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# Does Space Matter? The Case of the Housing Expenditure Cap[ℑ]

Yifan Gong, and Charles Ka Yui Leung[ℜ]

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HOOVER INSTITUTION
434 GALVEZ MALL
STANFORD UNIVERSITY
STANFORD, CA 94305-6010

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In our evaluation of the housing expenditure share cap, a macroprudential policy, we discover the importance of modeling space. First, the spatial considerations allow us to match the observed negative relationship between housing expenditure share and income without using non-homothetic preference. Second, it allows households to sort into segmented housing markets based on income. As a result, the cap policy causes a larger reduction in housing cost for low-income families than for high-income families in a spatial model. Depending on the assumption on households' preference, this mechanism leads to a smaller increase or even a modest decrease in welfare inequality in a spatial model than in a spaceless model.

Keywords: housing expenditure share, monocentric model of a city, spatial sorting, welfare inequality
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[ℜ] Correspondence: Gong, University of Nebraska-Lincoln, USA, ygong5@unl.edu ; Leung, City University of Hong Kong, Hong Kong, kycleung@cityu.edu.hk

# 1 Introduction

This paper argues that spatial considerations are essential for modeling housing markets and evaluating housing-related policies. We illustrate this point through a cap policy on the ratio of housing expenditures to disposable income, referred to as the housing expenditure share henceforth. We document some stylized facts regarding housing expenditure shares across different income classes. We can generate them through a homothetic CES utility function in a canonical monocentric city model or a "spaceless" model with non-homothetic preference. Yet, subject to the same expenditure cap policy, these two models have divergent welfare inequality predictions.

Our research is partly motivated by the booming housing research in recent years. After the 2008 Global Financial Crisis, many became aware that an increase in house prices could translate into an "excessive" amount of credit allocated to the real estate sector and increase the risk of having a financial crisis.[1] In response, many countries have implemented macroprudential policies, which help control the growth of house prices and housing-related credit.[2] According to IMF (2018), "...141 countries reported a total of 1,313 macroprudential measures, for an average of measures per country of 9.3" (p.6). The scale and diversity of all these macroprudential measures are enormous. It is almost impossible to assess all different macroprudential policies in a unifying framework.[3] To complement the literature on mortgage debt and risk-taking, this paper focuses on a policy that caps the housing expenditure share, a common concern of many policymakers.[4]

This type of cap policy has been explicitly and implicitly adopted in many contexts. For instance, in some affordable housing programs, the U.S. Department of Housing and Urban Development (HUD) imposes that rent "does not exceed 30 percent of the adjusted income of a family whose annual income equals 65 percent of the median income for the

[1]For a review of the literature, see Leung and Ng (2019), among others.

[2]See, e.g., Akinci and Olmstead-Rumsey (2018), Carreras, Davis, and Piggott (2018), Claessens, Ghosh, and Mihet (2014), and Gambacorta and Murcia (2020).

[3]Empirical works suggest that the relationship between macroprudential policies and inequality may be non-monotonic and depend on several factors (Biljanovska et al., 2023; Carpantier, Olivera, and Van Kerm, 2018; Casiraghi et al., 2018; Guerello, 2018). Favilukis, Mabille, and Van Nieuwerburgh (2019) combine the life-cycle dynamics with the spatial choice between two communities. They calibrate their model to match specific features of New York City.

[4]For instance, OECD (2019) states that "...Another common measure for housing affordability used here is the 'housing cost overburden rate', which measures the proportion of households or population that spend more than 40% of their disposable income on housing cost (in line with Eurostat methodology)." USA President Joe Biden also said that "every American in every zip code should have access to housing that is affordable - taking up no more than 30% of income so they have money left over to meet other needs." See Biden (2020) for more details.

area, as determined by HUD."[5] Among macroprudential policies implemented in many countries, caps are imposed on the debt servicing ratio in various types of stress tests (IMF, 2014).[6] In Appendix B, we show through an extended version of the model that this is equivalent to imposing a cap on the housing expenditure share in our static model.

However, the empirical evidence for the implications of such caps for housing markets and welfare is mixed. According to Biljanovska et al. (2023), many studies based on aggregate data generate statistically insignificant results, and studies based on micro-data tend to suggest more significant effects of macroprudential policies. One potential explanation is that some researchers assume away the spatial dimension of the macro-prudential policies, i.e., there is typically only one housing market in the model (e.g., Alpanda, Cateau, and Meh, 2018).[7] In reality, the housing markets are segmented even within the same city. This paper shows that models differ in spatial considerations and preference choices can deliver very different policy implications.

To substantiate our claim, we proceed in several steps. We start by establishing three stylized facts in the United States that guide our modeling exercise and are relevant to the welfare implication of the cap policy. The facts are (1) the approximate constancy in the working hours across income groups, (2) the negative correlation between the share of housing-related expenditure and income, and (3) the positive association between commute time and distance with income. Simultaneously satisfying these three facts imposes stringent restrictions on the equilibrium model. In particular, a spaceless model cannot produce the observed negative relationship between housing expenditure share and income if individuals have homothetic preference. In addition, because there is only one housing market, this spaceless model is silent on the observed spatial correlation between household income and residential locations. In contrast, we show that a canonical monocentric city model can simultaneously account for all three stylized facts even with homothetic preference (CES) (Brueckner, 1987; Mills, 1972; Muth, 1969).

Concerning the welfare implications, we find that imposing a cap on the housing ex-

[5] According to the California Department of Housing and Community Development, "State statutory limits are based on federal limits set and periodically revised by the U.S. Department of Housing and Urban Development (HUD) for the Section 8 Housing Choice Voucher Program. The comparable federal limit, more widely used, is 30 percent of gross income, with variations." For more details, see https://www.hcd.ca.gov/grants-and-funding/income-limits.

[6] For instance, the Hong Kong Monetary Authority reports that a household could borrow a mortgage loan of up to 157 times the households monthly income. After the stress-testing requirement came into effect on 15 September 2010, the maximum amount that the household could borrow was less than 80% of the situation before the HKMA's recent prudential measures were introduced (HKMA, 2010). Thus, the macroprudential measures effectively put a ceiling on the income share of housing-related expenditures for households without additional financial resources.

[7] There are exceptions. For instance, Acharya et al., 2022 show that caps on loan-to-value and loan-to-income ratios lead banks to reallocate mortgage credit from low- to high-income households and from urban to rural communities and change the relative house prices among different places as a result. This paper generates similar results in a structural model.

penditure share would lead to a higher welfare inequality in the *non-homothetic spaceless* model because relatively lower-income families are more likely to be constrained by the cap policy. On the other hand, the cap policy does not increase welfare inequality in the *homothetic monocentric city* model. We show this both analytically and quantitatively. If anything, the calibrated monocentric city model predicts a slight decrease in welfare inequality after the cap is imposed. The key mechanism behind this finding is a combination of general equilibrium rent adjustment and income-based spatial sorting. The intuition is as follows. Each "location" in a monocentric city is a geographically segregated housing market and agents self-select into different submarkets. Under the cap, the "market price" in each submarket adjusts differently. The low-income families enjoy disproportionately more significant reductions in endogenous equilibrium rent than high-income families. In our baseline model, the welfare gain from these rent reductions entirely offsets the welfare loss due to being constrained by the cap.

We explore the relevance of this key mechanism and the robustness of our welfare results in monocentric cities with a variety of alternative model settings. In Appendix B, we consider 1) a multi-period setting and 2) a setting with imperfect spatial sorting while keeping the utility function homothetic (CES). We show that our main conclusion that a housing expenditure cap has negligible effects on welfare inequality remains under these settings. In Section 5, we investigate the role of homothetic preference for our welfare results by considering a *non-homothetic monocentric city* model.[8] We find that, while the cap policy still aggravates welfare inequality in this model, the magnitude of this effect is smaller than that in the *non-homothetic spaceless* model because of the general equilibrium effect. Thus, we conclude that the key mechanism discussed above is always present with its importance depending on households' preferences. We discuss the implications of this finding for future research in the conclusion (Section 6) and provide a comprehensive summary of the value of modeling space in Section 5.3.

This paper builds on and complements several strands of the literature. On top of the few we briefly discussed above, the first literature that inspires this paper is the one on housing affordability and the evaluation of related government policies (Ben-Shahar, Gabriel, and Oliner, 2020; Gabriel and Painter, 2020; Quigley and Raphael, 2004). They typically employ a rich micro-data set and adopt a reduced-form estimation approach. The second literature concerns the design of macroprudential policies, such as Buch, Vogel, and Weigert (2018), Gadanecz and Jayaram (2017), etc. Third, this paper is related to the literature on how loan-to-value ratio policy (LTV), which is a different kind of macroprudential policy, would affect the housing market, such as Aastveity, Juelsrud, and Wold (2020), Armstrong, Skilling, and Yao (2019), Bekkum et al. (2019), Laufer and Tzur-Ilan (2021), Tzur-Ilan (2019). Fourth, this paper is related to the literature that

[8]After all, while many existing models adopt homothetic preference (Epple and Romer, 1991; Nechyba, 2000, 2003), there are also examples where non-homothetic preferences are employed (Albouy, Ehrlich, and Liu, 2016; Wang and Xie, 2022).

emphasizes the housing market heterogeneity at the city-level or regional-level (Beraja et al., 2019; Fratantoni and Schuh, 2003; Leung and Teo, 2011; Piazzesi, Schneider, and Stroebel, 2020; Sun and Tsang, 2018).

