{t+1}(\cdot)$ depends on σ and by setting σ to zero when computing the deterministic steady state.

Methods 1 and 2. Methods 1 and 2 find local approximations of today's consumption policy function $C_t(k_t, z_t)$ in period t from equations (38) and (39) given the next-period function $C_{t+1}(k_{t+1}, z_{t+1})$. The difference between the two methods lies only in the point around which the local approximation is taken and it is related to our implementation in Dynare.

In period t, Method 1 finds local approximation around a point $(k_t^*, 1)$ that solves the following system of two equations for c_t^* and k_t^*:

$$u'(c_t^*) = \beta u'(C_{t+1}(k_t^*, 1))\left[1-\delta + f_k(k_t^*, A_{t+1})\right], \tag{40}$$

$$k_t^* = (1-\delta) k_t^* + f(k_t^*, A_t) - c_t^*. \tag{41}$$

Here today's and tomorrow's capital are the same and equal to k_t^* because we assume that the growth rate of capital is one.

To understand Method 2, recall that the consumption decision function obtained by perturbation depends on the perturbation parameter σ and is given by $C_{t+1}(.,.;\sigma)$ in period $t+1$; see (32) for a general representation.[7] In Method 2, we perturb around a point that is computed taking $C_{t+1}(.,.;\sigma)$ without the effect of uncertainty, $\sigma = 0$; this approach is similar to finding a deterministic steady state first (as $\sigma = 0$). In other words, the approximation is conducted around a point $\left(k_t^\dagger, 1\right)$ that solves the following system of two equations for $c_t^\dagger$ and $k_t^\dagger$

$$u'(c_t^\dagger) = \beta u'(C_{t+1}\left(k_t^\dagger, 1; 0\right))\left[1-\delta + f_k\left(k_t^\dagger, A_{t+1}\right)\right], \tag{42}$$

$$k_t^\dagger = (1-\delta) k_t^\dagger + f\left(k_t^\dagger, A_t\right) - c_t^\dagger. \tag{43}$$

Evidently, the first-order perturbation solutions obtained by Method 1 and 2 are identical, as such solutions do not depend on uncertainty.[8]

[7] Note that the dependence of $C_{t+1}(k_t^*, 1)$ on σ is implicit in Method 1, i.e., we mean $C_{t+1}(k_t^*, 1; \sigma)$ there.

[8] Note, however, that higher-order approximations will differ between the two methods not only because the intercepts associated with uncertainty are distinct (equal to $C_{\sigma\sigma,t+1}(k_t^*, 1)\sigma^2$ and $C_{\sigma\sigma,t+1}\left(k_t^\dagger, 1; 0\right)\sigma^2$ for Method 1 and Method 2, respectively) but also because the points around we approximate differ.

Methods 3 and 4. These two methods explicitly account for time-varying growth rates $\{\gamma_{kt}\}_{t=1}^{T}$ (recall that for both Methods 1 and 2 we assume that growth rates are equal to unity). Similarly to the latter methods, our Methods 3 and 4 differ in points around which we find Taylor's expansions and parallel to Methods 1 and 2, respectively. To construct a path of growth rates $\{\gamma_{kt}\}_{t=1}^{T}$, both Methods 3 and 4 solve a deterministic version of the model. Namely, we shut down uncertainty by assuming $z_t = 1$ for all t, set $\tilde{c}_{T+1}$ and $\tilde{k}_{T+1}$ equal to the steady state of the stationary model in the terminal period, and solve the following system of equations:[9]

$$u'(\tilde{c}_t) = \beta u'(\tilde{c}_{t+1}) \left(1 - \delta + f\left(\tilde{k}_{t+1}, A_{t+1}\right)\right),$$
$$\tilde{k}_{t+1} = (1-\delta)\,\tilde{k}_t + f\left(\tilde{k}_t, A_t\right) - \tilde{c}_t.$$

Given the solution $\left\{\tilde{k}_{t+1}\right\}_{t=1}^{T}$, we compute the growth rates as $\gamma_{kt} = \tilde{k}_{t+1}/\tilde{k}_t$. Both Methods 3 and 4 take $\{\gamma_{kt}\}_{t=1}^{T}$ as given.

