The Downward Glide of the Natural Rate of Unemployment during Cyclical Recoveries Could Explain Flat Phillips Curves

Robert E. Hall and Marianna Kudlyak

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Abstract:

We propose that the natural rate of unemployment has an active role in the business cycle, in contrast to the prevailing view that the rate is essentially constant. We demonstrate that this tendency to treat the natural rate as near-constant would explain the surprisingly low slope of the Phillips curve—the small ratio of the increase in inflation to the decline in unemployment. We show that the natural rate closely tracked the actual rate during the long recovery that began in 2009 and ended in 2020. We explain how the common finding of research in the Phillips-curve framework of low—often extremely low—response of inflation to unemployment could be the result of fairly close tracking of the natural rate and the actual rate in recoveries. Our interpretation of the data contrasts to that of most Phillips-curve studies, that conclude that inflation has little relation to unemployment. We suggest that the flat Phillips curve is an illusion caused by assuming that the natural rate of unemployment has little or no movement during recoveries.

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1 Introduction

Following its introduction in Friedman (1968), the natural rate of unemployment became an essential element of thinking about the business cycle, inflation, and monetary policy. In the New Keynesian analysis that emerged from Friedman’s insights, the natural rate is the unemployment rate that would hold if the economy were in equilibrium, free of transitory forces that raise inflation above its normal low level or depress inflation below that level.

We investigate the relation between the actual rate of unemployment and the natural rate. We start by describing the relation between inflation and its two determinants in the Phillips curve, which we term the *inflation anchor* and *inflation pressure*. We review existing evidence and provide new evidence that the natural rate of unemployment, which is used to construct the unemployment gap in the inflationary pressure measure, is not a slow-moving measure reflecting demography, as often taken in the Phillips-curve literature, but, rather, the natural rate glides down during cyclical recoveries similarly to actual unemployment. We conclude that the gap between the actual and the natural unemployment rates stayed fairly close to zero during recoveries.

Our first exercise involves the cyclical recovery beginning in 2009 and ending in early 2020. During that recovery, inflation stayed close to the Fed’s target of two percent per year. We posit that the inflation anchor remained close to constant at two percent. Accordingly, the unemployment rate was close to the natural rate. Observed unemployment glided down from 10.0 percent at its maximum to 3.5 at its minimum. Our conclusion is that the natural rate glided down fairly closely to the same path.

Our second exercise extends the analysis to consider other cyclical recoveries, some with significant variations in actual inflation. Our additional evidence on the behavior of the natural rate of unemployment during recoveries builds on our finding that, historically, actual unemployment has moved in a systematic way (Hall and Kudlyak (2022a)), gliding smoothly downward at a low but steady proportional rate after an occasional sharp upward movement in an economic crisis.

We show that if the true natural rate is substantially positively correlated with the actual rate, Phillips curves estimated with constant or natural rates of unemployment uncorrelated with the actual rate will inevitably appear to be close to flat. We then review existing time-varying measures of the natural rate and observe that these measures mostly reflect long-run trend movement in actual unemployment rather than cyclical movements during recoveries as we argue, with a few notable exceptions. Finally, we review selected estimates of the Phillips curve, most of which rely on these time-varying measures.
We conclude that, during recoveries, unemployment by itself is not an informative indicator of inflationary pressure. Rather, it is the gap between the actual rate and the declining natural rate that is the logical candidate to measure inflationary pressure. Accordingly, the labor market can gradually tighten in the sense of the Diamond-Mortensen-Pissarides (DMP) model’s measure of labor-market tightness, the vacancy/unemployment ratio, while inflation remains at a constant low level.

In contrast to most research, we draw this conclusion not because the slope of the Phillips curve is close to zero, but because the unemployment gap in recoveries is close to zero.

A large Phillips-curve literature posits that the natural rate of unemployment is constant or weakly correlated with actual unemployment. Under that view, in recoveries with anchored inflation, the unemployment gap must be highly variable, because unemployment falls so far during recoveries. In those times, the slope of the Phillips curve must be close to zero. Under our contrary view, inflationary pressure in recoveries is low, while under the existing view, the inflationary response is low. The differentiating factor between the two views is flexibility of prices. The literature finds quite sticky prices, whereas our view can be consistent with relatively flexible prices.

Section 2 of the paper describes inflation and its relation with the inflationary anchor and inflationary pressure through the Phillips curve. Section 3 estimates the natural rate of unemployment during the recovery from 2009 to 2019, discusses inferring the path of the natural rate of unemployment from a model by Galí, Smets and Wouters (2011), and describes an important property of unemployment—it’s predictable but gradual decline during cyclical recoveries. Section 4 describes how a flatter Phillips curve can arise when the natural rate is mis-specified as uncorrelated with actual unemployment. In this section, we review existing time-varying measures of the natural rate and selected existing estimates of Phillips curve. Section 5 discusses the implications of the natural rate following the actual rate during recoveries. Section 6 discusses the potential effect of the pandemic on the Phillips curve.

2 The Inflation Process and the Phillips Curve

2.1 The sticky-price model of the inflation process

The New Keynesian inflation model views sellers as considering two forces in deciding how to set a price today that remains in force some time into the future. One is that inflation has a component that reflects the success of monetary policy in stabilizing inflation at a low and reasonably constant level and also reflects the failures of inflation control. We call this component the inflation anchor. It is the result of an inference that a price-setter makes
about how to set a price that will remain in effect for some time into the future. The basis for
the inference includes information about the likely success of the central bank in stabilizing
inflation in the near future, recent actual inflation, and the sources of that inflation. If
monetary policy loses its grip and high inflation sets in, the anchor rises to reflect that
development.

The second force operates at business-cycle frequencies. It captures the relation of in-
flation to economic activity. In the New Keynesian model, this force is measured by the
gap between the natural rate unemployment and the actual unemployment level. This gap
reflects inflationary pressure. The inflation response is the increase in inflation that accom-
panies the upward pressure on inflation when some expansionary force raises or lowers the
unemployment rate relative to the current natural rate. The level of inflationary pressure is
not observable directly because it depends on the natural rate, which is not observed.