Our paper complements the literature in different ways. First, we build a structural model and hence complement the literature based on the reduced-form approach.[9] Second, we abstract from the macroeconomic risk of the housing market and the dynamic considerations that the previous literature has studied. Instead, our baseline model focuses on a static environment where ex-ante heterogeneous agents would choose among infinitely many locations within a city. Thus, rather than investigating the effects of the cap policy on financial and housing market stability, this paper examines the importance of spatial considerations for understanding the cap policy's implications on welfare inequality.

## 2 Empirical Facts

This section documents several crucial empirical findings that motivate the mechanism we explore in this paper. The first stylized fact is that work hours are roughly constant concerning income; the hours worked on average are around 8 hours for all income groups. Table 2 shows the evidence for 2017.

Second, we document that the housing expenditure share is decreasing with income. Table 1 shows the average annual before-tax income, total expenditure, expenditure on housing, and the fraction of total expenditure spent on housing for each before-tax income decile of the population in 2016.[10]

A comparison of the first two rows of Table 1 shows that the pre-tax income and the post-tax (and transfer) expenditure can differ significantly, especially for lower-income households. This observation is also consistent with recent research that pre-tax and post-tax in the U.S. can be very different (Auten and Splinter, 2019; Splinter, 2020). Hence, we measure the housing expenditure share (ratio of housing expenditures to disposable income) by the fraction of total expenditure spent on housing throughout the paper because it considers the effect of progressive taxes and transfers on disposable income. This is conceptually valid because total expenditure equals disposable income in the static models we will present in later sections. As shown in the fifth row of Table 1, the housing expenditure share decreases from 0.41 to 0.30 in a roughly monotonic manner.

The third critical empirical finding is that more highly paid workers live farther away from their workplace. Using the 2010 American Community Survey (ACS) data, we first compute households' average commute time and distance in each annual wage and salary

[9]There is an emerging view that the reduced form approach and structural model approach are complementary to each other (Heckman, 2010; Todd and Wolpin, 2023). A merit of the "structural" approach is facilitating counterfactual analysis (Wolpin, 2013).

[10]In Table 1, we use pre-tax total income. Table 2 is based on a different dataset; hence, we use annual wage, which is the labor income. In the United States, mortgage applications are often based on labor income and the stable part of the capital income. Leung and Tang, 2023 employ labor income only.

Table 1: Average Total Expenditures and Housing Expenditures by Income Decile

| Item | Lowest 10 percent | Second 10 percent | Third 10 percent | Fourth 10 percent | Fifth 10 percent |
|---|---|---|---|---|---|
| (1) Income before taxes | $6,502 | $16,229 | $24,432 | $33,499 | $43,931 |
| (2) Average annual expenditures | $23,588 | $26,675 | $34,221 | $39,308 | $43,975 |
| (3) Average annual expenditures on housing | $9,567 | $10,961 | $12,829 | $14,271 | $15,511 |
| (4) Average annual expenditures on transportation | $1,037 | $1,218 | $1,530 | $1,848 | $2,139 |
| (3)/(2) | 0.4056 | 0.4109 | 0.3749 | 0.3631 | 0.3527 |

| Item | Sixth 10 percent | Seventh 10 percent | Eighth 10 percent | Ninth 10 percent | Tenth 10 percent |
|---|---|---|---|---|---|
| (1) Income before taxes | $57,192 | $73,568 | $94,739 | $127,268 | $269,644 |
| (2) Average annual expenditures | $51,351 | $59,395 | $70,411 | $87,432 | $136,873 |
| (3) Average annual expenditures on housing | $17,119 | $19,285 | $22,085 | $26,719 | $40,547 |
| (4) Average annual expenditures on transportation | $2,450 | $2,776 | $3,220 | $3,802 | $5,305 |
| (3)/(2) | 0.3334 | 0.3247 | 0.3137 | 0.3056 | 0.2962 |

Source: Consumer Expenditure Survey, 2016; U.S. Bureau of Labor Statistics.

Table 2: Hours Worked on an Average Day by Weekly Earnings

| Income Range | $0 - $590 | $591 - $920 | $921 - $1,440 | $1,441 and higher |
|---|---|---|---|---|
| Hours | 7.87 | 8.19 | 8.28 | 8.12 |

Source: American Time Use Survey, 2017; U.S. Bureau of Labor Statistics.

income distribution decile. We find that both commute time and distance are monotonically increasing with income. Recognizing that the differences in average commute time and distance across income groups might be partially driven by income-based geographic sorting, we also use a fixed-effects model to estimate the effect of income on commute time and distance.[11] Using decennial Panel data from the Census and the ACS from 1980 - 2010, we find that a 1,000 (2017) dollars increase in annual wage and salary income is associated with a 0.0359-minute increase in commute time, with this coefficient being significant at a 1% level. In 2017, the average commute speed is 23.42 miles per hour.[12] Therefore, in 2017, a 1,000-dollar increase in annual wage and salary income is associated with a 0.0140-mile increase in commute distance on average.[13] Another supporting evidence is that, as shown in the fourth row of Table 1, the average annual expenditures on transpiration increase with pre-tax income.

This paper develops a canonical monocentric model with an extended CES utility function to reproduce these empirical observations. More specifically, the utility function combines numeraire consumption goods, housing, and leisure to produce the final utility. While leaving the details for later sections, we provide some basic intuitions here. First, we show that wealthy families would reside in relatively remote areas with low housing/land prices. Intuitively, this is because the pecuniary component of commute cost is "less expensive" for more affluent households in a relative sense. Standard consumer theory predicts that the fraction of total expenditure spent on one commodity would increase with the price of this commodity if the elasticity of substitution between this commodity and other commodities is smaller than 1. Therefore, combining the features that (1) the unit price of housing is lower for more affluent households and (2) the elasticity of substitution between housing and numeraire consumption goods is greater than 1, we generate the pattern that the fraction of housing expenditure is decreasing in income. By the same token, as the elasticity of substitution between leisure and other commodities is unity, the work hours are roughly constant concerning income.

## 3 A Monocentric City Model: Baseline

We present a stylized static model that describes the resource allocation within a city. An absentee landlord owns all the land and allocates land through auction. The household

[11] We do not control for household characteristics such as age and family size in the regression. Consistent with our treatment of the association between income and housing expenditure share, we are trying to capture the "total" association between income and commute time, including the part that arises through these household characteristics.

[12] See National Household Travel Survey 2017, https://www.bts.gov/topics/national-household-travel-survey.

[13] While the magnitude of this association might seem small, it is close to what we find in our baseline model. In our baseline model, a 1,000-dollar increase in annual wage and salary income is associated with a 0.0175-mile increase in commute distance. In Appendix B, we consider an alternative model setting under which we can perfectly match this association by adding unobserved preference shocks to households' location decisions. We find quantitatively similar results in this alternative model.

that offers the highest bid acquires the land for each location if this highest bid exceeds an exogenous agricultural rent $P_a$.[14] Households then enjoy the housing service from their acquired land and commute to a location $r$ miles away from their home to work. This paper does not explicitly model housing development and uses the terms land and housing interchangeably. After paying for the land, households spend the rest of their income on non-durable consumption goods and commutes.

In terms of spatial configuration, in this section, we consider a monocentric city model in which the city is built around a central business district (CBD), i.e., all households commute to the city center for work. There are infinitely many housing markets in this model. These markets are defined by the distance $r$ to the CBD and have potentially different equilibrium rents. We first analytically show that this monocentric city model's spatial feature helps us generate the observed negative relationship between income and the housing expenditure share without non-homothetic preference. We then calibrate the baseline model to match the U.S. economy circa 2017.

### 3.1 Household's Problem

This subsection defines and solves a household's utility maximization problem. The solutions to the maximization problem have several relevant implications for the empirical findings in Section 2.

#### 3.1.1 Utility Function, Budget Constraint, and Household Heterogeneity

A household's utility is determined by its consumption on numeraire good $c$, housing lot size $h$, and leisure $l$. Formally, we assume that its preference is *homothetic* and its utility is given by:

$$U(c,h,l) = l^{1-\alpha}[\theta c^{1-\rho} + (1-\theta)h^{1-\rho}]^{\frac{\alpha}{1-\rho}}. \tag{1}$$

Our specification extends the Cobb-Douglas utility function typically used in previous work on computable spatial equilibrium models (e.g. Hanushek and Yilmaz, 2007, 2013). We can interpret it as a two-step aggregator. First, housing $h$ and consumption goods $c$ are combined in a general CES manner with an elasticity of substitution of $\frac{1}{\rho}$. Then, their aggregate enters with leisure $l$ in a Cobb-Douglas way to produce the final utility (Krusell et al., 2000; Ogaki and Reinhart, 1998).

