\[ r < g \]

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Debt sustainability: The most important macro question.

The hope:

\[ \frac{d}{dt} \left( \frac{b_t}{y_t} \right) = (r_t - g_t) \frac{b_t}{y_t} - \frac{s_t}{y_t}. \]

\( r < g \): Raise debt \((s < 0)\) then roll over \((s = 0)\), \(b/y\) declines?

“Fiscal expansion” with “no fiscal cost?” (Blanchard, Yellen v. 1)

Why is \( r < g \)? Will it last? (“Low interest rates & gov’t debt.”)

Does \( r < g \) work? (“\( r < g \)”.)

- \( r < g \) is fun, but irrelevant to US fiscal policy issues. Like seignorage, it is a small benefit to government finance. Large deficits need to be repaid with surpluses.
- Can see \( E(r) < E(g) \) yet debt = PV of surpluses. Grow out of debt strategy is like writing out of the money puts, calling it arbitrage. Don’t calibrate certainty models with uncertain data.
Steady trend since 1980.

Savings glut, fx reserves, QE, zero bound, liquidity, etc. Icing. Cake?

Basic economics? (Why do we not know?)
Basic economics

\[ r = \delta + \gamma (g - n) = \theta f'(k) \]

real \( r \) = impatience + (1-2) x (growth - pop.) = marg. product of capital

- Less \( g-n \)? Less capital intensive (services)? Fewer ideas (end of growth)? More tax and regulation (\( \theta \))? 
- Should we fear return to more \( g \)? (No. Surpluses easier.) 
- Gov gets \( g \), private uses \( g - n \). Less \( n \) is bad for government! 
- Savings from bulge of middle aged boomers? Will reverse.
Inflation

Quiz: What happened in 1980?
1980-1990 fear inflation return?
Post 1980 negative beta (rates and inflation decline in recession, exchange rate rises). Will that last/reverse?
(Though 1980 also inaugurated strong growth.)
Liquidity, privilege?
Does it work?

The hope:

\[
\frac{d}{dt} \left( \frac{b_t}{y_t} \right) = (r_t - g_t) \frac{b_t}{y_t} - \frac{s_t}{y_t}.
\]

\(r < g\): Raise debt (with \(s < 0\)) then roll over \((s = 0)\), \(b/y\) gently declines? “Fiscal expansion” with “no fiscal cost?”

Puzzles

Washington understands logic better than economists!
If US does not have to repay debt, why should citizens?
Never work, pay taxes? Checks, not infrastructure. Limit?
Theory wall between \(r < g\) manna and \(r > g\) austerity? 0.01%?

Obviously not. Conventional discussion:

1. \(r(b/y)\) rises. Crowd out? Liquidity premium? Borrow until \(r = g\)?
2. \(r\) rises. Lose \(\beta < 0\), reserve status, demographics, \(g\) changes.
3. There is a \(B/y\) limit. (Really a \(B/y + \) plan to pay it off limit.) 50-100 years of large \(b/y\) threatens doom loop. Dry powder. Another crisis, WWII starting at \(b/y = 200\%\)?

Today point 1: \(r < g\) is irrelevant to US fiscal policy issues.
Perpetual deficits with steady $b/y$?

The whole $r - g$ debate is irrelevant to current US fiscal policy issues.

Scenario 1: $r < g$ by 1% and $b/y = 100\%$ allow $s/y = -1\%$.

Not $s/y = -5\%$ in good times, $s/y = -25\%$ in 1/10 year crisis, and then entitlements, and then “one-time” expansion!

500\% $b/y$ forever to finance 5\% $s/y$?
One time expansion and grow out?

The whole $r - g$ debate is irrelevant to current US fiscal policy issues.

Scenario 2: “One-time $b/y$ expansion” and then $s = 0$.

$s = 0$ would be an austerity / conservative dream!

$r < g$ by 1% does not allow perpetually growing $b/y$!
Even if we could get to $s = 0$, $r < g$ by 1% takes a long time.
WWII exit was not painless

R, g and present values

Summary: \( r < g \) in perfect foresight modeling

- \( r < g \approx 1\%, \ b/y \approx 100\% \) shifts the *average* surplus to a slight perpetual deficit \( s/y \approx -1\% \), while it lasts.
- Any substantial *variation* in deficits about that average must be met by a substantial period of above average surpluses, to bring back debt to GDP in a reasonable time. Like seigniorage.
- A quantitative question. \( r < g \) of 10% would be different.
- *Point 1:* \( r < g \) debate is irrelevant to current US fiscal policy.

Point 2: Which \( r \)? Uncertainty, liquidity fundamentally change.