The source of the positive relation between the unemployment gap and the rate of infla-
tion in the basic New Keynesian model is the following: If unemployment is below the natural
rate, it means that the price level is below its equilibrium value. The reason is that the sticky
price level is below equilibrium and the consumers and firms, therefore, demand more than
the equilibrium levels of output. Their demands are automatically satisfied by producers be-
cause, according to a basic Keynesian principle, demand determines output. Higher output
implies lower unemployment. As time passes, the previously stuck prices become unstuck,
prices free up, the price level rises from their low levels back toward equilibrium. Inflation
continues higher than the anchored rate during this process.

2.2 Inflation and unemployment in the basic model

Figure 1 displays the basic model in a phase diagram. The unemployment rate \( u \) is on the
horizontal axis and the inflation rate \( \pi \) is on the vertical axis. The natural unemployment
rate, \( u^* \), is marked on the \( u \) axis and the anchored inflation rate, \( \pi^* \), is marked on the \( \pi \)
axis. The natural rate and the anchored rate are two key parameters of the model.

A conspicuous dot with coordinates \([u^*, \pi^*]\) marks the resting point of the economy, with
unchanging unemployment and inflation. A line rising from the high-unemployment, low-
inflation region describes the convergence toward the point of rest. A line declining from
the low-unemployment, high inflation region describes convergence from that region. Both
slopes are negative. Jointly, they trace out the Phillips curve of the model, inflation change
divided by unemployment change. That ratio, designated \( \phi \), is the slope of the Phillips
curve, the third key parameter of the model.
2.3 The Phillips curve

The model sketched above leads to the Phillips curve, an equation relating inflation to an inertial term and to a term involving economic activity. The Phillips curve is a key component of the New Keynesian class of macroeconomic models—see Woodford (2003) for a detailed exposition and Chapters 6 and 7 in Romer (2019) for a recent advanced textbook treatment of New Keynesian macroeconomics.

Following this logic, we consider a setup that embodies the properties described above:

\[ \pi_t - \pi^*_t = -\phi \cdot (u_t - u^*_t). \]  

(1)

Each of the three parameters introduced earlier has the property described there. For brevity, we omit random disturbances here. Note that we now consider the possibility that \( \pi^* \) and \( u^* \) change over time.

This equation is the Phillips curve. \( \pi_t \) is the actual rate of inflation; \( \pi^*_t \) is the inflation anchor; and \( \phi \) is a non-negative coefficient governing the strength of the response of inflation to pressure, measured as the gap between unemployment and the natural rate, \( u_t - u^*_t \). If \( \phi \) is small, prices are quite sticky and movements of \( u_t - u^*_t \) are large and persistent. If \( \phi \) is large, prices are flexible and \( u_t - u^*_t \) returns quickly to its normal value of zero.

The Phillips curve has the property that \( u_t = u^*_t \) when \( \pi_t = \pi^*_t \), that is, when inflation is at its anchored level.
Inflation also fluctuates for reasons apart from unemployment, notably from fluctuations in the supply of energy and agricultural products, and, more recently, in the pandemic, products with supplies cut back by the shutdowns. These fluctuations could be included in an extended model. In most empirical Phillips curves, they enter as separate additive terms.

As in Figure 1, the Phillips curve is generally displayed as a graph with the unemployment gap, \( u - u^* \), on the horizontal axis and the inflation gap, \( -\phi \cdot (\pi - \pi^*) \), on the vertical axis. The slope of the Phillips curve is \( \phi \) points of reduced inflation for each point of extra unemployment. We (and most other macroeconomists) talk about the slope of the curve being a positive number, \( \phi \), but, to tell the truth, the slope is the negative number \( -\phi \).

Although it is conventional to display unemployment on the horizontal axis of a Phillips curve and to treat the unemployment gap informally as an exogenous determinant of inflation, the variables under discussion here are obviously jointly determined.

If \( \phi \) is large, the Phillips curve is nearly vertical; even small values of the unemployment gap go with large effects on inflation. If \( \phi \) is small, the Phillips curve is nearly flat.

2.4 Price flexibility

We can rewrite the Phillips curve in aggregate-supply form as

\[
\begin{align*}
    u_t &= u^*_t - \frac{1}{\phi} \cdot (\pi_t - \pi^*_t). 
\end{align*}
\]

(2)

The coefficient \( \phi \), the downward slope of the Phillips curve, controls the influence of inflation on real activity, as measured by unemployment. Higher values of \( \phi \) make the model more like the real-business-cycle model, where real activity is not influenced by monetary factors such as inflation. In one polar case, with full monetary neutrality, \( \phi \) is large and unemployment tracks the natural rate. At the other end, \( \phi \) is small, and the Phillips curve is nearly flat. Large movements of unemployment are paired with small movements in inflation.

The parameter \( \phi \) captures the price-flexibility of the economy. If high, the economy has flexible prices and fits the real business cycle paradigm. If close to zero, prices are somewhat or fully sticky, and monetary factors have important involvement in the determination of unemployment and other real variables.

2.5 Uncovering the natural rate of unemployment, the inflation anchor, and the slope of the Phillips curve, from the data

The model sketched out above involves some interesting challenges to the macroeconomist seeking to use it as the starting point for practical implementation. Only two of its variables are unambiguously observable—unemployment and inflation. The natural unemployment
rate, the inflation anchor, and the slope of the Phillips curve, are theoretical constructs that have observable counterparts provided by modeling.

With respect to the natural rate of unemployment, we describe conditions that make it directly observable in the next section.

3 Defining and Measuring the Natural Rate of Unemployment

3.1 Our definition of the natural rate

The natural rate of unemployment, in our definition, is the unemployment rate at the point of rest in Figure 1. At this point, inflation equals the inflation anchor. The natural rate of unemployment also goes by the name non-accelerating-inflation rate of unemployment or NAIRU, which we take to be a synonym for the natural rate. The name captures a key property of that rate: Periods of stable inflation are times when the unemployment rate is at its natural level.