We now describe the budget constraint faced by the household. Consider a household with an hourly wage $w$, located $r$ miles away from the CBD.[15] The household allocates the $24 \times 7 = 168$ hours in a week to work, leisure ($l$ hours), and commuting ($br$ hours), where $b$ is the time cost per mile of weekly round-trip commute. Hence, the total income

[14] Alternatively, we could consider a city with exogenous boundaries, and the agricultural rent would be endogenously determined in equilibrium. Our main results are robust to this alternative specification.

[15] A household can only live in one location. Hence, the consumption set is not convex. The First Welfare Theorem does not apply here. See Rogerson (1988) and Shell and Wright (1993) for more discussion.

of this household is given by:

$$Income = [168 - l - br]w. \tag{2}$$

The household pays for the consumption goods and housing rents with income. We normalize the price of the composite consumption goods to 1 and denote the rent for one unit of housing at location $r$ as $P(r)$. To capture the pecuniary cost of commuting, we assume that the weekly round-trip commute costs $ar$ dollars. Formally, the total expenditure is given by:

$$Expenditure = c + P(r)h + ar. \tag{3}$$

As usual, total income should equal total expenditure at the equilibrium. Denoting $Y(r) \equiv (24 - br)w - ar$, the budget constraint implies the following:

$$Y(r) \equiv (168 - br)w - ar = wl + c + P(r)h. \tag{4}$$

Households in this model differ only in their hourly wages $w$. Limited by data availability and computational tractability, we categorize households into ten equal-sized groups based on hourly wages.[16] We refer to households whose hourly wages belong to the $i$th decile as Type $i$ households and add subscript $i$ to all relevant variables to differentiate among types. To ease the notation, we suppress the subscripts in most parts of this paper, and the readers should keep in mind that income levels differ across types of households.

### 3.1.2 Optimal Consumption Allocation and Indirect Utility Function

Taking equilibrium market rent $P^*(r)$ as given, a household living $r$ miles from the CBD maximizes the utility function (Equation 1) subject to the budget constraint (Equation 4):

$$V(P^*(r), r) = \max_{(c,h,l)} U(c, h, l) \text{ s.t. } Y(r) = wl + c + P^*(r)h, \tag{5}$$

where $Y(r) \equiv (168 - br)w - ar$.

Solving this maximization problem yields the following optimal choices of consumption, housing, and leisure:

$$h(P^*(r), r) = \frac{\alpha\kappa(P^*(r))Y(r)}{P^*(r)}, \tag{6}$$

$$c(P^*(r), r) = \alpha[1 - \kappa(P^*(r))]Y(r), \tag{7}$$

$$l(P^*(r), r) = \frac{(1-\alpha)Y(r)}{w} = (1-\alpha)(168 - br - \frac{ar}{w}), \tag{8}$$

where $\kappa(x; \theta, \rho) = \frac{(\frac{1-\theta}{\theta})^{\frac{1}{\rho}} x^{\frac{\rho-1}{\rho}}}{1+(\frac{1-\theta}{\theta})^{\frac{1}{\rho}} x^{\frac{\rho-1}{\rho}}}$.

[16] The income distribution of the U.S. is well-approximated by a log-normal distribution. Hence, each group's income range is different to generate equal-sized groups.

Finally, substituting the optimal choices shown in Equation (6)-(8) in the utility function shown in Equation (1) yields the indirect utility function $V(P^*(r), r)$:

$$V(P^*(r), r) = [1 - \kappa(P^*(r); \theta, \rho)]^{\frac{\alpha\rho}{\rho-1}} KY(r)(\frac{1}{w})^{1-\alpha}, \tag{9}$$

where $K = \alpha^\alpha(1-\alpha)^{1-\alpha}\theta^{\frac{\alpha}{1-\rho}}$.

### 3.1.3 Work Hours and Housing Expenditure Share

In our model, the weekly work hours are obtained by subtracting leisure time and time cost of commute, $br$, from 168 hours. Further, Equation (8) shows that the optimal leisure time is approximately a constant, $168\alpha$, if the time and monetary cost of commute, $ar$ and $br$, are small. Hence, we conclude that all types of households have roughly the same work hours (Stylized Fact 1) when the commute cost is low. In the calibration section later, we show $ar$ and $br$ are indeed small relative to income.

The $\kappa(P^*(r); \theta, \rho)$ function defined in the previous subsection provides a measure of housing expenditure share in the model. Equation (6) and (7) together imply that $\kappa(P^*(r); \theta, \rho) = \frac{P^*(r)h}{c+P^*(r)h}$. Noting that the total expenditure is equal to $c + P^*(r)h + ar$, $\kappa(P^*(r); \theta, \rho)$ measures the ratio of housing expenditures to total after-commute-cost expenditures and is hereafter referred to as the ACC housing expenditure share. Since $ar$ is relatively small compared to the total expenditure, $\kappa(P^*(r); \theta, \rho)$ is quite close to the housing expenditure share.

Note that $\kappa(P^*(r); \theta, \rho)$ does not depend on hourly wages. Since all households have the same preferences ($\theta$ and $\rho$), we simply write the $\kappa(P^*(r); \theta, \rho)$ function by suppressing the $\theta$ and $\rho$ arguments. In Appendix A, we show that $\kappa(P^*(r))$ is increasing in equilibrium rent $P^*(r)$ if the elasticity of substitution between consumption and housing is smaller than one.

**Proposition 1.** *If* $\rho > 1$*, then* $\frac{\partial\kappa(x)}{\partial x} > 0$.

As we will explain in the following subsection, this proposition is vital for generating the negative relationship between income and housing expenditure share.

## 3.2 Basic Analysis of the Equilibrium

This subsection defines and characterizes the equilibrium of our model. We will show that our monocentric city model is broadly consistent with the empirical evidence documented in Section 2. Also, these characterizations assist our calibration in a later section.

### 3.2.1 Bid-rent Functions and Market Rent Curves

As with many spatial equilibrium models, all households bid for land on a featureless plane. The common practice is to solve the bid-rent function, which expresses a household's willingness to pay for the equilibrium utility level $u_i^*$. For a Type $i \in \{1, 2, ..., 10\}$ household, the maximization problem can be mathematically expressed as follows:

$$\psi_i(u_i^*, r) = \max_{(c,h,l)} \left\{ \frac{Y_i(r) - c - w_i l}{h} | U\left(c, h, l\right) = u_i^* \right\}, \tag{10}$$

where $Y_i(r) \equiv (168 - br)w_i - ar$.

Technically speaking, this bid-rent maximization problem is the dual problem to the utility maximization problem defined in Equation (5). Hence, we can obtain the following *bid-rent function* by inverting the indirect utility function $V_i(P^*(r), r)$:

$$\psi_i(u_i^*, r) = \left\{ \left[ \frac{Y_i(r)K}{u_i^* w_i^{1-\alpha}} \right]^{\frac{\rho-1}{\alpha\rho}} - 1 \right\}^{\frac{\rho}{\rho-1}} (\frac{\theta}{1-\theta})^{\frac{1}{\rho-1}}, \tag{11}$$

where $K = \alpha^\alpha (1-\alpha)^{1-\alpha} \theta^{\frac{\alpha}{1-\rho}}$.

In the model, all of the lands are rented out via auctions. The ten types of households and agricultural workers can bid for any location indexed by its distance from the CBD, $r$.[17] For each location, the right of usage goes to the agent who offers the highest bid. Therefore, the equilibrium rent curve $P^*(r)$ is the upper envelope of the bid rent curves $\psi_i(u_i^*, r)$ of the ten types of households and the agricultural rent $P_a$. As the family moves away from the CBD, its bid rent declines and eventually hits 0. It means that beyond a certain distance $R_f^\star$, the agricultural rent $P_a$ dominates the bids offered by *all* of the households in the economy. Hence, no one resides there. We introduce the function $t^\star(r)$ to indicate the type of the residents at distance $r$. Formally, within the fringe distance $R_f^\star$, $t^\star(r)$ is given by:

$$t^\star(r) = \arg\max_i \psi_i(u_i^*, r). \tag{12}$$

The equilibrium rent $P^*(r)$ is then given by:

$$P^*(r) = \max( \sum_{i \in \{1,2,\ldots,10\}} \psi_i(u_i^*, r) I(t^\star(r) = i), P_a), \tag{13}$$

where $I(\cdot)$ is an indicator function that takes the value 1 when the condition in the bracket is satisfied and 0 otherwise.