In period t, Method 3 perturbs the solution around a point $(k_t^*, 1)$ that solves for k_t^* and c_t^* the following system of two equations:

$$u'(c_t^*) = \beta u'(C_{t+1}(\gamma_{kt} k_t^*, 1))(1 - \delta + f(\gamma_{kt} k_t^*, A_{t+1}), \tag{44}$$
$$\gamma_{kt} k_t^* = (1-\delta)\, k_t^* + f(k_t^*, A_t) - c_t^*. \tag{45}$$

Note that a variable k_{t+1}^* is replaced by $\gamma_{kt} k_t^*$ meaning that we take into account growth when computing the point of approximation. In turn, Method 4 finds a perturbation solution around a point $\left(k_t^{\dagger}, 1; 0\right)$ and finds $c_t^{\dagger}$ and $k_t^{\dagger}$ by solving

$$u'(c_t^{\dagger}) = \beta u'(C_{t+1}\left(\gamma_{kt} k_t^{\dagger}, 1; 0\right))1 - \delta + f\left(\gamma_{kt} k_t^{\dagger}, A_{t+1}\right), \tag{46}$$
$$\gamma_{kt} k_t^{\dagger} = (1-\delta)\, k_t^{\dagger} + f\left(k_t^{\dagger}, A_t\right) - c_t^{\dagger}. \tag{47}$$

Method 5. Method 5 is close to Method 3, but the path for growth rates is computed iteratively. We begin by exogenously fixing the path $\left\{\gamma_{k,t}\right\}_{t=1}^{T}$ and obtaining the policy functions for a stochastic version of the model; this is similar to Method 3. As a next step, we simulate the model with the realized values of shocks which are set to zero, we compute the growth rates of capital over this simulated path, and we obtain the policy functions for a stochastic version of the model. We can repeat this step as many times as necessary. We do not offer any counterpart of Method 5 (i.e., Method 6) that corrects for volatility as it is the case of the methods above because the stochastic growth path is computed in a stochastic version of the model, in which the growth path is obtained endogenously.

Numerical results. In this section, we present the results of our numerical analysis. We assume the standard utility and production functions:

$$u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma}, \quad f(k, A) = A^{1-\alpha} k^{\alpha}.$$

For all the experiments, we fix the parameters $\{\alpha, \beta, \delta, \rho\}$ at the following values:

$$\alpha = 0.36, \quad \beta = 0.99, \quad \delta = 0.025, \quad \rho = 0.95.$$

[9] To implement this step in Dynare, we just solve a system of equations backward in terms of variables $\left\{\tilde{c}_t, \tilde{k}_t\right\}$.

	Method 1		Method 2		Method 3		Method 4		Method 5	
First-order solution										
Errors, in $\log_{10}$ units										
Horizon		Mean	Max			Mean	Max		Mean	Max
$[0, 50]$		-1.41	-1.13			-1.50	-1.22		-1.53	-1.23
$[0, 100]$		-1.24	-0.97			-1.33	-1.11		-1.35	-1.12
$[0, 150]$		-1.14	-0.77			-1.25	-1.08		-1.27	-1.08
$[0, 175]$		-1.07	-0.58			-1.22	-0.94		-1.23	-0.97
$[0, 200]$		-1.04	-0.58			-1.19	-0.94		-1.20	-0.97
Running time, in seconds										
Solution	161.57		157.60		294.28		288.03		317.38	
Simulation	0.0387		0.0275		0.0346		0.0293		0.0271	
Second-order solution										
Errors, in $\log_{10}$ units										
Horizon	Mean	Max	Mean	Max	Mean	Max	Mean	Max	Mean	Max
$[0, 50]$	-2.28	-2.03	-2.83	-2.53	-3.48	-2.95	-2.79	-2.24	-3.51	-2.92
$[0, 100]$	-2.12	-1.90	-2.77	-2.53	-3.30	-2.93	-2.46	-2.05	-3.30	-2.91
$[0, 150]$	-2.05	-1.80	-2.75	-2.53	-3.26	-2.88	-2.33	-2.03	-3.26	-2.88
$[0, 175]$	-2.00	-1.71	-2.71	-2.15	-3.14	-2.23	-2.38	-2.03	-3.14	-2.23
$[0, 200]$	-2.04	-1.71	-2.61	-1.79	-3.07	-2.23	-2.43	-2.03	-3.09	-2.23
Running time, in seconds										
Solution	167.99		167.18		308.87		296.46		337.60	
Simulation	0.0256		0.0249		0.0326		0.0312		0.0348	

Notes: Mean and Max are, respectively, the average and maximum of absolute difference between the P-EFP and exact solutions (in $\log_{10}$ units) on a stochastic simulation of 200 observations.

