$r < g$

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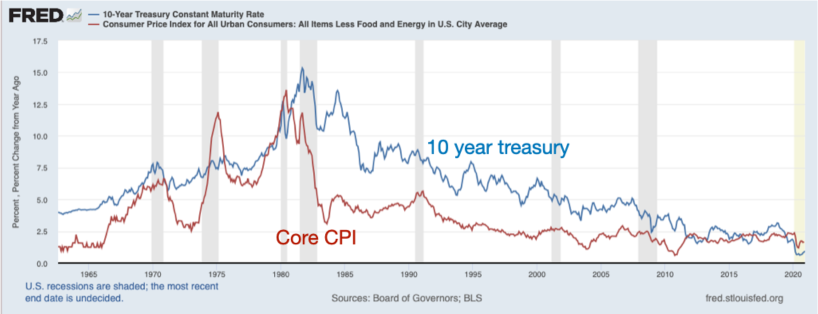
Preview

- Debt sustainability: The most important macro question.
- The hope:

$$\frac{d}{dt}\left(\frac{b_t}{y_t}\right) = (r_t - g_t)\frac{b_t}{y_t} - \frac{s_t}{y_t}.$$

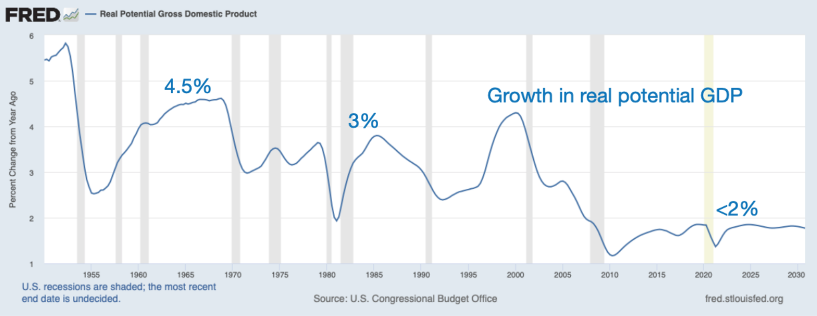
  $r < g$: Raise debt ($s < 0$) then roll over ($s = 0$), b/y declines? "Fiscal expansion" with "no fiscal cost?" (Blanchard, Yellen v. 1)
- Why is $r < g$? Will it last? ("Low interest rates & gov't debt.")
- Does $r < g$ work? ("$r < g$".)
  - $r < g$ is fun, but irrelevant to US fiscal policy issues. Like seignorage, it is a small benefit to government finance. Large deficits need to be repaid with surpluses.
  - Can see $E(r) < E(g)$ yet debt = PV of surpluses. Grow out of debt strategy is like writing out of the money puts, calling it arbitrage. Don't calibrate certainty models with uncertain data.

r fact



- Steady trend since 1980.
- Savings glut, fx reserves, QE, zero bound, liquidity, etc. Icing. Cake?
- Basic economics? (Why do we not know?)