- Liquidity, uncertainty: Many \( r \) to choose from.
- \( r = \text{rate of return on government debt} \leq g \text{ average growth, but present values converge and debts must be paid.} \)
- Do *not* measure \( r \) from a world with liquidity and uncertainty, and use it in perfect foresight modeling!
- Discontinuity at \( r = g \)? (Manna vs. austerity)
The hidden put option example

\[ u'(c) = c^{-\gamma}; \log(c_{t+1}/c_t) \sim \text{i.i.d. Normal} \]

\[ \gamma = 2, \delta = 0, \sigma = 0.15 \]

\[ g = E[\log(c_{t+1}/c_t)] = 3\% \]

\[ r^f = \exp(\delta + \gamma g - \gamma^2 \sigma^2 / 2) = 1.5\% \]

► Borrow \( b_0 \), roll it over forever at \( r^f = r \) on gov’t debt.
► Apply \( r^f, g \) to a risk free model: free lunch.

\[ \frac{b_t}{y_t} = \left( \frac{1 + r^f}{1 + g} \right)^t \rightarrow 0 \]

► Satisfies the false certainty TV condition

\[ \beta^T \left( \frac{c_T}{c_0} \right)^{-\gamma} b_T = e^{-\delta T} e^{-\gamma g T} b_0 e^{(\delta + \gamma g - \gamma^2 \sigma^2 / 2) T} = e^{-\gamma^2 \sigma^2 T / 2} b_0 \rightarrow 0 \]

► But \( b_0 \), roll forever, violates the real TV condition

\[ E \left[ \beta^T \left( \frac{c_T}{c_0} \right)^{-\gamma} b_T \right] = \frac{1}{(1 + r^f)^T} b_0 (1 + r^f)^T = b_0 \neq 0 \]

\( b_0 > 0 \) requires surpluses, especially in high \( u' \) states.
Ex-post consumption and debt paths

\[ r^f = 1.5 \ g = 3\%, \ \sigma = 15\% \]

- Borrow 1, roll over at \( r^f < g \) forever. Certainty: \( g \) beats \( r^f \).
- Uncertainty: there are sample paths of low consumption (high \( u' \)).
Certainty: $b/y$ declines. Uncertainty: Some low growth sample paths lead to huge $b/y$, fiscal adjustment in (unlikely) low $c$, high $u'$ state. $r < g$ strategy is like writing out of the money puts.
Liquidity value of government debt

Example: All debt is money. G debt return \( r = -\pi < g \). No magic.

Steady state can finance small deficit.

\[
(\pi + g) \frac{M}{Py} = -s \frac{1}{y}
\]

but big deficits need to be repaid by later surpluses.

PV? Discount with \( r^f \), i.e. with \( e^{-\delta t} u'(c_t) \), suppose \( r^f > g \):

\[
\frac{M_t}{P_t y_t} = \int_{\tau=t}^{T} e^{-(r^f-g)(\tau-t)} \left( \frac{s_\tau}{y_\tau} + i_\tau \frac{M_\tau}{P_\tau y_\tau} \right) d\tau + e^{-(r^f-g)(T-t)} \frac{M_T}{P_T y_T}
\]

Debt = PV of surpluses, including seigniorage. Terminal value converges. Can fund \( s < 0 \). Big \( s < 0 \) need to be repaid by \( s > 0 \).

Discount with gov’t debt return \( r = -\pi < g \):

\[
\frac{M_t}{P_t y_t} = \int_{\tau=t}^{T} e^{-(\pi-g)(\tau-t)} \left( \frac{s_\tau}{y_\tau} \right) d\tau + e^{-(\pi-g)(T-t)} \frac{M_T}{P_T y_T}.
\]

Explosive “bubble,” negative PV.

Same \( M/(Py) < \infty \). Which is useful? “Mine bubble”? See \( s \) limit?
The technical problem

- You can discount one-period payoffs with ex-post returns.

\[ 1 = E_t \left( \frac{\beta u'(c_{t+1})}{u'(c_t)} R_{t+1} \right) = E_t \left( R_{t+1}^{-1} R_{t+1} \right). \]

- You cannot always discount infinite payoffs with ex-post returns.

\[ p_t = E_t \sum_{j=1}^{T} \frac{\beta^j u'(c_{t+j})}{u'(c_t)} d_{t+j} + E_t \frac{\beta^j u'(c_{t+j})}{u'(c_t)} p_{t+T} \]

Each term converges, yet

\[ p_t = E_t \sum_{j=1}^{T} \prod_{k=1}^{j} \frac{1}{R_{t+k}} d_{t+j} + E_t \prod_{k=1}^{T} \frac{1}{R_{t+k}} p_{t+T} \]

terms can explode in opposite directions.

\[ \frac{p_t}{d_t} = E_t \sum_{j=1}^{T} \prod_{k=1}^{j} \frac{1}{R_{t+k}} d_{t+k} + E_t \prod_{k=1}^{T} \frac{1}{R_{t+k}} \frac{d_{t+k}}{d_{t+k-1}} p_{t+T} \]

Roughly, \( r < g \) is the condition for \( r < g \) to fail!
Bohn’s (1995) example – uncertainty

- \( \frac{c_{t+1}}{c_t} \sim \text{iid.} \frac{1}{1+r_f} = E \left[ \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma} \right] \). \( r_f < g \) is possible.