Our definition of the natural rate is essentially the same as one expressed in terms of the neutral value of inflation pressure. Our definition may differ from others based on a hypothesized absence of frictions in price and wage formation. Our definition recognizes that positive unemployment prevails when the economy is at rest, owing to normal turnover in the labor market. Defining the natural rate of unemployment is essential, given the myriad of definitions in the literature as summarized by Rogerson (1997).

3.2 Inferring the natural rate of unemployment during the periods of anchored inflation

We can solve the Phillips curve for the natural rate of unemployment:

\[ u_t^* = u_t - \frac{\pi_t - \pi_t^*}{\phi}. \]  

(3)

The natural rate is the actual unemployment rate with an adjustment for the effect on unemployment inferred from the price gap \( \pi_t - \pi_t^* \).

This equation is only useful if there is information from some other source about the inflation anchor \( \pi_t^* \). Inflationary pressure, \( u_t - u_t^* \), is not observable directly. Had we known \( \pi_t^* \) and \( \phi \), we could have inferred the natural rate of unemployment \( u_t^* \) from inflation and actual unemployment.

In this section, we start by noting a special case where outside information is arguably available that permits calculation of \( u_t^* \) from equation (3).
For any admissible value of $\phi$, equation (3) shows that $u_t^* = u_t$ if actual inflation equals anchored inflation—the equation embodies our definition of the natural rate. The issue becomes, what configuration of theory and data would make a finding plausible that actual inflation was equal to anchored inflation at a particular time? Friedman himself associated the source of inertia in inflation with expectations of future inflation, and that theme has resonated in most discussions of inflation dynamics ever since. One determinant of expected inflation is the central bank’s target rate of inflation. Another is the success of the bank in achieving its target. The place to look for actual inflation close to anchored is a moderately lengthy historical episode where inflation was close to constant at a low level consistent with the central bank’s stated objective.

The recovery in the US economy starting in November 2009 and lasting until the pandemic terminated the recovery partway through March 2020, deserves consideration as an episode when inflation was close to the target rate of 2 percent, adopted formally by the Fed in 2012. The anchor rate coincided fairly closely with the target rate.

Under these two assumptions about the US labor market in 2009 to 2020, we construct Figure 2 to demonstrate the resulting inferences about the natural rate during that period. The figure plots the actual unemployment rate and indicates with red dots the months when inflation was close to the anchored rate of two percent. Specifically, it indicates the months when the year-over-year percentage change of the price of personal consumption expenditures (PCE) was between 1.5 and 2.5 percent. The figure shows that during the 2009-2020 period, there were numerous months when the natural rate closely tracked actual unemployment, according to the logic developed in this section.

Note that the figure applies only to the one recovery with exceptionally stable inflation. In the other recoveries, actual inflation was sufficiently variable that we cannot make a similar inference. And there is no case of a recession with constant inflation, so there is no direct information about the natural rate during recessions, using this approach. In Section 4.3.2, we discuss existing econometric approaches to solving this problem.

3.3 Inferring the path of the natural rate of unemployment from a model

General-equilibrium models provide another way to determine the path of the natural rate of unemployment. The idea is to construct a counterfactual solution to a version of the model that describes an economy satisfying a definition of the natural rate of unemployment. For example, the version could impose the condition that the actual rate of inflation equals the anchored rate and both equal the central bank’s inflation target.
Figure 2: The Actual Unemployment Rate and the Estimated Natural Rate during the 2009-2019 Recovery

Galí et al. (2011) (GSW) is the leading example of this approach to extracting the natural rate of unemployment from a general-equilibrium model. In GSW’s case, the model is a widely used New Keynesian model developed by Smets and Wouters. The model includes a sub-model that deals explicitly with unemployment along the lines of the DMP framework. The calculation runs as follows: “We (GSW) ... assess the role of wage rigidities as a factor underlying observed unemployment fluctuations by comparing the observed unemployment rate to its estimated natural counterpart, where the latter is defined as the unemployment rate that would be observed in the absence of nominal wage rigidities...” (pages 348-349). Their counterfactual is somewhat different from ours, but we believe that its results are indicative of one based on ours or other definitions of the natural rate.

Figure 3 plots the resulting calculated natural rate from 1966, first quarter, through 2015, third quarter.¹ The path of the natural rate captures the bulge of unemployment in the 1970s, and it also moves substantially in harmony with actual unemployment at other times, especially after the serious recessions starting in 1981 and 2007.

The estimates of the natural rate by GSW convey the same basic message as we found in a single historical episode in the previous sub-section: The natural unemployment rate is not a constant or low-amplitude slow-moving smooth time series. Instead, we find that the natural rate is a changing force that was substantially positively correlated with actual unemployment, in the long time series considered by GSW, and extremely highly correlated in the recovery from the financial crisis of 2008 and 2009.

### 3.4 Some evidence from the behavior of actual unemployment

Equation (2), replicated here,

\[ u_t = u_t^* - \frac{1}{\phi} \cdot (\pi_t - \pi_t^*), \]

implies that systematic movements of actual unemployment resemble the corresponding movements of the natural rate. This holds unless the \( \frac{1}{\phi} \cdot (\pi_t - \pi_t^*) \) component is large, which happens when inflation exhibits substantial deviations from the anchor, or when the coefficient \( \phi \) is small. As before, we proceed under the assumption that \( \phi \) is not zero, that is, that inflation does respond to the unemployment gap—the Phillips curve is not completely flat.

We characterize the behavior of actual unemployment from Hall and Kudlyak (2022a), in which we undertake a close examination of the behavior of actual unemployment over the period from 1948 to 2019 using data from the Current Population Survey. We find that

¹This figure displays data supplied to us by Galí covering three more years than the published Figure 10.
Figure 3: The Actual Unemployment Rate and the Natural Rate based on Galí et al. (2011)

Note: Data provided to the authors by Galí; quarterly, Q1 1966—Q3 2015.
rather than vibrating around a fixed natural rate, the observed behavior of unemployment comprises (1) occasional sharp upward movements in times of economic crisis, and (2) an inexorable downward glide at a low but reliable proportional rate at all other times. The glide continues until unemployment reaches a low barrier of approximately 3.5 percent or until another economic crisis interrupts the glide.