Table 2: Difference of a simulated solution path from the balanced growth path in log10 units

We vary the values of the remaining parameters $\{\gamma, \sigma_\varepsilon, \gamma_A, T\}$; in the benchmark case, we set them to the following values:

$$\gamma = 5, \quad \sigma_\varepsilon = 0.03, \quad \gamma_A = 1.01, \quad T = 200.$$

We simulate the model's solution for different values of the terminal date T. For all simulations, we use the same initial condition (k_0, z_0) and the same sequence of productivity shocks $\{z_t\}_{t=1}^{\mathcal{T}}$.

To see whether our perturbation-based method computes accurate solutions, we obtain an (almost) exact solution by exploiting the property of balanced growth. For this purpose, we first introduce labor-augmenting technical change into the model, then derive the first-order conditions, and finally, detrend them. The resulting stationary model is solved by a very accurate standard projection method with Smolyak grid, third-order polynomial approximation, and 10-node Gauss-Hermite quadrature (the maximum residuals in the model's equations are of order 10^{-9} in $\log_{10}$ units). We compare the simulated series generated by such a projection method with those of our perturbation method on a fixed sequence of shocks of length $\mathcal{T}$.

In Table 2, we report absolute unit-free mean and maximum differences between our approximate and balanced growth ("exact") solutions (in $\log_{10}$ units) on a simulated path $[0, \mathcal{T}]$ with $\mathcal{T} \in \{50, 100, 150, 175, 200\}$. We consider both, first- and second-order approximations.

As is evident from the table, the first-order perturbation solutions are significantly less accurate than the second-order solutions; the difference between the two can reach two orders of magnitude. However, in terms of running times (both solution and simulation), the two solutions are roughly comparable. It is not faster to obtain a first- than second-order solution because each perturbation step takes just few seconds and the largest share of time is spent on finding different decision rules for each period. For second-order approximations, the most basic method, Method 1, yields very accurate solutions: the mean difference from the exact solution is at most 1 percent across the considered simulation lengths, while the maximum difference reaches 1.5 percent. The ranking of the methods in terms of accuracy varies with time horizon $\mathcal{T}$. For example, for $\mathcal{T} = 200$, Method 1 is the least accurate method, followed by Method 2, and then by Methods 4 and 3 (we look at the maximum errors). However, the ranking between Methods 2 and 4 reverses when the other $\mathcal{T}$s in the table are considered. Methods 3 and 5 are about the same in terms of accuracy and they are the most accurate.