To understand how different agents are distributed spatially, we consider the spatial order of two adjacent types of households. It is determined by their bid rent curves' relative steepness at the intersection point. The one with the steeper curve resides closer to the CBD. In other words, the condition for the equilibrium location of Household 1 being farther from the CBD than that of Household 2 is $\frac{\partial \psi_1(\cdot)/\partial r}{\partial \psi_2(\cdot)/\partial r} < 1$. In Appendix A, we show that $\frac{\partial \psi_i(\cdot)}{\partial r}$ is always negative and that $|\frac{\partial \psi_i(\cdot)}{\partial r}|$ is larger for households with lower hourly wage $w_i$. Hence, the following proposition holds.

**Proposition 2.** *Households with higher hourly wage $w_i$ live farther from the CBD at the equilibrium.*

[17] Following the urban economics literature, the agricultural workers are assumed to be self-sustained, except for the participation of the land auction. They would not affect any other aspect of the model economy.

The equilibrium rent $P^*(r)$ decreases in the distance $r$. Combining Proposition 2 and 1, we conclude that households with higher hourly wages spend a smaller fraction of their total after-commute-cost expenditures on housing services.

**Proposition 3.** *Consider two arbitrary households. We denote their types as $i$ and $j$, respectively. Let $r_i^*$ and $r_j^*$ denote their distance from the CBD at the equilibrium. If $\rho > 1$ and $w_i > w_j$, then $\kappa(P(r_i^*)) < \kappa(P(r_j^*))$ at the equilibrium.*

#### 3.2.2 Population Density and Land Market Clearance

Proposition 3 shows how agents are allocated across the city given the equilibrium rent schedule. Now we explain how the equilibrium rent schedule is determined. Following the literature, the total number of households for each type $i$, $i \in \{1, 2, ..., 10\}$, is exogenously given at $\overline{N_i}$ in this model. Suppose that Type $i$ households in equilibrium occupy the locations $r$ miles from the CBD, and $L(r)$ represents the amount of land available per unit distance. Our model has $L(r) = 2\pi r$.[18]

The clearing of the land market means that within the fringe distance $R_f^\star$, the following equation holds, $L(r) = h_i(P^*(r), r)m_i(P^*(r), r)$, where $m_i(P^*(r), r)$ is the equilibrium number of households per unit distance, assuming that distance $r$ is occupied by Type $i$ household and $u_i^\star$ is the equilibrium utility of Type $i$ household. All households find residence locations, implying the following *population constraint.*

$$\int_0^\infty m_i(P^*(r), r)I(t^\star(r) = i)dr = \overline{N}_i, \text{ for } i \in \{1, 2, ..., 10\}. \tag{14}$$

It is easy to verify that the household number distribution function $m(r)$ is given by:

$$m(r) = \sum_{i \in \{1,2,...,10\}} m_i(P^*(r), r)I[t^\star(r) = i]. \tag{15}$$

#### 3.2.3 Stationary Equilibrium

We now define the general equilibrium of this model economy. In the stationary equilibrium, no household has an incentive to relocate. It can be formally defined as follows:

**Definition 1.** An equilibrium is a set of utility levels $\{u_1^*, u_2^*, ..., u_{10}^*\}$, market rent curve $P^*(r)$, and type function $t^*(r)$ that show the equilibrium occupant of the location at distance $r$ that satisfy the following conditions.

• Households offer their bids according to Equation (11). The land is rented out through an auction. The household that offers the highest bid wins a particular location

[18] In reality, one unit of land might correspond to more housing units at locations closer to the city's center. One way to incorporate this possibility is to let $L(r) = 2\pi r\mathcal{D}(r)$, while $\mathcal{D}(r)$ is decreasing in $r$. It is important to note that our main welfare result (Proposition 4) remains in this case because its validity does not depend on the choice of $L(r)$.

if the bid is higher than the agricultural rent. Otherwise, the land is used for agriculture. Type function $t^*(r)$ records the auction's winner.

• Market rent $P^*(r)$ is determined as the upper envelope of bids from all types of households and the agricultural rent according to Equation (13).

• All Type $i$ households attain the same utility level, i.e., $u_i^* = V_i(P^*(r), r)$ for all $r$ such that $t^*(r) = i$.

• Each household rents a certain amount of land according to Equation (6). The land market clears, and the population constraint (Equation 14) holds.

## 3.3 Calibration

### 3.3.1 Parameter Set

This section shows that our baseline model can be calibrated to match the stylized facts of the U.S. circa 2017 documented in Section 2. The parameters of our model can be divided into three categories, which are (1) macroeconomic environment parameters $P_a$ and $\overline{N}_i$, (2) budget constraint parameters $w_i$, $a$, and $b$, and (3) preference parameters $\alpha$, $\theta$, and $\rho$. Below we describe the calibration of each category of parameters. Table 3 summarizes our calibration exercise.

We start with the macroeconomic environment parameters, i.e., agricultural rent $P_a$ and the number of households $\overline{N}_i$. In 2017, the average population and area of cities with more than 100,000 people were 303,322 and 94.08 square miles.[19] In our model, all households commute to the CBD for work. In the U.S., the employment-to-population rate is consistently around 60%. Hence, we fix the total number of households at $303,322 \times 0.6 = 181,993$. Each income group constitutes 10% of the total population implying that $\overline{N}_i = 18,199$ for all $i$. Given the total number of households (and other parameters), agricultural rent $P_a$ determines the city's size. The lower $P_a$ is, the larger the city is. We set agricultural rent to $P_a = \$505,340$ per square mile per week to match the endogenous calibration targets for the fringe distance $R_f^*$, which is $\sqrt{94.08/\pi}$ miles.

We then describe the calibration of the budget constraint parameters. Note that our model is static and does not feature income tax. Hence, instead of calibrating hourly wage $w_i$ to match annual before-tax income (reported in the first row of Table 1), we calibrate $w_i$ to match total yearly expenditures (reported in the second row of Table 1). Recall that Table 2 shows that the hours worked on average are around 8 hours for all income groups. Hence, under a reasonable assumption that a typical worker works five days each week, the weekly and annual work times are around $5 \times 8 = 40$ hours and $40 \times 52 = 2080$ hours, respectively. Therefore, we obtain the hourly wages for each income group by dividing the average total annual expenditures reported in the second row of Table 1 by 2080.

The weekly per mile pecuniary cost $a$ and time cost $b$ are chosen to match average transportation expenditure and commute time. The average annual spending on trans-

[19] Authors' calculation based on the Census data.

portation for each income group has been reported in Table 1. Dividing those numbers by 52 (weeks) yields the calibration targets for the group-specific averages of $ar$. In the U.S., the average one-way commute time is 26.1 minutes, according to 2017 Census data. We assume each household does a round-trip commute to the CBD for five days weekly. Hence, the calibration target for the average $br$ is $26.1 \times 2 \times 5/60 = 4.35$ hours. The calibrated values of $a$ and $b$ are 17.46 and 1.56, respectively.

The last set of parameters to be determined are the preference parameters. Equation (8) shows that $\alpha = 1 - \frac{l}{168-br-\frac{ar}{w}} = 1 - \frac{24-br-n}{168-br-\frac{ar}{w}}$, where $n$ represents the weekly hours of work. As documented above, $n$ is roughly 40 hours and does not vary much across households. The average commute time $br$ is 4.35 hours. The average $\frac{ar}{w}$ can be obtained this way. We first calculate the ratio of the group-specific averages of $ar$ to group-specific weekly wage $w_i$. We then take the average over income groups. The result is 1.81 hours. Hence, we set $\alpha$ to be $1 - \frac{168-4.35-40}{168-4.35-1.81} = 0.236$.

Finally, we calibrate parameters $\theta$ and $\rho$ to match the observed pattern of the housing expenditure share. Intuitively speaking, $\rho$ determines the speed at which this fraction decreases with total income; for any fixed $\rho$, $\theta$ pins down the overall level of this fraction. We search for the set of $\theta$ and $\rho$ that minimizes the sum of the squared differences between model-implied shares for each income group and their data counterparts. We find that $\theta = 0.0068$ and $\rho = 2.85$.

### 3.3.2 Baseline Equilibrium

Table 4 summarizes the baseline equilibrium outcomes. Panel A of Table 4 shows that the calibrated model can closely match data on total expenditures, housing expenditures, housing expenditure share, work hours, commute time, and pecuniary commute cost. Figure 1 shows that our model can match the observed negative relationship between income, or equivalently total expenditures, and the housing expenditure share. The red dashed line is obtained by plotting the fifth row of Panel A of Table 4 against the first row of Panel A of Table 4. The housing expenditure share is roughly monotonically decreasing in total weekly spending. The solid blue line, obtained by plotting the sixth row of Panel A against Panel A's second row, represents the model-implied counterpart of the red line. Figure 1shows these two lines are very close.

Panel B of Table 4 presents the additional results. Notably, the second row of Panel B shows that more affluent households reside farther away from the CBD, where the rent is lower. More affluent households can afford housing with larger lot sizes with higher disposable income, as shown in the first row of Panel B. Finally, the last row of Panel B reports the average value of $\kappa$ for each income group. Comparing them to the sixth row of Panel A, we find that $\kappa$ is systematically larger than but quite close to the housing expenditure share. This is consistent with our interpretation of $\kappa$ in Section 3.1.2.