Figure 4 shows our main evidence. It displays the log of the unemployment rate during the 10 completed recoveries since 1948, with the recession spells of sharply rising unemployment left blank. The key fact about recoveries is apparent in the figure: Unemployment declines smoothly but slowly throughout most recoveries most of the time. In the log plot, the recoveries appear as impressively close to straight lines. In our analysis, we confirm the linearity and estimate the slopes of these lines. We estimate that the annual reduction of log unemployment during recoveries over 1949-2019 was 0.10. The interpretation of this estimate is that during a recovery an expected annual reduction of unemployment is approximately 10 percent of the unemployment value at the beginning of the year.

The fact that actual unemployment behaves the same way in all recoveries makes it plausible that the natural rate behaves the same way.

A mechanism behind the steady but slow downward glide of actual unemployment during recoveries provides a cue why the natural rate cannot be too far from actual unemployment
during that time. In Hall and Kudlyak (2022b), we show that despite high variation in monetary and fiscal policy, and in productivity and labor-force growth during the 70 years, there was little variation in the rate of decline of unemployment during recoveries. Why has the US economy recovered so consistently from every recession in the past 70 years? Our thesis is that the economy has a powerful tendency to self-recover from serious adverse shocks. A natural force causes job-seekers to match with available jobs and to lower unemployment. The process is slow because a typical crisis breaks worker-firm employment relationships, and the process of creating new stable firm-worker relationships is time consuming. Workers who lost jobs often circle through short-term jobs, spells of unemployment, and spells out of the labor force, before finding stable employment (Hall and Kudlyak (2019)).

In Hall and Kudlyak (2022b), we suggest that high unemployment itself slows down the search and matching process through congestion. For example, in times of higher unemployment, employers incur higher costs to select suitable prospective workers from among the many applicants, thereby lowering job-finding for many other job-seekers in addition to those who lost jobs in the crisis. The bulge of unemployment created by a crisis at the beginning of a recovery creates a negative feedback to labor market tightness, endogenously slowing the recovery. Hall (2013) observes that an implication of the DMP model is that there is no fixed natural rate of unemployment which the actual unemployment rate revolves around; rather, the observed level of unemployment varies according to driving forces and is always an equilibrium.

To summarize, unemployment recovers steadily but slowly after each recession because of a self-recovery property of the labor market. The process is slow due to self-impediment that arises from high unemployment itself. As an example, in the case of a recovery where the unemployment rate is 9 percent, following a serious crisis, no forces outside the labor market can lower unemployment to 5 percent next year. The natural rate must be close to 9 percent in this case.

4 The Natural Rate and the Flat Phillips Curve

Our discussion above makes it clear that determining the path of the natural unemployment is a challenge. But that path is an intrinsic element of the Phillips curve—every study of the Phillips curve rests on a specification of the natural rate. A simple specification, adopted by many authors, is to take the natural rate to be a constant, or, equivalently, to omit the natural rate altogether. Another specification takes a long-run trend in the actual unemployment rate the natural rate of unemployment. Such time-varying measures do not capture the cyclical variation in the natural rate described in Sections 3.2-3.4. In this section, we discuss
a potential bias toward understatement of the slope of the Phillips curve resulting from the absence of realistic variation over time in the natural rate of unemployment embodied in most specifications of the Phillips curve. In this section, we show that if the true natural rate of unemployment is highly correlated with the actual rate, Phillips curves estimated with constant natural rates or natural rates uncorrelated with actual rate will inevitably be close to flat.

4.1 Implications of mis-specifying the natural rate as uncorrelated with actual unemployment

We are studying the Phillips curve from Equation (1), replicated here

\[ \pi_t - \pi_t^* = -\phi \cdot (u_t - u_t^*) \].

We presume that we have solved the problem of measuring \( \pi_t^* \), and focus on the unobserved \( u_t^* \).

We now demonstrate that taking the natural rate of unemployment as constant or near-constant has profound implications for estimation of the Phillips curve if the true model implies a material positive correlation between the natural and actual rates.

Consider a Phillips-curve regression that does not include the natural unemployment rate:

\[ \pi_t - \pi_t^* = -\tilde{\phi} u_t \].

(4)

The regression coefficient is

\[ \tilde{\phi} = \frac{-\text{Cov}(\pi_t - \pi_t^*, u_t)}{V(u_t)}. \]

(5)

Substituting the model for \( \pi_t \) with the true time-varying natural rate of unemployment yields

\[ \tilde{\phi} = \frac{\text{Cov}(u_t - u_t^*, u_t)}{V(u_t)} \cdot \phi = \left( 1 - \frac{\text{Cov}(u_t^*, u_t)}{V(u_t)} \right) \cdot \phi = (1 - R) \cdot \phi. \]

(6)

\( R \) is the regression coefficient of \( u_t^* \) on \( u_t \). It is an index of the relevance of the natural rate. If \( R = 0 \), the natural rate is irrelevant and the regression coefficient \( \tilde{\phi} \) will be an unbiased estimate of the Phillips-curve slope, \( \phi \). If \( R = 1 \), \( \tilde{\phi} = 0 \)—the Phillips curve appears to be totally flat, even if the true slope is robustly positive. The natural rate is highly relevant.

Note that \( R \) is not sensitive to the overall level of the natural rate, because the constant part of \( R \) is absorbed by the constant that would normally be part of the functional form of the Phillips curve. \( R \) is sensitive to the co-variation of the natural rate and the actual unemployment rate.
Our discussion of the omission of a time-varying natural rate from a regression for the slope of the Phillips curve is an application of the standard analysis of the bias from an omitted variable, on the right-hand side of a regression.