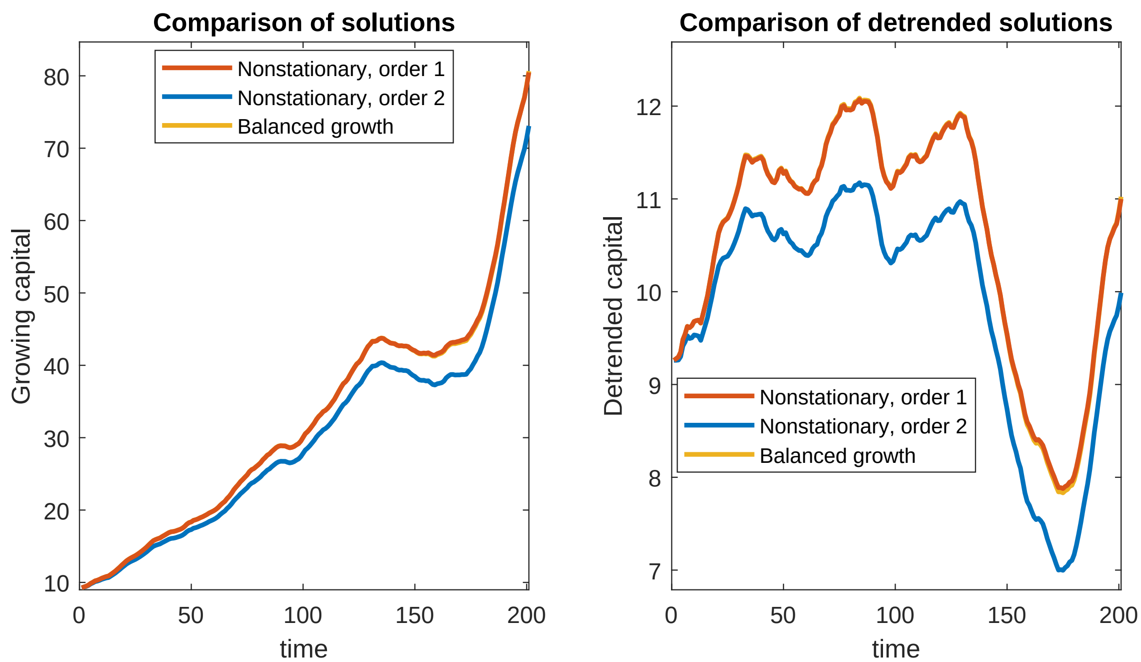

Figure 10: Comparison of the nonstationary P-EFP solutions computed by Method 5 and the balanced growth solution

Figure 10 plots our first- and second-order solutions for capital of the nonstationary model (produced by Method 5), as well as the exact solution of the balanced growth model (produced by the standard projection method); the left panel displays the growing solutions, while the right panel contains the detrended solutions. One striking feature of our solutions is that its second-order approximation is virtually identical to the exact solution (blue and yellow lines coincide). In turn, the first-order solution is a visible upward shift of the other two solutions, and therefore, can imply substantial inaccuracy.

An important question is: How does our perturbation solutions compare to the existing methods that can solve nonstationary models?

In Table 3, we make a comparison of our perturbation method to three other methods, an extended path method of Fair and Taylor (1983) method, a naive method and a global EFP method of Maliar et al. (2020). Fair and Taylor's (1983) method solves for a path of variables and not functions (as our method does). A naive method finds a different solution for each period t under the assumption that the t-period level of technology prevails in each subsequent period. For each of the methods, we use $T = 200$ in the solution procedure, and we simulate the model for $\mathcal{T} \in \{50, 100, 150, 175, 200\}$.

As is seen from the table, among the three alternative methods, the ranking of the methods is always the same: the naive method is the least accurate and the global EFP is the most accurate, with Fair and Taylor's (1983) method being in between. The latter reaches a notorious accuracy of 0.0001 percent for $\mathcal{T} = 50$; the residuals increase to 3.5% for $\mathcal{T} = 200$. The main finding in the table is that for $\mathcal{T} = 175$ our second-order method is almost as accurate as third-degree solution obtained with the global EFP method, and for $\mathcal{T} = 200$, the second-order solution overpasses the third-degree global EFP solution by a half order

Table 3: Comparison of the P-EFP to the other methods

	Fair-Taylor (1983) method	Naive method	Global EFP	P-EFP	
Type of approximation	path	path	3rd order	1st order	2nd order
Maximum errors, in $\log_{10}$ units					
$[0, 50]$	-1.29	-1.04	-6.35	-1.27	-2.24
$[0, 100]$	-1.18	-0.92	-4.76	-1.11	-2.05
$[0, 150]$	-1.14	-0.89	-3.22	-1.07	-2.03
$[0, 175]$	-1.14	-0.89	-2.47	-0.94	-2.03
$[0, 200]$	-1.14	-0.89	-1.51	-0.94	-2.03
Running time, in seconds					
Solution	1.2(+4)	28.9	199.4	317.4	337.6
Simulation	-	2.6	0.0244	0.0271	0.0348
Total	1.2(+4)	31.5	199.4	317.4	337.6

Note: Maximum errors are the maximum of the absolute difference between the given and exact solutions (in log10 units) on a stochastic simulation of $\mathcal{T}$ observations.

of magnitude. Moreover, our perturbation solution is not only more accurate for longer $\mathcal{T}$ but also much faster. This is because of perturbation used as a basis of the method.