## Table 3: Calibration Targets and Results

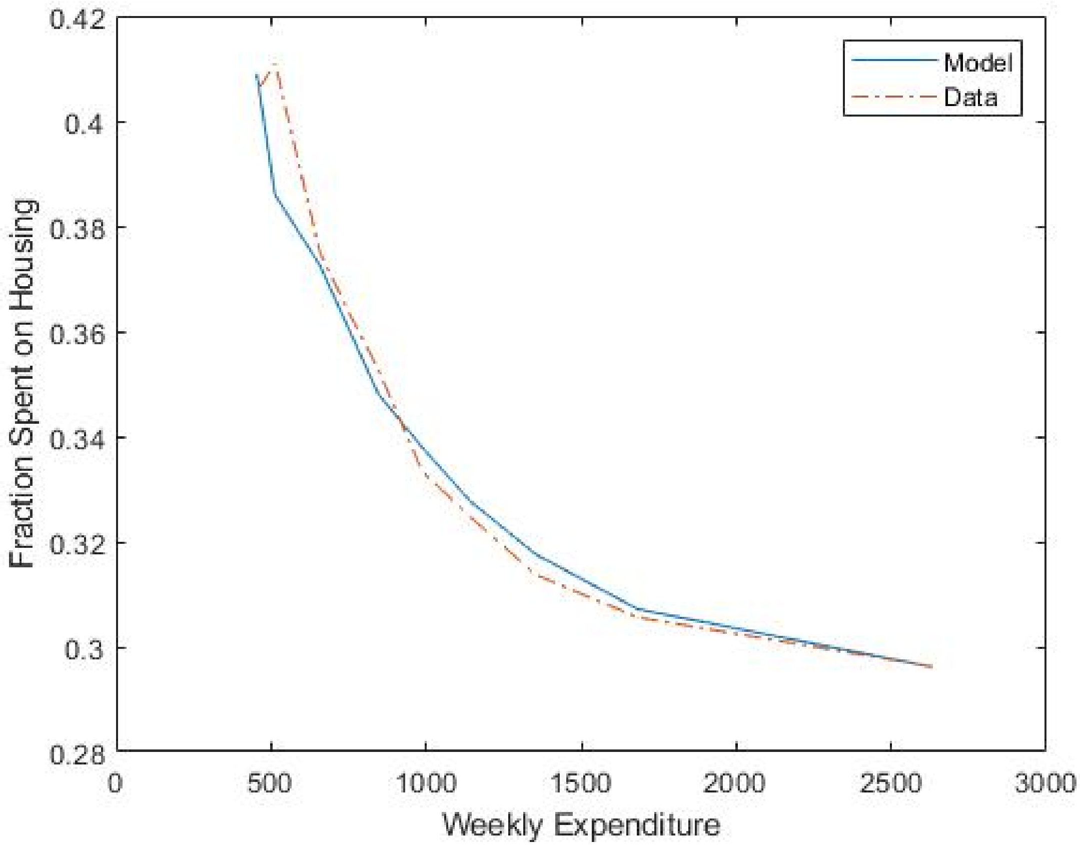
| Parameter | Interpretation | Targeted Moment | Value: CES Spatial | Non-homothetic Spaceless | Non-homothetic Spatial |
|---|---|---|---|---|---|
| **Macroeconomic Environment Parameters** | | | | | |
| $P_a$ | Weekly Agricultural Rent (dollars per square mile) | Average Size of U.S. Cities | 505,340 | 703,070 | 644,910 |
| $\overline{N}_i$ | Population Size of Type $i$ | Average Working Population of U.S. Cities | | 18,199 | |
| **Budget Constraint Parameters** | | | | | |
| $w_i$ | Hourly Wage Rate (dollars) | Total Expenditures by Income Type | | Varies by Income Type | |
| $a$ | Weekly Commute Cost per Mile (dollars) | Average Transportation Expenditure | | 17.46 | |
| $b$ | Weekly Commute Cost per Mile (hours) | Average Commute Time | | 1.56 | |
| **Preference Parameters** | | | | | |
| $\alpha$ (CES) | Importance of Consumption relative to Leisure | Weekly Hours of Work | 0.236 | N.A. | |
| $\theta$ (CES) | Importance of Numerare Goods relative to Housing | Relationship between Housing Expenditure Share and Hourly Wage Rate $w_i$ | 0.0068 | N.A. | |
| $\rho$ (CES) | Inverse of the Elasticity of Substitution between Numerare Goods and Housing | | 2.85 | N.A. | |
| $\beta_C$ (Non-homothetic) | Importance of Numerare Goods relative to Housing | | N.A. | 0.7191 | |
| $\bar{c}$ (Non-homothetic) | "Free" Consumption of Numerare Goods (dollars) | | N.A. | 240 | |

Note: Here we report type-specific hourly wages: $w_1 = 11.34$, $w_2 = 12.82$, $w_3 = 16.45$, $w_4 = 18.90$, $w_5 = 21.14$, $w_6 = 24.69$, $w_7 = 28.56$, $w_8 = 33.85$, $w_9 = 42.03$, $w_{10} = 65.80$.

Table 4: Summary of Baseline Equilibrium Outcomes

| Item | Lowest 10 percent | Second 10 percent | Third 10 percent | Fourth 10 percent | Fifth 10 percent | Sixth 10 percent | Seventh 10 percent | Eighth 10 percent | Ninth 10 percent | Tenth 10 percent |
|---|---|---|---|---|---|---|---|---|---|---|
| **Panel A: Model-fit** | | | | | | | | | | |
| Average Weekly Expenditures | | | | | | | | | | |
| Data | \$454 | \$513 | \$658 | \$756 | \$846 | \$988 | \$1,142 | \$1,354 | \$1,681 | \$2,632 |
| Model | \$456 | \$519 | \$664 | \$763 | \$852 | \$991 | \$1,142 | \$1,345 | \$1,657 | \$2,554 |
| Average Weekly Housing Expenditures | | | | | | | | | | |
| Data | \$184 | \$211 | \$247 | \$274 | \$298 | \$329 | \$371 | \$425 | \$514 | \$780 |
| Model | \$186 | \$200 | \$247 | \$274 | \$297 | \$335 | \$374 | \$427 | \$509 | \$756 |
| Fraction of Expenditures Spent on Housing | | | | | | | | | | |
| Data | 0.4056 | 0.4109 | 0.3749 | 0.3631 | 0.3527 | 0.3334 | 0.3247 | 0.3137 | 0.3056 | 0.2962 |
| Model | 0.4090 | 0.3859 | 0.3725 | 0.3597 | 0.3481 | 0.3379 | 0.3277 | 0.3176 | 0.3072 | 0.2962 |
| Average Weekly Work Hours | | | | | | | | | | |
| Data | around 40 hours | | | | | | | | | |
| Model | 40.17 | 40.46 | 40.38 | 40.36 | 40.31 | 40.15 | 39.98 | 39.74 | 39.42 | 38.81 |
| Average Weekly Commute Time | | | | | | | | | | |
| Data | around 4.35 hours | | | | | | | | | |
| Model | 1.01 | 1.90 | 2.61 | 3.30 | 3.95 | 4.61 | 5.30 | 6.04 | 6.85 | 7.92 |
| Average Weekly Pecuniary Commute Cost | | | | | | | | | | |
| Data | \$20 | \$23 | \$29 | \$36 | \$41 | \$47 | \$53 | \$62 | \$73 | \$102 |
| Model | \$11 | \$21 | \$29 | \$37 | \$44 | \$52 | \$59 | \$68 | \$77 | \$89 |
| **Panel B: Additional Outcomes** | | | | | | | | | | |
| Average Lot-size (unit: $10^{-4}$ miles$^2$) | | | | | | | | | | |
| Model | 1.6268 | 1.9479 | 2.6137 | 3.1254 | 3.6306 | 4.4020 | 5.2905 | 6.5291 | 8.4751 | 14.0459 |
| Average Weekly Rent (unit: per miles$^2$) | | | | | | | | | | |
| Model | \$1,145,300 | \$1,028,100 | \$946,900 | \$877,900 | \$817,100 | \$760,900 | \$707,100 | \$654,400 | \$600,600 | \$538,400 |
| $\kappa$ | | | | | | | | | | |
| Model | 0.4364 | 0.4096 | 0.3960 | 0.3838 | 0.3726 | 0.3619 | 0.3512 | 0.3402 | 0.3286 | 0.3157 |

Figure 1: Total Annual Expenditure and the Fraction Spent on Housing




# 4 A Monocentric City Model: Housing Expenditure Share Cap

In this section, we examine the welfare implications of imposing a cap $\bar{\kappa}$ on the after-commute-cost (ACC) housing expenditure share, $\frac{P^*(r)h}{c+P^*(r)h}$, in the context of the monocentric city model described in Section 3. In Appendix B, we show that our results are robust to adding dynamics and unobserved preference shock to the baseline static deterministic model.