The denominator in $R$, $V(u_t)$, is observed directly and is robustly positive because unemployment rises briskly in recessions and falls reliably in recoveries. The big question is the covariance of the natural rate $u^*_t$ with actual unemployment. If the covariance is zero—possibly because $u^*_t$ is constant over time—there is no bias. If $u_t$ tracks the natural rate $u^*_t$ almost perfectly, $R$ will be almost 1, and the estimate of $\tilde{\phi}$ will be essentially zero, even if the true value of $\phi$ is quite positive.

One particularly salient point from this analysis is the following: If the true natural rate is highly correlated with the actual rate, Phillips curves estimated with constant or nearly constant natural rates of unemployment uncorrelated with the actual rate will inevitably be close to flat.

Our finding that, in the long expansion of 2009 through early 2020, actual unemployment tracked the natural rate closely, shows that the bias was almost certainly substantial during that period. $R$ for that period is very close to one.

We can use the results from Galí et al. (2011) to illustrate our analysis of the bias from failing to consider the movements of the natural rate. Suppose that we studied the Phillips curve in the GSW model by regressing an appropriate version of $\pi_t - \pi^*_t$ on $u_t$. The regression coefficient for $u^*_t$ on $u_t$ is $R = 0.60$ in the 1966-2015 sample, so the estimated slope of the Phillips curve is depressed to $1 - R = 0.40$ times its true value during that period.

The natural rate does not account for all of the movement of actual unemployment—the gap between actual and natural unemployment accounts for some of the cyclical movements. The key conclusion is that the movements of the natural rate constitute a central factor in the economics of the Phillips curve and employment volatility. The natural rate and the actual rate of unemployment move together—in some episodes, notably in the recovery from the financial crisis, the regression coefficient is close to 1. On the average over a lengthy period, GSW’s results set it to $R = 0.60$.

4.1.1 Non-linear specifications

Nonlinear specifications for the Phillips curve suffer from similar bias. The analysis of the bias follows standard results in econometrics.
4.2 Changes over time in the relevance of the natural rate of unemployment

Increases over time in the relevance of the natural rate might account for the apparent flattening of the Phillips curve. To check this possibility in the Galí et al. (2011) results, we replicated the estimates in Figure 3 in the form of coefficients from rolling regressions covering 20 forward quarters from the date shown on the horizontal axis. Figure 5 shows the movements of $R$ calculated in this way. In general, $R$, is cyclical but untrended over the long run. There is a sign of an increase in $R$ that would explain the flattening of the Phillips curve over the period from 1994 through 2010.
Another approach to the issue is based on del Negro, Lenza, Primiceri and Tambalotti (2020) (DLPT), a recent careful empirical study of the response of inflation and the unemployment rate to what they call an unemployment shock, in an 8-variable vector autoregression. The authors argue that the Phillips curve has flattened since 1990 and seek to explain the flattening. Lines 1, 2, and 3 of Table 1 display their results. Line 1 shows the response of unemployment to the shock for the first half of the sample through 1989, and for the second half, starting in 1990. The response is measured four quarters after the shock. Line 2 shows the responses of inflation, and line 3 shows ratio of the price response to the unemployment response. We interpret the ratio as the slope of the Phillips curve, in units of the percentage-point decline in inflation per percentage point increase in unemployment. According to these results, the estimated slope of the Phillips curve was 1.0 in the earlier period, and declined to 0.3 in the later period. These findings are in line with the substantial literature, which is surveyed extensively in the DLPT paper.

DLPT includes CBO’s time series for the natural rate of unemployment as one of its 8 basic indicators and as their measure of “natural unemployment”, but does not report the functions for the responses to the natural rate impulses. It does report the impulse response function from unemployment to the CBO series for the natural rate, which suggests almost no volatility.

We use our earlier derivations of the relevance of the natural rate based on leaving out the natural rate and the estimates of the natural rate series from GSW. Lines 4 and 5 of Table 1 provide information about the possibility that the econometric downward bias in the slope of the Phillips curve, caused by omitting data on the natural rate, accounts for the large decline in the slope of the estimated Phillips curve from the pre-1990 to post-1990 period. Line 4 reports the relevance statistic $R$, which we derived from GSW, as potentially

<table>
<thead>
<tr>
<th>Line</th>
<th>Variable</th>
<th>Before 1990</th>
<th>After 1990</th>
<th>Source</th>
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<tr>
<td>1</td>
<td>Unemployment (positive)</td>
<td>0.3</td>
<td>0.3</td>
<td>DLPT</td>
</tr>
<tr>
<td>2</td>
<td>Inflation (negative)</td>
<td>0.3</td>
<td>0.1</td>
<td>DLPT</td>
</tr>
<tr>
<td>3</td>
<td>Slope of Phillips curve regression</td>
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<td>0.3</td>
<td>row 2/row 1</td>
</tr>
<tr>
<td>4</td>
<td>Relevance of natural rate</td>
<td>0.6</td>
<td>0.9</td>
<td>GSW (see text)</td>
</tr>
<tr>
<td>5</td>
<td>True slope of Phillips curve</td>
<td>2.5</td>
<td>2.5</td>
<td>row 3/(1-row 4)</td>
</tr>
</tbody>
</table>

Table 1: Calculations Relating to the Downward Bias in the Estimated Slope of the Phillips Curve, based on the estimates from del Negro et al. (2020) (DLPT) and Galí et al. (2011) (GSW)
applicable to the earlier period in DLPT’s results. Line 5 shows the implied value of the slope of the Phillips curve adjusted for the given $R$. For the earlier period, the adjusted estimate, $\phi$, is $2.5 = 1.0/(1 - 0.6)$.

We then calculate what the value of $R$ should be for the later period so that the adjusted Phillips curve slopes are the same in both periods. With $R = 0.9$, the slope in the later period $\phi = 2.5 = 0.3/(1 - 0.9)$ equals the slope in the earlier period. While we do not know the value of $R$, the hypothesis that the relevance of the natural rate of unemployment for actual unemployment has increased in the later period has some support in Figure 5. It shows that during the long recovery from the 2007-09 recession, $R$, calculated from GSW data, reached its all-time maximum.