Appendix B

In this section, we present sensitivity experiments.

A gradual increase in inflation target implemented with probability. In Figures 11 and 12, we present the supplementary experiments for Section 3.2. Namely, we consider two experiments that are parallel to the one in Figure 3, where a gradual increase in the inflation target happens with probability of 50 percent. In Figures 11 and 12, such a gradual change occurs with probabilities 75 and 25 percent, respectively. As is seen from the figures, a larger probability of implementing a higher inflation target leads to slightly larger expansionary effects on output, consumption, investment, and commodity exports. Although the qualitative patterns are the same, the anticipation effects (changes up to the fifth period when the actual change takes place) are visibly larger with 75 percent probability than with 25 percent probability.

A negative supply shock and a switch to price-level targeting. In Figure 7, we focus on a switch to price-level targeting after a negative foreign demand shock. Here, we present a supplementary experiment for Section 3.5. namely, we consider a negative supply shock instead.

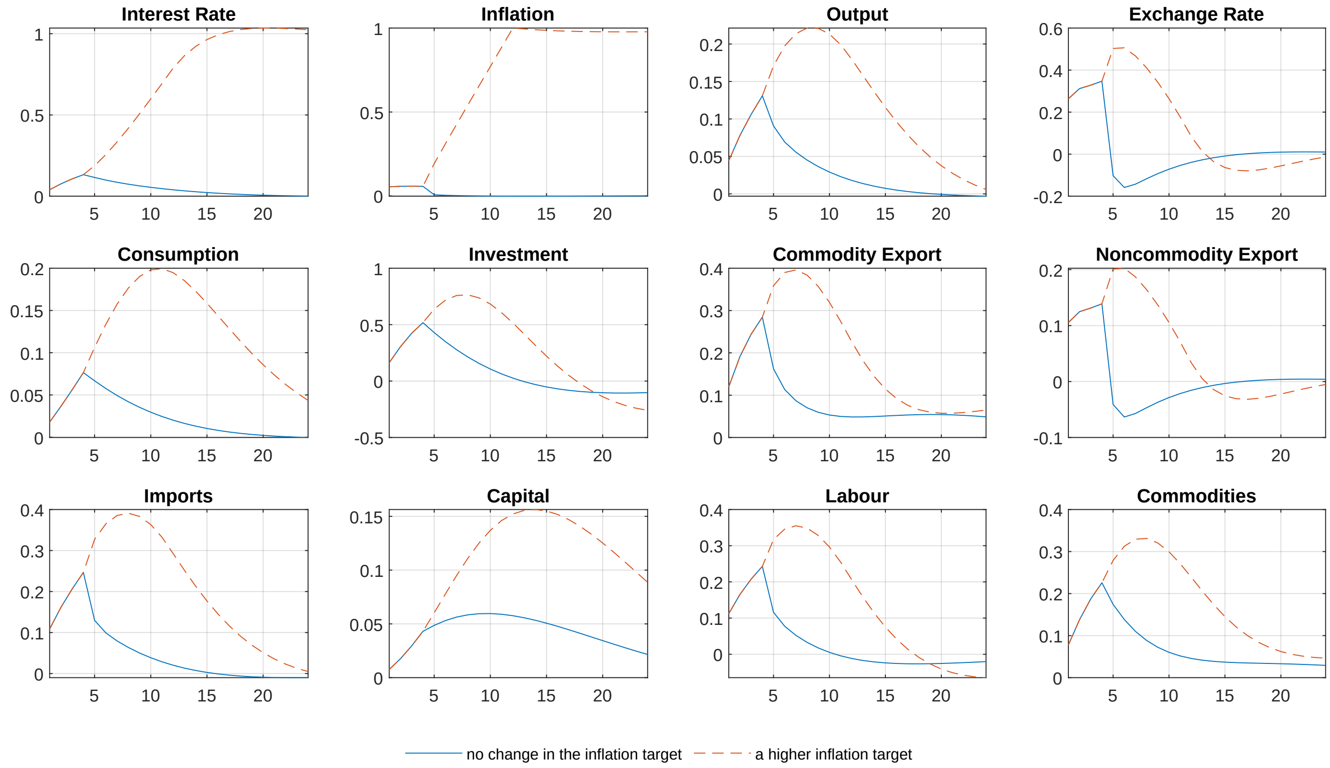

Figure 11: A gradual increase in the inflation target (75% probability)