## 4.1 Optimal Consumption and Indirect Utility Function

When a cap of $\bar{\kappa}$ is imposed on $\frac{P*(r)h}{c+P*(r)h}$, we modify the utility maximization problem defined in Equation (5) accordingly to obtain the indirect utility function $\tilde{V}_i(P^*(r), r)$:

$$\tilde{V}_i(P^*(r), r) = \max_{(c,h,l)} U(c, h, l)$$
$$\text{s.t. } Y_i(r) = w_i l + c + P^*(r)h, \text{ and } \frac{P^*(r)h}{c + P*(r)h} \leq \bar{\kappa}, \tag{16}$$

where $Y_i(r) \equiv (168 - br)w_i - ar$.

When the cap is not binding, this utility maximization problem is identical to the one defined in Equation (5). As shown in Section 3.1.2, in this case, the optimal value of $\frac{P^*(r)h}{c+P*(r)h}$ is equal to $\kappa(P^*(r)) = \frac{(\frac{1-\theta}{\theta})^{\frac{1}{\rho}} P^*(r)^{\frac{\rho-1}{\rho}}}{1+(\frac{1-\theta}{\theta})^{\frac{1}{\rho}} P^*(r)^{\frac{\rho-1}{\rho}}}$, which is monotonically increasing in the market rent $P^*(r)$. Hence, the cap binds when $P^*(r)$ is sufficiently high. It is easy to verify that the cutoff value for $P^*(r)$ is $(\frac{\kappa}{1-\kappa})^{\frac{\rho}{\rho-1}}(\frac{\theta}{1-\theta})^{\frac{1}{\rho-1}}$.

When the cap binds, the choices of consumption, housing, and leisure are given by:

$$\tilde{h}_i(P^*(r), r) = \frac{\alpha\bar{\kappa}Y_i(r)}{P^*(r)}, \tag{17}$$

$$\tilde{c}_i(P^*(r), r) = \alpha(1-\bar{\kappa})Y_i(r), \tag{18}$$

$$\tilde{l}_i(P^*(r), r) = \frac{(1-\alpha)Y_i(r)}{w_i} = (1-\alpha)(168 - br - \frac{ar}{w_i}). \tag{19}$$

Comparing these choices to their counterparts in the unconstrained case (Equation 6-8), we find that under a binding housing expenditure cap, households spend less money on housing and more money on consumption, and enjoy the same amount of leisure time. Moreover, we can combine results for both the constrained and the unconstrained cases in the following expression:

$$\tilde{V}_i(P^*(r), r) = [(1-\tilde{\kappa})^{1-\rho} + \frac{1-\theta}{\theta}\frac{\tilde{\kappa}^{1-\rho}}{P^*(r)^{1-\rho}}]^{\frac{\alpha}{1-\rho}} KY_i(r)(\frac{1}{w_i})^{1-\alpha}, \tag{20}$$

where $\tilde{\kappa} = \min(\kappa(P^*(r)), \bar{\kappa})$ represents the ACC housing expenditure share chosen by the household at the equilibrium.[20]

## 4.2 Bid-rent Function and Spatial Order under Housing Expenditure Cap

This section studies how agents are distributed in the monocentric city under the housing expenditure cap. Recall that the spatial order of two adjacent types of households is determined by the relative steepness of the bid-rent function $\tilde{\psi}_i(u_i^*, r)$ at the intersection, and the household with a steeper bid-rent function lives closer to the CBD.

When the cap is non-binding, this bid-rent function is identical to the one shown in Equation (11). Hence, as discussed in Section 3.2.1, among the unconstrained households, those with higher hourly wages $w_i$ live farther away from the CBD.

When the cap binds, we obtain the bid-rent function $\tilde{\psi}_i(u_i^*, r)$ by inverting the indirect utility function $\tilde{V}_i(P^*(r), r)$ shown in Equation (20) with $\tilde{\kappa}$ taking the value of the cap $\bar{\kappa}$:

$$\tilde{\psi}_i(u_i^*, r) = \bar{\kappa}\left\{\left[\frac{Y_i(r)K}{u_i^* w_i^{1-\alpha}}\right]^{\frac{\rho-1}{\alpha}} - (1-\bar{\kappa})^{1-\rho}\right\}^{\frac{1}{\rho-1}}(\frac{\theta}{1-\theta})^{\frac{1}{\rho-1}}. \tag{21}$$

As with the unconstrained case, we can show that 1) $\frac{\partial\tilde{\psi}_i(\cdot)}{\partial r}$ is always negative, and 2) $|\frac{\partial\tilde{\psi}_i(\cdot)}{\partial r}|$ is larger for households with lower hourly wage $w_i$ in the constrained case as well. Hence, those with higher hourly wages $w_i$ also live farther away from the CBD among the constrained households.

[20] We obtain the indirect utility function for this constrained case by substituting the choices shown in Equation (17)-(19) in the utility function shown in Equation (1).

To sum up, we conclude that Proposition 2, which states that households with higher hourly wage $w_i$ live farther from the CBD at the equilibrium, remains valid under the housing expenditure cap.[21]

### 4.3 Welfare Analysis

This subsection analyzes and quantifies the welfare implications of imposing a cap on the ACC housing expenditure share. A particular focus is examining the cap's implication for welfare/utility inequality. Since utility is unit-free, direct cross-sectional comparisons of individual utility do not convey meaningful information. Hence, we adopt a widely used consumption-equivalent measure (CE) to quantify the cap's effect on the utility level for each type of household (Cooley and Hansen, 1989). We focus on the cross-sectional relationship between CE and income level. For example, if low-income households tend to have smaller gains or more losses in CE under the cap, we would conclude that the cap policy leads to a more considerable welfare inequality. Formally, the consumption-equivalent measure $\xi$ is defined as follows:

**Definition 2.** Consider a household whose numeraire goods consumption, housing consumption, and leisure time in the baseline equilibrium are $c_{base}$, $h_{base}$, and $l_{base}$, respectively. Further, denote $u_{cap}$ as the utility level this household obtains under a cap on the ACC housing expenditure share. The consumption-equivalent measure $\xi$ for this household satisfies the following equations:

$$U((1+\xi)c_{base}, (1+\xi)h_{base}, l_{base}) = u_{cap}.$$

Definition 2 states that $\xi$ measures the faction of numeraire goods and housing consumption that a household needs to obtain in the baseline to achieve the same level of utility in the economy with the cap policy. A positive $\xi$ represents a utility gain, while a negative $\xi$ represents a utility loss.

It is easy to verify that, for any given cap $\bar{\kappa}$, households of the same type have the same $\xi$. We then denote the consumption-equivalent measure for type $i$ household as $\xi_i$. The Appendix shows that when a commute's monetary and time costs are negligible, $\xi_i$ is roughly the same for all households. Formally, the following proposition holds.

**Proposition 4.** *For any $\bar{\kappa} \in (0, 1]$ and any Type $i$ and Type $j$ households, we have*

$$\lim_{a->0^+, b->0^+} \xi_i - \xi_j = 0.$$

Proposition 4 implies that, in contrast to the single housing market, the welfare costs incurred by different types of households are similar under the policy cap inflicted in a

[21] Noting that $\frac{\partial \tilde{\psi}_i(\cdot)}{\partial r}$ is negative for all households, we also conclude that the equilibrium rent $P^*(r)$ is decreasing in the distance from the CBD $r$. Hence, Proposition 1 ($\frac{\partial \kappa(x)}{\partial x} > 0$) implies that the housing expenditure share, $\tilde{\kappa} = \min(\kappa(P^*(r)), \bar{\kappa})$, is weakly monotonically decreasing in the distance $r$ and in the hourly wage $w_i$ at the equilibrium.

monocentric city model when commuting costs are negligible. The intuition is as follows. In a monocentric city model, there are infinitely many housing markets indexed by the distance $r$. Because the equilibrium exhibits perfect sorting by income, constrained low-income households only need to compete with other constrained low-income households for residential land, resulting in a lower equilibrium rent for these locations. Hence, the utility loss due to constrained housing expenditure share is compensated by the utility gain from lower market rent for the constrained households.

Next, we quantify the effect of the cap on equilibrium utility based on our calibrated model. We consider five different values for $\bar{\kappa}$, 20%, 25%, 30%, 35%, and 40%. For each value, we solve for the equilibrium utility and the consumption-equivalent measure $\xi_i$ for all types of households.

Panel A of Table 5 reports the results. As shown in Section 3.3, commute costs are quite small in our calibrated model. Hence, consistent with Proposition 4, we find very similar $\xi_i$ across household types for all $\bar{\kappa}$ that we consider. If anything, we find that $\xi_i$ is generally smaller or more negative for relatively higher-income households.[22] Both the theoretical and quantitative results suggest that restricting the fraction of total income a household can spend on housing consumption does not lead to higher welfare inequality in a monocentric city environment.