We propose that the US labor market experienced an increase in the relevance of the natural rate as measured by $R$, the regression coefficient for the natural rate on the actual rate of unemployment. This supports our hypothesis that the scenario in Table 1 accounts for at least some of the apparent decline in the slope of the Phillips curve after 1990.

4.3 Selected empirical studies

4.3.1 Estimates of the long-run path of the natural rate of unemployment

The notion that the natural unemployment rate changes slowly over time along with the composition of the labor force has been influential from the beginning. The Congressional Budget Office publishes a frequently updated estimate of the time path of the natural unemployment rate with demographic adjustment, and many Phillips-curve studies have adopted the CBO’s path. That path rises gradually to a maximum in the 1970s and declines thereafter—see Figure 6.

In 2021, the CBO changed the name of the path, to the noncyclical rate of unemployment. The new name clarifies that the counterfactual underlying the calculation is the absence of cyclical movements, not the absence of all low and medium-frequency fluctuations. The clarification differentiates the CBO’s estimate from those calculating the natural rate as the result of passing the actual rate through a band-pass filter, such as Hodrick-Prescott, and retaining only the low-frequency component. Other recent estimates of the long-run trend in unemployment are Barnichon and Matthes (2017), Tasci (2018), Barnichon and Mesters (2018), and Hornstein and Kudlyak (2019).

Using a search and matching model, Daly, Hobijn, Sahin and Valletta (2012) estimated that during the 2007-09 recession the natural rate of unemployment rose to a value in the range of 5.5 to 6.6 percent.
4.3.2 Joint estimation of the natural rate and the Phillips curve

Gordon (1997) estimated a time-varying natural rate from a statistical model comprising an inflation equation with the unemployment gap and an equation for the natural rate, which follows a random walk. From 1955 through 1995, the estimated natural rate varied between 5.4 and 6.5 percent—see the series under the author’s preferred smoothness parameter in his Figure 2. It declined by a percentage point between the mid-1980s and mid-1990s. See also Gordon (1998).

Bok, Crump, Nekarda and Petrosky-Nadeau (2023) (BCNP) present a model that combines a Phillips curve and the extraction of the natural rate of unemployment in a three-equation state-space model. The model draws upon related earlier work by Laubach (2001), Crump, Eusepi, Giannoni and Sahin (2019) and Crump, Eusepi, Giannoni and Sahin (2022). In BCNP, the unemployment gap follows an AR(1) process and the natural rate follows a random walk. Figure 2 of BCNP shows the unemployment rate and the estimated natural rate, labeled as the “preferred stable-price unemployment rate”. Their estimated natural rate is substantially positively correlated with the actual rate as compared to the CBO’s natural rate, so, in some years, the gap $u_t - u^*_t$ is small. However, the gap is large in the re-

Crump et al. (2019) take the natural rate of unemployment as the sum of a secular trend component and a cyclical component. Having estimated the trend, they estimate the cyclical component from a forward-looking Phillips curve model under the assumption of an AR(2) process for the unemployment gap.
covery following the recession of 2007 through 2009, a period where our Figure 2 shows that the gap appears to be quite small. Using BCNP’s estimate of the natural rate, we calculate the regression coefficient for $u^*_t$ on $u_t$ to be $R = 0.47$ over 1985 through 2019. During the period from Q1 1985 to Q3 2015, for which we have both GSW’s and BCNP’s estimates of the natural rate, we calculate $R$ to be 0.46 and 0.45, respectively.

Such statistical models of the unemployment gap or the natural rate may overstate the unemployment gap following recessions. As we suggest in Section 3.4, the natural rate may jump upward to come close to matching the high level of unemployment coming out of a crisis. Then, for a protracted period, the actual unemployment rate and the natural rate glide down together, implying that the gap is small throughout the recovery. Accommodating such possibilities for the natural rate requires relaxing the assumption of an AR process with a constant-variance error for the unemployment gap.

### 4.3.3 Policymakers’ views

Policymakers have faced a great deal of uncertainty about the natural rate of unemployment. Orphanides (2002) and Orphanides and Williams (2013), discuss the magnitude of informational problems and disagreement over the natural rate of unemployment among policymakers in real time. Staiger, Stock and Watson (1997) find that it is measured quite imprecisely. A similar conclusion is reached in a comment by Davis (2019). Hetzel (2022), in a book on the history of the Federal Reserve system, expresses skepticism about the role of the Phillips curve as a structural model of the economy to guide the choice of monetary policy, evoking the Lucas critique (Lucas (1976)) and the lack of certainty about the natural rate of unemployment. Crump, Nekarda and Petrosky-Nadeau (2020) discuss the range of the natural unemployment rate benchmarks used by policymakers.

In a departure from the prevailing view among policymakers and other observers, that the natural rate moves slowly and does not track the cycle, Lacker (2012) took the opposite view: “There is a reference unemployment rate to which it’s most appropriate to compare the current unemployment rate for the purposes of assessing current policy...The most common term for this reference rate is “the natural rate” of unemployment...There is a clear intuition for having the unemployment yardstick for monetary policy vary with economic conditions...Estimates of [the natural rate] invariably impose the assumption that it varies only slowly and does not respond to many transitory shocks...These are reasonable strategies for estimating the long-run unemployment rate, but by design, they will fail to capture important variations in the natural rate, especially variations over the business cycle.”
4.3.4 Other aspects of the natural rate and the Phillips curve

A substantial literature starts from the assumption that the natural rate of unemployment is constant or only slowly moving, reflecting movements in a long-run trend in actual unemployment, and, therefore, uncorrelated or only weakly correlated with actual unemployment. Another branch of the literature notes that there have been several instances when large movements in the unemployment rate have coincided with small changes in the inflation rate. Using a smooth trend as the measure of the natural rate, studies typically find a flat Phillips curve, especially in recent decades. Some studies have invoked nonlinearities, whereby the slope differs when unemployment is high versus low, or time-variation of the Phillips curve across time-periods.