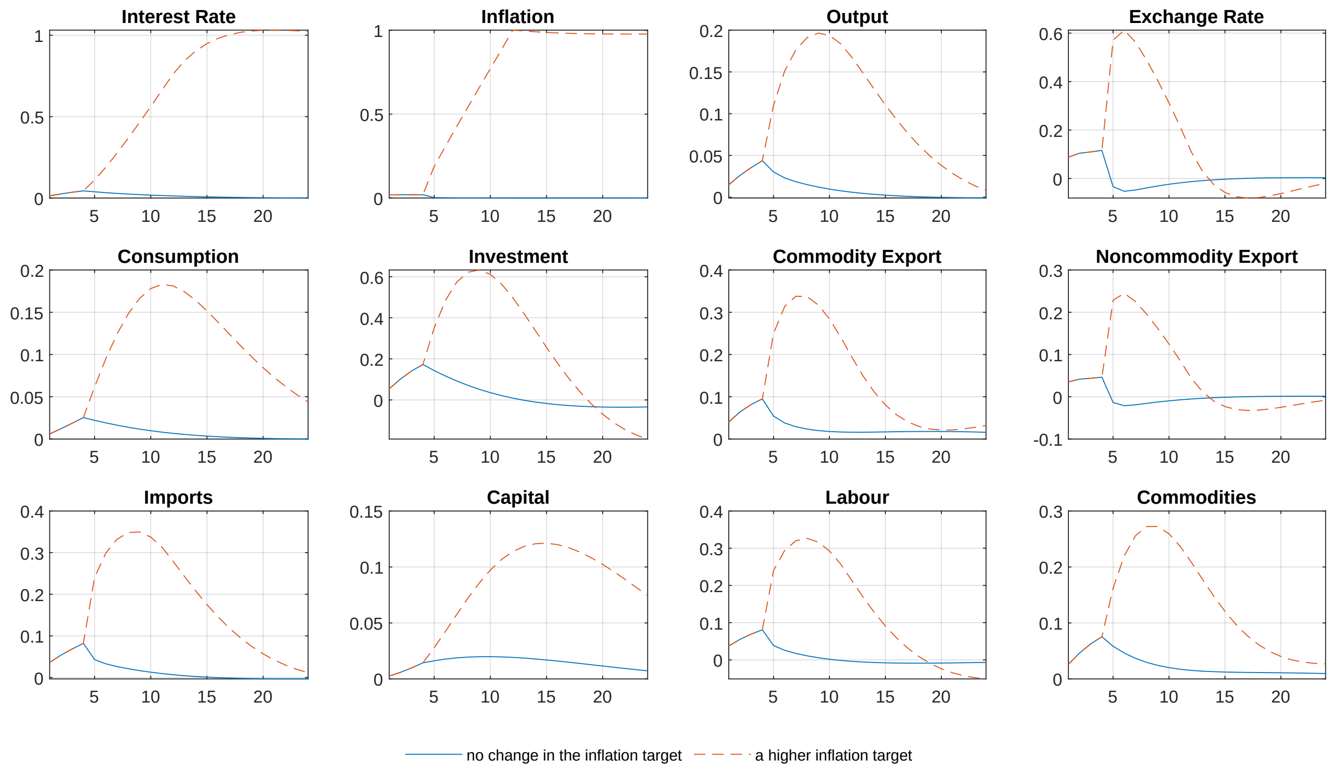

Figure 12: A gradual increase in the inflation target (25% probability)