Basic economics

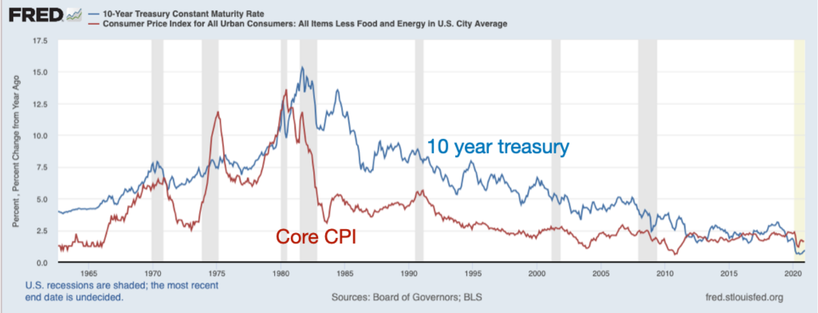


$$r = \delta + \gamma(g - n) = \theta f'(k)$$

real r = impatience + (1-2) x (growth - pop.) = marg. product of capital

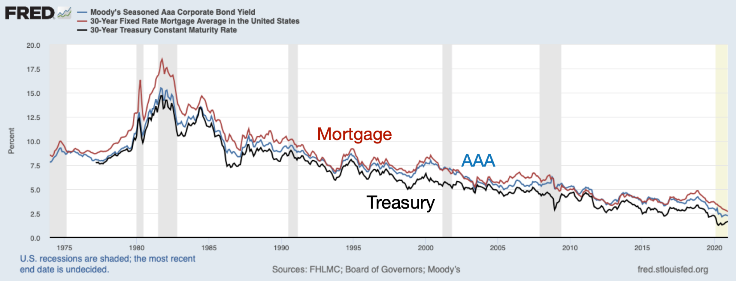
- Less g-n? Less capital intensive (services)? Fewer ideas (end of growth)? More tax and regulation (θ)?
- Should we fear return to more g? (No. Surpluses easier.)
- Gov gets g, private uses $g - n$. Less n is bad for government!
- Savings from bulge of middle aged boomers? Will reverse.

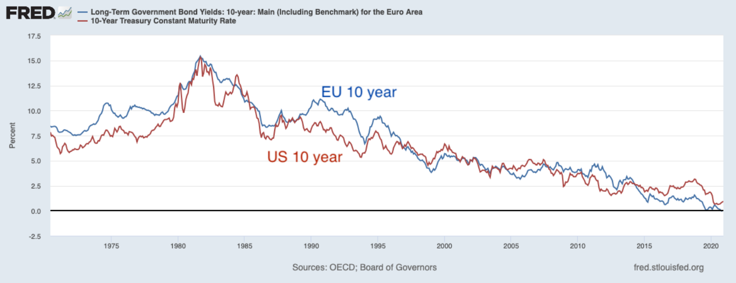
Inflation



- Quiz: What happened in 1980?
- 1980-1990 fear inflation return?
- Post 1980 negative beta (rates and inflation decline in recession, exchange rate rises). Will that last/reverse?
- (Though 1980 also inaugurated strong growth.)

Liquidity, privilege?





Does it work?

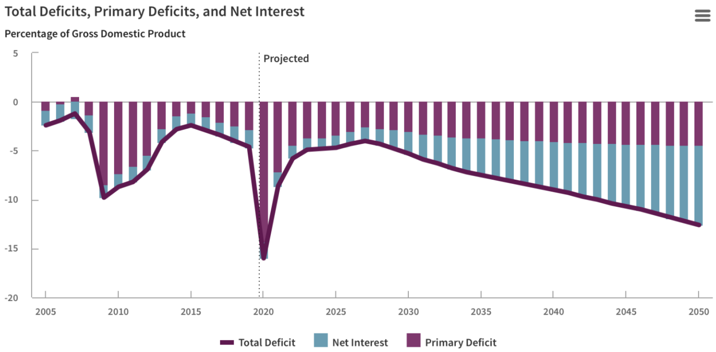
- The hope:

$$\frac{d}{dt}\left(\frac{b_t}{y_t}\right) = (r_t - g_t)\frac{b_t}{y_t} - \frac{s_t}{y_t}.$$

  $r < g$: Raise debt (with $s < 0$) then roll over ($s = 0$), b/y gently declines? "Fiscal expansion" with "no fiscal cost?"
- Puzzles
  - Washington understands logic better than economists!
  - If US does not have to repay debt, why should citizens?
  - Never work, pay taxes? Checks, not infrastructure. Limit?
  - Theory wall between $r < g$ manna and $r > g$ austerity? 0.01%?
- Obviously not. Conventional discussion:
  1. $r(b/y)$ rises. Crowd out? Liquidity premium? Borrow until $r = g$?
  2. r rises. Lose $\beta < 0$, reserve status, demographics, g changes.
  3. *There is a B/y limit*. (Really a B/y + plan to pay it off limit.) 50-100 years of large b/y threatens doom loop. Dry powder. Another crisis, WWII *starting* at $b/y = 200\%$?
- Today point 1: $r < g$ *is irrelevant to US fiscal policy issues.*

Perpetual deficits with steady b/y?