Hooper, Mishkin and Sufi (2020), review earlier research and contribute their own national and cross-state evidence on the flattening of the Phillips curve. They conclude that “national data going back to the 1950s and 60s yield strong evidence of negative slopes and significant nonlinearity in those slopes, with slopes much steeper in tight labor markets than in easy labor markets. The evidence of both slope and nonlinearity weakens dramatically based on macro data since the 1980s for the price Phillips curve.” For the aggregate analysis, they use the CBO’s measure of the national natural unemployment rate. For the state-level results, they use bins of actual state data because no estimates of the state-level natural rate were available at the time of their work.

Hazell, Herreno, Nakamura and Steinsson (2022) is a recent contribution that finds very flat state-level Phillips curves for non-tradeable goods and services at the state level, using state-level unemployment as the measure of the state-level gap and the CBO’s natural-rate measure for the aggregate analysis.\(^3\)

Most recently, using the CBO’s measure in the aggregate analysis and the unemployment rate rather than the unemployment gap in the MSA-level analysis (because “there are no estimates of the natural rate of unemployment ... for the city-level unemployment data”), Smith, Timmermann and Wright (2023) study time-variation in the slope of the Phillips curve and find two regime changes: prior to 1972, the estimated slope is 0.51 units of price decline per unit of unemployment increase; this slope steepens 1972-2001 to 0.87; after, 2001 break, the slope of the Phillips curve becomes essentially zero.

\(^3\)From a standard New Keynesian model, the authors derive an estimating equation in which the log of the price of nondurables in a given state, adjusted for a time effect applying in all states and a state effect applying in all years, is determined by a downward effect based on the discounted value of future state level unemployment, and by a positive effect based on the log of the discounted value of future state-level nondurables prices. The coefficient on unemployment is the semi-elasticity of the downward slope of the Phillips curve. It is estimated to be 0.0062 log points of inflation per percentage point of unemployment, with a standard error of 0.0028 (Table 1).
Leduc and Wilson (2017) measure the city-level natural rate of unemployment as a 10-year trailing average of the city’s actual unemployment. They estimate the slope of the cross-city wage Phillips curve using seven-year rolling regressions and find a steady flattening of the Phillips curve slope starting with the 2001–2007 sample.

Dotsey, Fujita and Stark (2018), measuring the unemployment gap by the deviations of actual unemployment from its Hodrick–Prescott trend, conclude that “using the Phillips curve may add value to the monetary policy process during downturns. [...] We find no evidence for relying on the Phillips curve during normal times, such as those currently facing the U.S. economy.” (p.90) Ashley and Verbrugge (2023), using the long-run trend of unemployment from Tasci (2018) or from CBO as the measure of the natural rate, estimate what they call a “persistence-dependent” version of the Phillips curve that varies across three phases of the business cycle and find that in the recovery phase, inflation is unrelated to the unemployment gap. Using the CBO’s measure, Doser, Nunes, Rao and Sheremirov (2023) estimate a piecewise-linear specification and document that the data favor a model with two regions, with the response of inflation to an increase in unemployment slower in the region where unemployment is already high. Barnes and Olivei (2003) estimate a piecewise linear specification of the Phillips curve. Using state fixed effects, Leduc, Marti and Wilson (2019) estimate nonlinear wage Phillips curve in the state-level unemployment data and find some evidence of steeper Phillips curve in hot labor markets. Other recent studies of the Phillips curve include Laubach (2001), Stella and Stock (2013), Coibion and Gorodnichenko (2015), Cecchetti, Feroli, Hooper, Kashyap and Schoenholtz (2017), and Jorgensen and Lansing (2019).

Stock and Watson (2010) present evidence consistent with our conclusion that, in recoveries, the natural rate follows the smoothly declining path of the actual rate. They show that inflation takes a step downward early in a recession, but then remains unrelated to unemployment changes as the business cycle progresses through recovery—see their Figure 2. Constant inflation with declining unemployment suggests that the natural rate of unemployment is declining in parallel with actual unemployment, according to equation (3).
5 Potential Ambiguity about the Slope of the Phillips Curve and the Size of the Unemployment Gap

5.1 Econometric identification of the natural rate and the slope parameter

The Phillips curve sets the excess of the rate of inflation over its anchor to the product of the slope parameter and the unemployment gap:

$$\pi_t - \pi_t^* = \phi_t \cdot (u_t - u_t^*)$$.

In this section, we explore the case that arises if $\pi_t - \pi_t^*$ and $u_t$ are observed with reasonable accuracy but $\phi_t$ and $u_t^*$ are unknown. This case interests investigators who are skeptical about the identification of the natural rate $u_t^*$. As our review of research on this topic shows, the general approach to identification involves a one-dimensional condition involving $u_t^*$ alone, often assuming that it is a constant or a function of slow-moving demographics, or, more recently, to posit a state-space model as in the BCNP’s work and its predecessors.

Absent a convincing identifying condition, the available information identifies a range of paths of $\phi_t$ paired with the corresponding paths of $u_t^*$ that satisfy the Phillips curve, given the observed $\pi_t - \pi_t^*$ and $u_t$. Near one end, $\phi_t$ is close to zero and $\pi_t - \pi_t^*$ is correspondingly large. Near the other end, $\phi_t$ is large and the unemployment gap $u_t - u_t^*$ is small.

The range of opinions about the recovery from 2009 through early 2020 illustrates this point. One view, widely present in the literature, is that the Phillips-curve slope, $\phi_t$, was small and even vanishing during the recovery, while the unemployment gap, $u_t - u_t^*$ was large and negative. We call this the low and sticky view of the slope of the Phillips curve.

The other view is that the slope, $\phi$, was material during the recovery, while the unemployment gap, $u_t - u_t^*$ was small and positive. We call this the flexible view of the Phillips curve. Section 3 of this paper makes the case for this interpretation.

Both views fit the specified data. Additional data helping to reveal $u_t^*$ or $\phi$ would be needed to determine which view is correct.