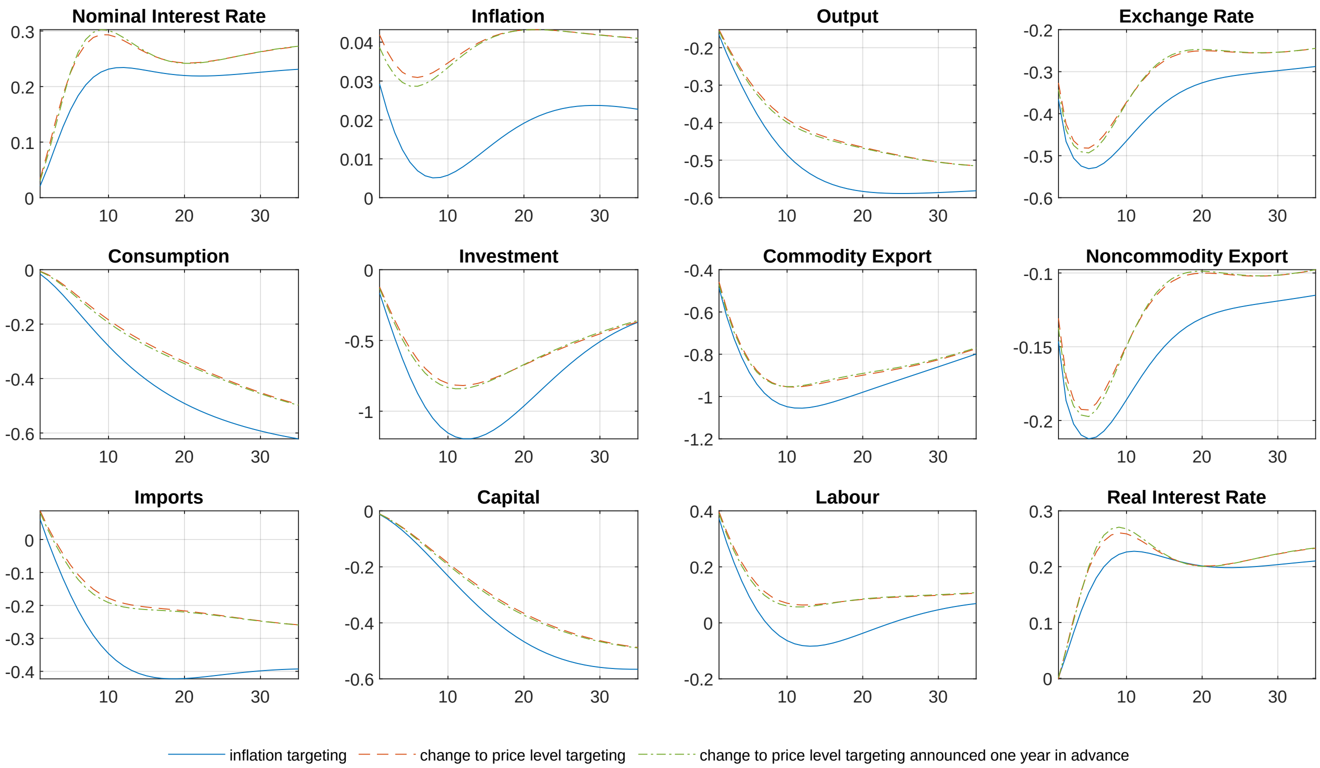

Figure 13: A negative supply shock and a switch to price-level targeting

Online appendix

For the reader's convenience, we provide a description of the calibration procedure, which is similar to LMM (2020).

5.1 Calibration

The model contains 61 parameters to be calibrated. Whenever possible, we use the same values of parameters in the scaled-down model as those in the full-scale model, and we choose the remaining parameters to reproduce a selected set of observations from the Canadian time series data. In particular, our calibration procedure targets the ratios of six nominal variables to nominal GDP $P_t^y Y_t$, namely, consumption $P_t C_t$, investment $P_t^i I_t$, noncommodity export $P_t^{nc} X_t^{nc}$, commodity export $P_t^{com} X_t^{com}$, import $P_t^m M_t$, total commodities $P_t^{com} COM_t$, and labor input $W_t L_t$. Furthermore, we calibrate the persistence of shocks so that the standard deviations of the selected bToTEM variables coincide with those of the corresponding ToTEM variables, namely, those of domestic nominal interest rate R_t, productivity A_t, foreign demand Z_t^f, foreign commodity price p_t^{comf}, and foreign interest rate r_t^f. The parameters choice is summarized in Tables 4 and 5 below.