# 5 Spaceless Model and Non-homothetic Preference

As discussed in the introduction, when only one housing market exists ("spaceless" model), non-homothetic preference is needed to generate the negative relationship between income and housing expenditure share. In this section, we use a simple Stone-Geary type non-homothetic utility function as an example to illustrate this (Geary, 1950; Stone, 1954). We show that a cap on the housing expenditure share in this spaceless non-homothetic model has drastically different implications for welfare inequality than the monocentric city CES model. We also consider a case where we combine non-homothetic preference with space and discuss the implications of these results for the value of explicitly modeling space.

## 5.1 Spaceless Non-homothetic Model

With a standard Stone-Geary utility function with leisure, numeraire good, and housing consumption as components, leisure, and consequently work hours, varies with income. Hence, to be consistent with the first stylized fact, we consider a simpler version where all households work the same amount of time (40 hours per week), and a household's utility is only determined by its consumption on numeraire good $c$ and housing lot size

[22] In many cases, we see $\xi_i$ is small but positive for all types of households. It is important to note that this does not represent an economy-wide Pareto improvement. Equilibrium market rents in these cases are lower than in the baseline, meaning the absentee landlord bears the cost.

Table 5: Consumption Equivalent Measure $\xi_i$ (%) for Alternative Housing Expenditure Share Cap $\bar{\kappa}$

| Item / Cap $\bar{\kappa}$ | Lowest 10 percent | Second 10 percent | Third 10 percent | Fourth 10 percent | Fifth 10 percent | Sixth 10 percent | Seventh 10 percent | Eighth 10 percent | Ninth 10 percent | Tenth 10 percent |
|---|---|---|---|---|---|---|---|---|---|---|
| **Panel A: Monocentric City Model: CES Preference** | | | | | | | | | | |
| 40% | 0.07 | 0.06 | 0.04 | 0.04 | 0.04 | 0.03 | 0.03 | 0.03 | 0.03 | 0.02 |
| 35% | 0.56 | 0.54 | 0.48 | 0.44 | 0.41 | 0.38 | 0.35 | 0.33 | 0.31 | 0.28 |
| 30% | 1.65 | 1.62 | 1.53 | 1.47 | 1.43 | 1.37 | 1.31 | 1.25 | 1.18 | 1.08 |
| 25% | 1.56 | 1.52 | 1.40 | 1.33 | 1.27 | 1.18 | 1.10 | 1.00 | 0.89 | 0.70 |
| 20% | -2.91 | -2.97 | -3.14 | -3.24 | -3.33 | -3.46 | -3.58 | -3.73 | -3.90 | -4.21 |
| **Panel B: Spaceless Model: Non-Homothetic Preference** | | | | | | | | | | |
| 40% | -0.19 | -0.06 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 35% | -1.22 | -0.83 | -0.27 | -0.12 | -0.04 | 0 | 0 | 0 | 0 | 0 |
| 30% | -3.31 | -2.65 | -1.55 | -1.14 | -0.88 | -0.59 | -0.40 | -0.24 | -0.12 | -0.01 |
| 25% | -6.71 | -5.77 | -4.08 | -3.39 | -2.94 | -2.40 | -2.00 | -1.63 | -1.27 | -0.81 |
| 20% | -11.81 | -10.56 | -8.24 | -7.26 | -6.59 | -5.78 | -5.16 | -4.55 | -3.94 | -3.10 |
| **Panel C: Monocentric City Model: Non-Homothetic Preference** | | | | | | | | | | |
| 40% | -0.13 | 0.02 | 0.05 | 0.04 | 0.04 | 0.03 | 0.03 | 0.02 | 0.01 | 0.01 |
| 35% | -0.91 | -0.49 | 0 | 0.13 | 0.17 | 0.17 | 0.14 | 0.11 | 0.08 | 0.05 |
| 30% | -2.33 | -1.64 | -0.68 | -0.34 | -0.15 | 0.04 | 0.14 | 0.21 | 0.24 | 0.21 |
| 25% | -4.78 | -3.84 | -2.40 | -1.84 | -1.49 | -1.11 | -0.86 | -0.64 | -0.47 | -0.30 |
| 20% | -8.86 | -7.64 | -5.68 | -4.89 | -4.36 | -3.79 | -3.38 | -3.01 | -2.68 | -2.28 |

$h$. Formally, we assume that its preference is *non-homothetic* and its utility is given by:

$$U(c,h) = (c+\bar{c})^{\beta_c} h^{1-\beta_c}, \tag{22}$$

where $\bar{c}$ is "free" consumption of numeraire goods, which, as shown below, can help generate a decreasing (with income) housing expenditure share.

Because this model is spaceless, all households reside at a location $r^*$ miles away from their workplace, the CBD. The (weekly) budget constraint faced by a household with an hourly wage of $w$ is given by

$$40w = c + P^*(r^*)h + ar^*, \tag{23}$$

where, as before, $P^*(r^*)$ is the equilibrium market rent and $ar^*$ is the pecuniary cost of commuting.

Solving the household's problem yields the following equations for the optimal housing expenditure and the indirect utility function:

$$P^*(r^*)h(P^*(r^*), r^*) = (1-\beta_c)(40w - ar^* + \bar{c}), \tag{24}$$

$$V(P^*(r^*), r^*) = \beta_c^{\beta_c}(1-\beta_c)^{1-\beta_c}(40w - ar^* + \bar{c})[P^*(r^*)]^{\beta_c - 1}. \tag{25}$$

The optimal after-commute-cost (ACC) housing expenditure share in this model is then given by:

$$\frac{P^*(r^*)h(P^*(r^*), r^*)}{40w - ar^*} = (1-\beta_c)(1 + \frac{\bar{c}}{40w - ar}). \tag{26}$$

It is trivial to see that this housing expenditure share is decreasing in hourly wage $w$, which is consistent with our second stylized fact.

Next we examine the implications of imposing a cap of $\bar{\kappa}$ on the ACC housing expenditure share, $\frac{P^*(r^*)h(P^*(r^*),r^*)}{40w-ar^*}$. Notice that because our model features an open city, the equilibrium rent equals the exogenous agricultural rent $P_a$ with or without the housing expenditure cap. Hence, if the cap is not binding, the utility maximization problem faced by a particular household is identical to the baseline case. Consider a household with an hourly wage of $w_i$. If the optimal ACC housing expenditure share is lower than $\bar{\kappa}$, the cap is not binding for this household. In this case, this household will attain the same level of utility as in the baseline. On the other hand, if this share is greater than $\bar{\kappa}$, this household cannot allocate consumption expenditures optimally. As a result, it will attain utility that is strictly lower than the baseline level.

Because high-income (higher $w$) households have a lower optimal housing expenditure share, they are less likely to be constrained by the housing expenditure share cap $\bar{\kappa}$ compared to low-income households. Therefore, high-income households may not be affected for a moderate $\bar{\kappa}$, while low-income families may be constrained and attain lower utility (than in the baseline). Since high-income households have relatively higher utility than low-income households in the baseline ($V(P^*(r^*), r^*)$ is increasing in $w$), this policy cap on the ACC housing expenditure share would increase utility/welfare inequality.

We also quantitatively examine the welfare implications of the cap policy in this spaceless model. We calibrate the model to match the same targeted moments as in Section 3.3.1. The only three parameters calibrated to have different values from those for the monocentric city model are the agricultural rent $P_a$ and preference parameters $\beta_c$ and $\bar{c}$. $P_a$ is the minimum market rent in the monocentric city model and the unique market rent in the spaceless model. Hence, to keep the city's size constant, $P_a$ in the spaceless model needs to be larger than its counterpart in the monocentric city model. The calibrated weekly agricultural rent $P_a$ is \$703,070 per mile$^2$. The preference parameters, $\beta_c$ and $\bar{c}$, are chosen to match the relationship between housing expenditure share and total weekly expenditure/income (Figure 1) as closely as possible. Their calibrated values are $\beta_c = 0.7191$ and $\bar{c} = \$240$, respectively.[23]

Panel B of Table 5 reports the welfare results from this spaceless model. Consistent with the discussion above, a household experiences a drop in welfare (negative CE measure) only when its optimal ACC housing expenditure share is higher than $\bar{\kappa}$. Since low-income families have a higher optimal ACC housing expenditure share, they are more heavily (negatively) influenced by the cap. As a result, the consumption-equivalent measure $\xi_i$ tends to be much more negative for low-income households. For example, a 20% cap imposes 3.10% welfare loss (in terms of $\xi_i$) for families in the top decile of the income distribution and 11.81% welfare loss for households in the bottom decile of the income distribution.

### 5.2 Spatial Non-homothetic Model

The model considered in Section 5.1 differs from the spatial CES model (Section 3) in two ways: 1) the preference is non-homothetic and 2) there is no space, so there is only one housing market. In this section, we examine the contribution of these two model features to the differences in the welfare results we see in the two models (Panels A and B of Table 5) by considering a spatial model with non-homothetic preference. Expressly, we assume a household's problem is that described in Section 5.1 while keeping all the other model settings the same as those described in Section 3.