5.2 Further discussion

In the flexible view of the Phillips curve, low unemployment does not necessarily signal high inflationary pressure. During recoveries, the unemployment gap $u_t - u_t^*$ is close to zero, based on the evidence that during recoveries natural rate of unemployment glides down together with actual unemployment and they are likely close to each other. Our finding means that the unemployment gap is close to zero and, therefore, inflationary pressure is weak. We do not take a position on the gap during contractions when unemployment is rising rapidly.
Our finding is that it is hard or impossible to see the effects of the unemployment gap because, during recoveries, the gap is essentially zero.

Under the flexible view, during recoveries the gap is zero. That is, during recoveries an economy resembles a real business cycles economy, with $\frac{1}{\phi}$ being relatively low and the Phillips curve being steep. The flexibility of prices is the key differentiating factor. Our view requires that prices are somewhat flexible, so the Phillips curve is reasonably steep, whereas the opposing view posits stickier prices and a flatter Phillips curve.4

The most radical potential conclusion about the relation between the observed rate of unemployment and the natural rate is that there is no difference—observed unemployment is at its natural level all the time. This conclusion would cut the heart out of the Phillips curve and the distinctive features of the New Keynesian model. It would deny unemployment any role as a measure of inflationary pressure. That conclusion goes beyond the evidence, however. We have relatively few observations of stable inflation in times of rising or really high unemployment. We believe that a reasonable interpretation of the evidence is that, during long, slow, reliable recoveries with gradually declining unemployment, unemployment is close to its natural rate and is not a measure of inflationary pressure. Under those conditions, there is no meaningful unemployment gap.

6  The Effect of the Pandemic on the Phillips Curve

During the long recovery from 2009 to 2019, inflation became powerfully anchored at just below 2 percent per year. In this stable environment, sellers adapted their price-setting procedures to stability. The Phillips curve became relatively flat because sellers tended to leave prices unchanged for extended periods—relatively few sellers responded to change each month.

The pandemic created a completely different environment for pricing decisions. The pandemic created turbulence with rapid new developments. A quick response to each new development was required. Policy responses to the pandemic included expansionary monetary and fiscal policy. Another important development was a reduction in output supply, due to idling of important sectors of the economy, notably hospitality. The turbulence that the pandemic brought to seller’s economic situations made it necessary to make more frequent prices changes than in the tranquil pre-pandemic times. Prices changed frequently.

We can explain this situation in a standard supply-and-demand framework, stating the behavior of inflation and output in terms of an aggregate supply curve (the Phillips curve

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4Hornstein (2008), in the introduction to the special issue of the Richmond Fed’s Economic Quarterly on the New Keynesian Phillips curve, writes that there is some support for the New Keynesian Phillips curve in aggregate data, but there is no agreement on the extent of nominal price rigidities in the aggregate economy.
written in terms of the output gap rather than the unemployment gap) and an aggregate demand curve. The aggregate supply-aggregate demand diagram has inflation on the vertical axis and the output gap on the horizontal axis. The former slopes upward. The latter slopes downward—people buy lower volumes of output if higher inflation pushes the price level higher. When inflation is strongly anchored, aggregate supply is flat. A shift of the aggregate demand curve to the right raises output while leaving inflation almost unchanged. Under pandemic conditions, with a weakened anchor, aggregate supply slopes upward more. The additional inflation resulting from a policy expansion is higher and the output expansion is lower. Additionally, the restrictions on supply shifted the aggregate supply function to the left. Again, the contribution to inflation from this shift is greater with a more vertical supply curve.

In the Phillips curve framework, an increase in turbulence represents a regime change—the Phillips curve becomes more sensitive to changes in unemployment. That is, in tranquil times the Phillips curve is relatively flat—any shifts in demand show up mostly as quantity changes, while in turbulent times, the shifts in demand have large effects on inflation. One possible explanation behind the post-pandemic inflation is an increased volatility of the inflation anchor—\( \pi_t^* \) in the Phillips curve. As discussed above, the inflation anchor also depends on the current speed of adjustment of prices—if information becomes more volatile, sellers will choose to change prices more frequently.

Hall (2023) argues that the New Keynesian Phillips curve becomes steeper in volatile times. He establishes that sectors with standard New Keynesian price stickiness are vulnerable to rapid transitions from stickiness to flexibility, as sellers elect to reset their prices and abandon anchoring. He argues that the logic of the New Keynesian model of the Phillips curve links inflation to volatility, because a larger fraction of sellers are pushed out of their regions of inaction when volatility is elevated. See also Blanco, Boar, Jones and Midrigan (2023) and work cited.

The sticky-price model introduced above might not be a good description of the Covid economy. Of course, the Covid economy is far from a situation when prices change daily. But the disarray that the pandemic brought to people’s life might have made it easier to accept more frequent prices changes than in the tranquil pre-pandemic times. It might very well be that the prices that we thought were sticky turned much more flexible. The Covid economy might be closer to a flexible-price economy, an economy without price inertia, versus a sticky-price pre-pandemic economy with the strong demand.

If the environment changes toward lower inflation, either because the constraints from the pandemic relax or because the Fed adopts a contractionary policy to combat inflation, the steep slope of the Phillips curve will remain and become an advantage. The steep slope
is a result of change in the environment and will remain until low steady inflation is anchored again. Disinflation can occur with a smaller bulge of unemployment than would occur under the pre-pandemic, more tightly anchored Phillips curve.

Sargent (1982) studies the disinflations in four economies as they overcame high inflation rates and achieved price stability with little dislocation of economic activity. Their Phillips curves became vertical. The steeper is the Phillips curve, the less the cost of disinflation. In times of rapid change, especially those involving fiscal or monetary reforms, the real cost of disinflation can be smaller than in more tranquil times.

7 Conclusions

We conclude that during recoveries, the natural rate of unemployment likely glides down together with the actual rate. $u - u^*$ stayed fairly close to zero in the 2009-2019 recovery, given an inflation anchor of 2 percent.

The pandemic loosened the powerful anchoring of the inflation rate that prevailed between 2009 and 2019. According to standard Phillips-curve theory, the flatness of the curve during that era gave way to a steeper curve in the pandemic. That means that the cost in terms of elevated unemployment of a policy to restore price stability may be lower than it would have been if inflation had become anchored at its current high rate.
References


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