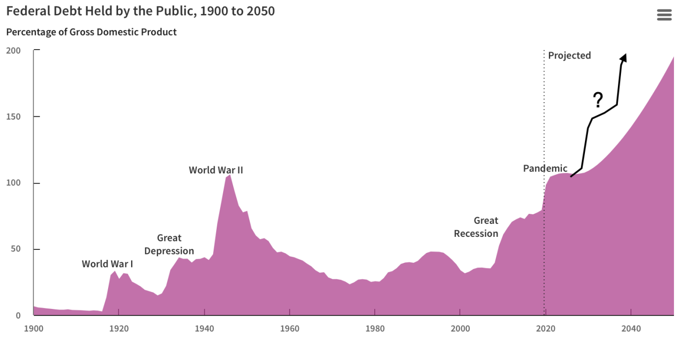
The whole $r - g$ debate is irrelevant to current US fiscal policy issues.



- Scenario 1: $r < g$ by 1% and $b/y = 100\%$ allow $s/y = -1\%$.
- Not $s/y = -5\%$ in good times, $s/y = -25\%$ in 1/10 year crisis, and then entitlements, *and then* "one-time" expansion!
- 500% b/y forever to finance 5% s/y?

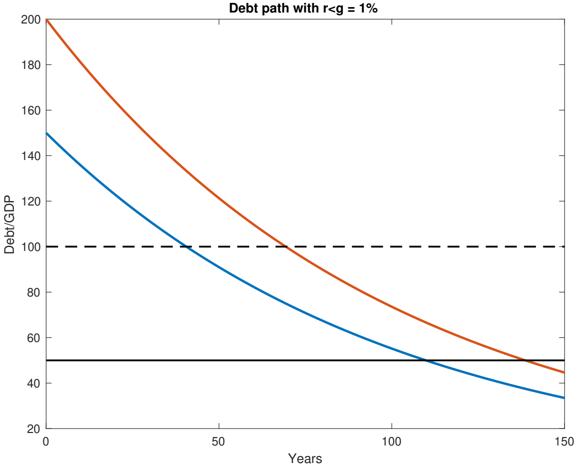
One time expansion and grow out?

The whole $r - g$ debate is irrelevant to current US fiscal policy issues.



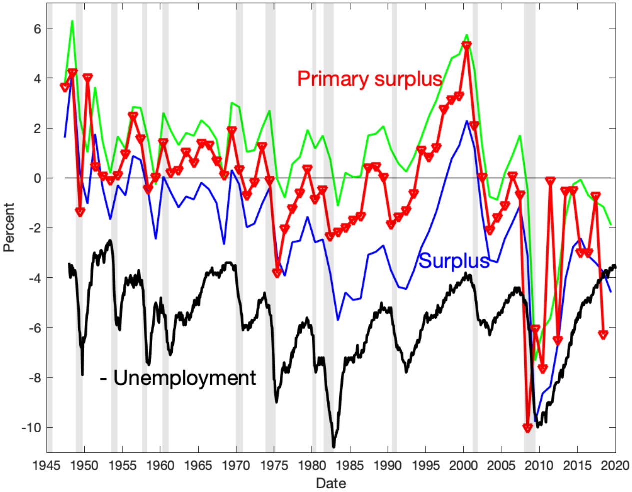
- Scenario 2: "One-time b/y expansion" *and then* $s = 0$.
- $s = 0$ would be an austerity / conservative dream!
- $r < g$ by 1% does not allow perpetually growing b/y!

Reversion takes a long time



- Even if we could get to $s = 0$, $r < g$ by 1% takes a long time.

WWII exit was not painless



- WWII exit featured steady primary surpluses. Plus 1947-1952, 1970s, inflation, capital controls, financial repression. Worse in UK

R, g and present values

Summary: $r < g$ in perfect foresight modeling

- $r < g \approx 1\%$, $b/y \approx 100\%$ shifts the *average* surplus to a slight perpetual deficit $s/y \approx -1\%$, while it lasts.
- Any substantial *variation* in deficits about that average must be met by a substantial period of above average surpluses, to bring back debt to GDP in a reasonable time. Like seigniorage.
- A quantitative question. $r < g$ of 10% would be different.
- *Point 1: $r < g$ debate is irrelevant to current US fiscal policy.*

Point 2: Which r? Uncertainty, liquidity fundamentally change.