- Government keeps a constant debt/GDP. Borrows \( c_t \), repays \((1 + r^f) c_t \) at time \( t + 1 \). \( b_t = c_t \). See as present value?

- \( s_t = (1 + r^f) c_{t-1} - c_t \). Discounting with marginal utility,

\[
b_t = E_t \sum_{j=1}^{T} \beta^j \left( \frac{c_{t+j}}{c_t} \right)^{-\gamma} s_{t+j} + E_t \beta^T \left( \frac{c_{t+T}}{c_t} \right)^{-\gamma} c_{t+T} \]

\[
= E_t \sum_{j=1}^{T} \beta^j \left( \frac{c_{t+j}}{c_t} \right)^{-\gamma} [(1 + r^f) c_{t+j-1} - c_{t+j}] + E_t \beta^T \left( \frac{c_{t+T}}{c_t} \right)^{-\gamma} c_{t+T} \]

\[
b_t = \left\{ c_t - E_t \left[ \beta^T \left( \frac{c_{t+T}}{c_t} \right)^{-\gamma} c_{t+T} \right] \right\} + E_t \left[ \beta^T \left( \frac{c_{t+T}}{c_t} \right)^{-\gamma} c_{t+T} \right] \]
Bohn’s (1995) example

Discounting with gov’t bond return \( = r^f \),

\[
b_t = \sum_{j=1}^{T} \left( \prod_{k=1}^{j} \frac{1}{R_{t+k}} \right) s_{t+j} + \left( \prod_{k=1}^{T} \frac{1}{R_{t+k}} \right) c_{t+T} = \\
= \sum_{j=1}^{T} \frac{(1 + r^f) c_{t+j-1} - c_{t+j}}{(1 + r^f)^j} + \frac{1}{(1 + r^f)^T} c_{t+T} \\

b_t = \left[ c_t - \frac{c_{t+T}}{(1 + r^f)^T} \right] + \frac{c_{t+T}}{(1 + r^f)^T}.
\]

Taking expected value,

\[
b_t = c_t \left[ 1 - \frac{(1 + g)^T}{(1 + r^f)^T} \right] + c_t \frac{(1 + g)^T}{(1 + r^f)^T}
\]
Bohn’s (1995) example

- Discounting with marginal utility $c_t^{-\gamma}$, terms converge

$$b_t = \left\{ c_t - E_t \left[ \beta^T \left( \frac{c_{t+T}}{c_t} \right)^{-\gamma} c_{t+T} \right] \right\} + E_t \left[ \beta^T \left( \frac{c_{t+T}}{c_t} \right)^{-\gamma} c_{t+T} \right].$$

- Discounting with government bond return = $r^f$, offsetting explosions

$$b_t = c_t \left[ 1 - \frac{(1 + g)^T}{(1 + r^f)^T} \right] + c_t \frac{(1 + g)^T}{(1 + r^f)^T}.$$ 

$$b_t = \left\{ c_t - E_t \left[ \beta^T \left( \frac{c_{t+T}}{c_t} \right)^{-\gamma} E_t c_{t+T} \right] \right\} + E_t \left[ \beta^T \left( \frac{c_{t+T}}{c_t} \right)^{-\gamma} \right] E_t c_{t+T}.$$ 

- Both right. Which is more useful?

- At least be careful about offsetting infinite limits! Can miss $b/y = 1$, not $\infty$, that deficits are repaid in PV terms. No “mineable bubbles”!
Discontinuity at \( r = g \)?

\( r = g \) divides bond vigilantes from garden of Eden? Divides \( \int \) forward to present value, repay, vs. \( \int \) back to debt just accumulates past?

Look at flows.

- \( r - g = +0.01\% \) with \( b/y = 100\% \) means \( s/y = 0.01\% = \$2 \) billion.
- \( r - g = -0.01\% \) means \( s/y = -0.01\% = -\$2 \) billion.
- \( This \) transition is clearly continuous.

Look at growing out of “one time” expansion

- \( r - g = -0.01\% \), means \( b/y=150\% \) resolves with \( s = 0 \) back to \( b/y = 50\% \) in 11,000 years.
- \( r - g = +0.01\% \) means \( b/y=150\% \) grows to 450\% in 11,000 years, on the way to \( \infty \).
- “Wealth effect” in transversality condition, is likely the same.

Lessons

- Economic meaning of solving integrals forward vs. backward should be continuous.
- Economically sensible reading: Small \( r < g \) is not discontinuously different from small \( r > g \).
Lessons

- $r < g \approx 1\%$ is fun but irrelevant for US fiscal problems.
- $r < g \approx 1\%$ allows steady small deficits like seignorage. Larger deficits need to be repaid with subsequent surpluses.
- Grow out of debt opportunity is like writing out of the money put options and calling it arbitrage.
- With liquidity or uncertainty, discounting with ex post return can lead to terminal condition and PV that explode in opposite directions, while discounting with marginal utility is well behaved.
- If you do it, be careful. Discounting with marginal utility is safer.
- Do not pluck $r$ measures from the world and use risk free models for quantitative questions.