Parameter	Symbol	Value	Source
Rates			
– real interest rate	$\bar{r}$	1.0076	ToTEM
– discount factor	β	0.9925	ToTEM
– inflation target	$\bar{\pi}$	1.005	ToTEM
– nominal interest rate	$\bar{R}$	1.0126	ToTEM
– ELB on the nominal interest rate	R^{elb}	1.0076	fixed
Output production			
– CES elasticity of substitution	σ	0.5	ToTEM
– CES labor share parameter	δ_l	0.249	calibrated
– CES capital share parameter	δ_k	0.575	calibrated
– CES commodity share parameter	δ_{com}	0.0015	calibrated
– CES import share parameter	δ_m	0.0287	calibrated
– investment adjustment cost	χ_i	20	calibrated
– fixed depreciation rate	d_0	0.0054	ToTEM
– variable depreciation rate	$\bar{d}$	0.0261	ToTEM
– depreciation semielasticity	ρ	4.0931	calibrated
– real investment price	ι_i	1.2698	ToTEM
– real noncommodity export price	ι_x	1.143	ToTEM
– labor productivity	$\bar{A}$	100	normalization
Price setting parameters for consumption			
– probability of indexation	θ	0.75	ToTEM
– RT indexation to past inflation	γ	0.0576	ToTEM
– RT share	ω	0.4819	ToTEM
– elasticity of substitution of consumption goods	ε	11	ToTEM
– Leontieff technology parameter	s_m	0.6	ToTEM
Price setting parameters for imports			
– probability of indexation	θ^m	0.8635	ToTEM
– RT indexation to past inflation	γ^m	0.7358	ToTEM
– RT share	ω^m	0.3	ToTEM
– elasticity of substitution of imports	ε^m	4.4	
Price setting parameters for wages			
– probability of indexation	θ^w	0.5901	ToTEM

– RT indexation to past inflation	γ^w	0.1087	ToTEM
– RT share	ω^w	0.6896	ToTEM
– elasticity of substitution of labor service	ε^w	1.5	ToTEM
Household utility			
– consumption habit	ξ	0.9396	ToTEM
– consumption elasticity of substitution	μ	0.8775	ToTEM
– wage elasticity of labor supply	η	0.0704	ToTEM
Monetary policy			
– interest rate persistence parameter	ρ_r	0.83	ToTEM
– interest rate response to inflation gap	ρ_π	4.12	ToTEM
– interest rate response to output gap	ρ_y	0.4	ToTEM
Other			
– capital premium	κ^k	0.0674	calibrated
– exchange rate persistence parameter	$\varkappa$	0.1585	ToTEM
– foreign commodity price	$\bar{p}^{comf}$	1.6591	ToTEM
– foreign import price	$\bar{p}^{mf}$	1.294	ToTEM
– risk premium response to debt	ς	0.0083	calibrated
– export scale factor	γ^f	18.3113	calibrated
– foreign demand elasticity	ϕ	0.4	calibrated
– elasticity in commodity production	s_z	0.8	calibrated
– land	F	0.1559	calibrated
– share of other components of output	υ_z	0.7651	calibrated
– share of other components of GDP	υ_y	0.311	calibrated
– adjustment cost in commodity production	χ_{com}	16	calibrated
– persistence of potential GDP	φ_z	0.75	calibrated

Table 4: Calibrated parameters in endogenous model's equations

Parameter	Symbol	Value	Source
Shock persistence			
– persistence of interest rate shock	φ_r	0.25	ToTEM
– persistence of productivity shock	φ_a	0.9	fixed
– persistence of consumption demand shock	φ_c	0	fixed
– persistence of foreign output shock	φ_{zf}	0.9	fixed
– persistence of foreign commodity price shock	φ_{comf}	0.87	calibrated
– persistence of foreign interest rate shock	φ_{rf}	0.88	calibrated
Shock volatility			
– standard deviation of interest rate shock	σ_r	0.0006	calibrated
– standard deviation of productivity shock	σ_a	0.0067	calibrated
– standard deviation of consumption demand shock	σ_c	0.0001	fixed
– standard deviation of foreign output shock	σ_{zf}	0.0085	calibrated
– standard deviation of foreign commodity price shock	σ_{comf}	0.0796	calibrated
– standard deviation of foreign interest rate shock	σ_{rf}	0.0020	calibrated

Table 5: Calibrated parameters in exogenous model's equations