- Liquidity, uncertainty: Many r to choose from.
- $r =$ rate of return on government debt may $< g$ average growth, but present values converge and debts must be paid.
- Do *not* measure r from a world with liquidity and uncertainty, and use it in perfect foresight modeling!
- Discontinuity at $r = g$? (Manna vs. austerity)

The hidden put option example

$$u'(c) = c^{-\gamma}; \log(c_{t+1}/c_t) \sim \text{i.i.d. Normal}$$
$$\gamma = 2, \delta = 0, \sigma = 0.15$$
$$g = E[\log(c_{t+1}/c_t)] = 3\%$$
$$r^f = exp(\delta + \gamma g - \gamma^2\sigma^2/2) = 1.5\%$$

- Borrow b_0, roll it over forever at $r^f = r$ on gov't debt.
- Apply r^f, g to a risk free model: free lunch.

$$\frac{b_t}{y_t} = \left(\frac{1+r^f}{1+g}\right)^t \to 0$$

- Satisfies the false certainty TV condition

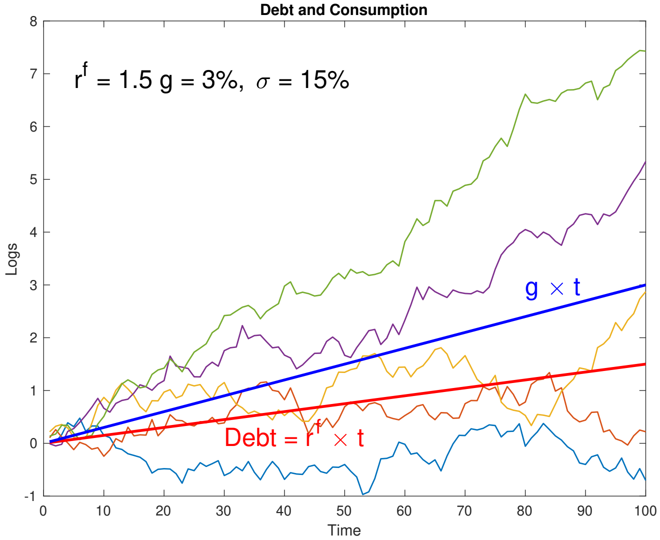
$$\beta^T \left(\frac{c_T}{c_0}\right)^{-\gamma} b_T = e^{-\delta T} e^{-\gamma g T} b_0 e^{(\delta+\gamma g - \gamma^2\sigma^2/2)T} = e^{-\gamma^2\sigma^2 T/2} b_0 \to 0$$

- But b_0, roll forever, violates the real TV condition

$$E\left[\beta^T \left(\frac{c_T}{c_0}\right)^{-\gamma} b_T\right] = \frac{1}{(1+r^f)^T} b_0 (1+r^f)^T = b_0 \neq 0$$

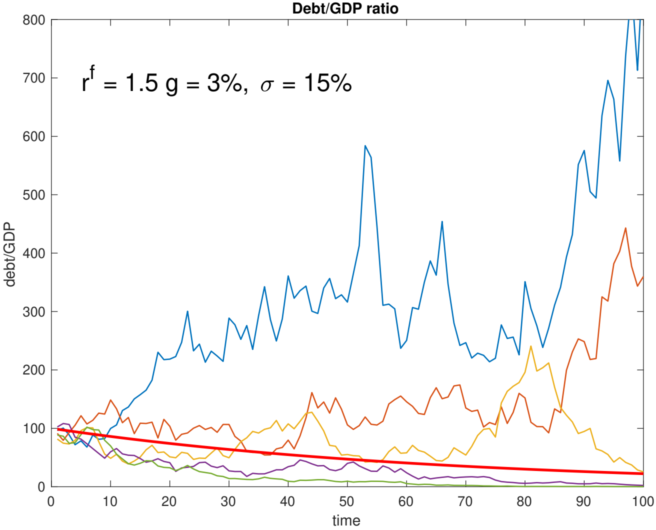
$b_0 > 0$ requires surpluses, especially in high u' states.