Appendix C.2 shows that this spatial non-homothetic model can generate the same spatial order as the spatial CES model - wealthier households live farther away from the CBD. We focus on quantitatively evaluating the welfare implications of imposing a housing expenditure share cap, $\bar{\kappa}$. Most calibrated parameters of the spatial non-homothetic model are the same as those of the spaceless non-homothetic model. The only exception is the weekly agricultural rent $P_a$, calibrated to be 644,910 per mile$^2$.

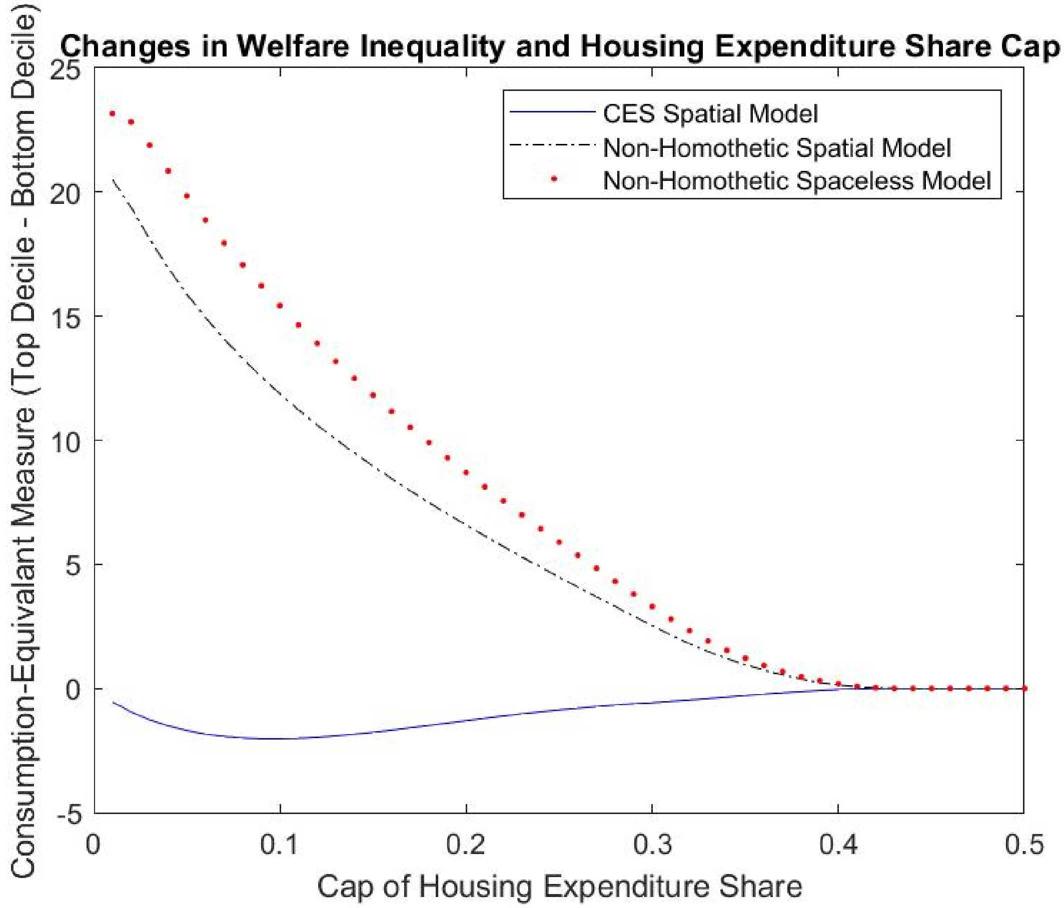
We again measure welfare changes using the consumption-equivalent ($\xi_i$) defined in Definition 2. Panel C of Table 5 reports the welfare results. The general pattern is qualitatively similar to those of the spaceless model (Panel B). When a cap is imposed on the housing expenditure share, poorer households suffer a larger welfare drop than more

[23] See Appendix C.1 for the fit of the spaceless model.

affluent households. For example, a 20% cap imposes 2.28% welfare loss for households in the top decile of the income distribution and 8.86% welfare loss for households in the bottom decile of the income distribution, which results in a larger welfare inequality.

However, the quantitative differences are worth noting. Again using a 20% cap as an example, the welfare drop in the spatial model is smaller than that in the spaceless model (3.10% for the top decile and 11.81% for the bottom decile). We then turn our attention to the effect of the cap on welfare inequality. One measure of welfare inequality change is the difference between the top income decile's and bottom income decile's CE, $\xi_{10} - \xi_1$. This difference's positive (negative) value indicates an increase (decrease) in welfare inequality. For a 20% cap, this difference equals 6.58% (8.86% - 2.28%) in the spatial model and 8.71% (11.81% - 3.10%) in the spaceless model. In Figure 2, we plot this measure of welfare inequality change for a wide range of $\bar{\kappa}$ and for all three models we have considered. For all levels of $\bar{\kappa}$, Figure 2 shows that while, unlike the case in the spatial CES model, we still see aggravated welfare inequality due to the cap policy in the spatial non-homothetic model, the magnitude of this effect is smaller compared to that in the spaceless non-homothetic model.

Figure 2: Changes in Welfare Inequality and Housing Expenditure Share Cap




The quantitative difference between the welfare results in the spatial non-homothetic model and the spaceless non-homothetic model stems from the general equilibrium price adjustment and income-based spatial sorting discussed in Section 4.3. The welfare inequality still increases in the spatial non-homothetic model because the general equilibrium channel is not strong enough when households have (non-homothetic) Stone-Geary preference. In this case, market rent only affects welfare through its influence on housing affordability. In contrast, in the CES model, market rent influences housing affordability and determines the optimal housing expenditure share. This second additional effect strengthens the general equilibrium channel, allowing it to entirely offset the aggravating impact of the cap on welfare inequality in the spatial CES model.

### 5.3 Discussion: The Value of Modeling Space

In examining the effect of a housing expenditure share cap, the value of modeling space is three-fold. First, it enables our model to match the empirical relationship between commute distance and household income. In contrast, by definition, a spaceless model is silent on this empirical finding.

Second, because of the general equilibrium channel discussed above, keeping household preference unchanged, the aggravating effect of the cap on welfare inequality is always more potent in a model without space than in a model with space. This is the case even when the preference is non-homothetic (Stone-Geary).

Third, perhaps more importantly, by explicitly modeling space, we can generate the negative relationship between income and housing expenditure share using homothetic preference (CES). This presents an important *unidentification* issue because both the spatial CES model and the spatial Stone-Geary model can match the three stylized facts documented in Section 2 but have drastically different policy predictions. Intuitively, these two models represent two extremes. The optimal housing expenditure share is determined entirely by equilibrium market rents in the former and entirely by hourly wages in the latter. In reality, this share likely depends on both. That means the true effect of the cap policy on welfare inequality is somewhere between the blue solid line and the black dashed line in Figure 2. To point-identify this effect, one needs first to bring in more data to recover households' preferences precisely.

## 6 Conclusion

This paper reinforces the intuitive and essential point that spatial consideration matters in economic analysis. We frame the statement in the context of the housing expenditure share cap. Variants of this policy have been explicitly and implicitly adopted as some components of macroprudential policies worldwide. We document three cross-sectional, stylized facts: the housing expenditure share decreases with income, the apparent constancy of work hours across income groups, and the positive relationship between income and the commute distance from the CBD. While non-homothetic preference is needed for a spaceless model to match the first two facts, a simple monocentric city model can match all three facts with both homothetic and non-homothetic preferences.

Moreover, because all households face the same housing price, the spaceless model would predict that the cap policy leads to a disproportionately high welfare loss for low-income families constrained by the cap, resulting in substantially larger welfare inequality. The monocentric city model has a different prediction. Because of income-based spatial sorting and geographically differentiated housing price adjustments, constrained low-income families experience a disproportionally more considerable drop in endogenous equilibrium rents than unconstrained high-income families. Depending on households' preferences, the welfare gain from this mechanism can partially (non-homothetic preference) or entirely (homothetic preference) offset the welfare loss due to the cap policy, resulting in a smaller increase (non-homothetic preference) or even a mild reduction (homothetic preference) in welfare inequality after the cap is imposed.

Our results suggest that policymakers should incorporate spatial considerations in the design of macroprudential policies. Moreover, our finding that the exact welfare results depend on households' preferences highlights the importance for future research to precisely estimate a flexible utility function that allows the housing expenditure share to depend on both household income and housing rents/prices.

We have considered several extensions in this paper, including introducing idiosyncratic location preference shock and extending to a more realistic, dynamic setting. Future research can explore other extensions, including collateral constraints, dynamic location choice, endogenous housing supply, endogenous tenure choice, multi-community within a city, and portfolio choices.[24] These new features can facilitate a more comprehensive and in-depth analysis of macroprudential policies.

[24]See Beraja et al. (2019), Combes, Duranton, and Gobillon (2021), Hanushek and Yilmaz (2007, 2013), Leung and Teo (2011), Piazzesi, Schneider, and Stroebel (2020), and Yao (2023), among others.

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