Ex-post consumption and debt paths



- Borrow 1, roll over at $r^f < g$ forever. Certainty: g beats r^f. Uncertainty: there are sample paths of low consumption (high u').

Ex post debt to GDP ratio



- Certainty: b/y declines. Uncertainty: Some low growth sample paths lead to huge b/y, fiscal adjustment in (unlikely) low c, high u' state. $r < g$ *strategy is like writing out of the money puts.*

Liquidity value of government debt

- Example: All debt is money. G debt return $r = -\pi < g$. No magic.
- Steady state can finance small deficit.

$$(\pi + g)\frac{M}{Py} = -\frac{s}{y}$$

but big deficits need to be repaid by later surpluses.

- PV? Discount with r^f, i.e. with $e^{-\delta t}u'(c_t)$, suppose $r^f > g$:

$$\frac{M_t}{P_t y_t} = \int_{\tau=t}^{T} e^{-\left(r^f - g\right)(\tau - t)} \left(\frac{s_\tau}{y_\tau} + i_\tau \frac{M_\tau}{P_\tau y_\tau}\right) d\tau + e^{-(r^f - g)(T-t)} \frac{M_T}{P_T y_T}$$

Debt = PV of surpluses, including seignorage. Terminal value converges. Can fund $s < 0$. Big $s < 0$ need to be repaid by $s > 0$.

- Discount with gov't debt return $r = -\pi < g$:

$$\frac{M_t}{P_t y_t} = \int_{\tau=t}^{T} e^{-(-\pi - g)(\tau - t)} \left(\frac{s_\tau}{y_\tau}\right) d\tau + e^{-(-\pi - g)(T-t)} \frac{M_T}{P_T y_T}.$$

Explosive "bubble," negative PV.

- Same $M/(Py) < \infty$. Which is *useful*? "Mine bubble"? See s limit?

The technical problem

- You can discount one-period payoffs with ex-post returns.

$$1 = E_t\left(\frac{\beta u'(c_{t+1})}{u'(c_t)} R_{t+1}\right) = E_t\left(R_{t+1}^{-1} R_{t+1}\right).$$

- You cannot always discount infinite payoffs with ex-post returns.

$$p_t = E_t \sum_{j=1}^{T} \frac{\beta^j u'(c_{t+j})}{u'(c_t)} d_{t+j} + E_t \frac{\beta^j u'(c_{t+j})}{u'(c_t)} p_{t+T}$$

each term converges, yet

$$p_t = E_t \sum_{j=1}^{T} \prod_{k=1}^{j} \frac{1}{R_{t+k}} d_{t+j} + E_t \prod_{k=1}^{T} \frac{1}{R_{t+k}} p_{t+T}$$

terms *can* explode in opposite directions.

$$\frac{p_t}{d_t} = E_t \sum_{j=1}^{T} \prod_{k=1}^{j} \frac{1}{R_{t+k}} \frac{d_{t+k}}{d_{t+k-1}} + E_t \prod_{k=1}^{T} \frac{1}{R_{t+k}} \frac{d_{t+k}}{d_{t+k-1}} \frac{p_{t+T}}{d_{t+T}}$$

Roughly, $r < g$ is the condition for $r < g$ to fail!

Bohn's (1995) example – uncertainty

- $\frac{c_{t+1}}{c_t} \sim$ iid. $\frac{1}{1+r^f} = E\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}\right]$. $r^f < g$ is possible.
- Government keeps a constant debt/GDP. Borrows c_t, repays $(1+r^f)c_t$ at time $t+1$. $b_t = c_t$. See as present value?
- $s_t = (1+r^f)c_{t-1} - c_t$. Discounting with marginal utility,

$$b_t = E_t \sum_{j=1}^{T} \beta^j \left(\frac{c_{t+j}}{c_t}\right)^{-\gamma} s_{t+j} + E_t \beta^T \left(\frac{c_{t+T}}{c_t}\right)^{-\gamma} c_{t+T}$$

$$= E_t \sum_{j=1}^{T} \beta^j \left(\frac{c_{t+j}}{c_t}\right)^{-\gamma} \left[(1+r^f)c_{t+j-1} - c_{t+j}\right] + E_t \beta^T \left(\frac{c_{t+T}}{c_t}\right)^{-\gamma} c_{t+T}$$

$$b_t = \left\{c_t - E_t\left[\beta^T \left(\frac{c_{t+T}}{c_t}\right)^{-\gamma} c_{t+T}\right]\right\} + E_t\left[\beta^T \left(\frac{c_{t+T}}{c_t}\right)^{-\gamma} c_{t+T}\right]$$

Bohn's (1995) example

- Discounting with gov't bond return $= r^f$,

$$b_t = \sum_{j=1}^{T} \left(\prod_{k=1}^{j} \frac{1}{R_{t+k}} \right) s_{t+j} + \left(\prod_{k=1}^{T} \frac{1}{R_{t+k}} \right) c_{t+T} =$$

$$= \sum_{j=1}^{T} \frac{(1+r^f)c_{t+j-1} - c_{t+j}}{(1+r^f)^j} + \frac{1}{(1+r^f)^T} c_{t+T}$$

$$b_t = \left[c_t - \frac{c_{t+T}}{(1+r^f)^T} \right] + \frac{c_{t+T}}{(1+r^f)^T}.$$

Taking expected value,

$$b_t = c_t \left[1 - \frac{(1+g)^T}{(1+r^f)^T} \right] + c_t \frac{(1+g)^T}{(1+r^f)^T}$$

Bohn's (1995) example

- Discounting with marginal utility $c_t^{-\gamma}$, terms converge

$$b_t = \left\{ c_t - E_t\left[\beta^T \left(\frac{c_{t+T}}{c_t}\right)^{-\gamma} c_{t+T} \right] \right\} + E_t\left[\beta^T \left(\frac{c_{t+T}}{c_t}\right)^{-\gamma} c_{t+T} \right].$$

- Discounting with government bond return $= r^f$, offsetting explosions

$$b_t = c_t \left[1 - \frac{(1+g)^T}{(1+r^f)^T} \right] + c_t \frac{(1+g)^T}{(1+r^f)^T}.$$

$$b_t = \left\{ c_t - E_t\left[\beta^T \left(\frac{c_{t+T}}{c_t}\right)^{-\gamma} \right] E_t c_{t+T} \right\} + E_t\left[\beta^T \left(\frac{c_{t+T}}{c_t}\right)^{-\gamma} \right] E_t c_{t+T}.$$

- Both right. Which is more useful?
- At least be careful about offsetting infinite limits! Can miss $b/y = 1$, not ∞, that deficits *are* repaid in PV terms. No "mineable bubbles"!

Discontinuity at $r = g$?

$r = g$ divides bond vigilantes from garden of Eden? Divides $\int$ forward to present value, repay, vs. $\int$ back to debt just accumulates past?

Look at flows.

- $r - g = +0.01\%$ with $b/y = 100\%$ means $s/y = 0.01\% =$ \$2 billion.
- $r - g = -0.01\%$ means $s/y = -0.01\% =$ -\$2 billion.
- *This* transition is clearly continuous.

Look at growing out of "one time" expansion

- $r - g = -0.01\%$, means b/y=150% resolves with $s = 0$ back to $b/y = 50\%$ in $11,000$ years.
- $r - g = +0.01\%$ means b/y=150% grows to 450% in $11,000$ years, on the way to ∞.
- "Wealth effect" in transversality condition, is likely the same.

Lessons

- Economic meaning of solving integrals forward vs. backward should be continuous.
- Economically sensible reading: Small $r < g$ is not discontinuously different from small $r > g$.

Bottom line

Lessons

- $r < g \approx 1\%$ is fun but irrelevant for US fiscal problems.
- $r < g \approx 1\%$ allows steady small deficits like seignorage. Larger deficits need to be repaid with subsequent surpluses.
- Grow out of debt opportunity is like writing out of the money put options and calling it arbitrage.
- With liquidity or uncertainty, discounting with ex post return *can* lead to terminal condition and PV that explode in opposite directions, while discounting with marginal utility is well behaved.
- If you do it, be careful. Discounting with marginal utility is safer.
- Do not pluck r measures from the world and use risk free models for quantitative questions.